SENIOR SECONDARY COURSE

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PHYSICS

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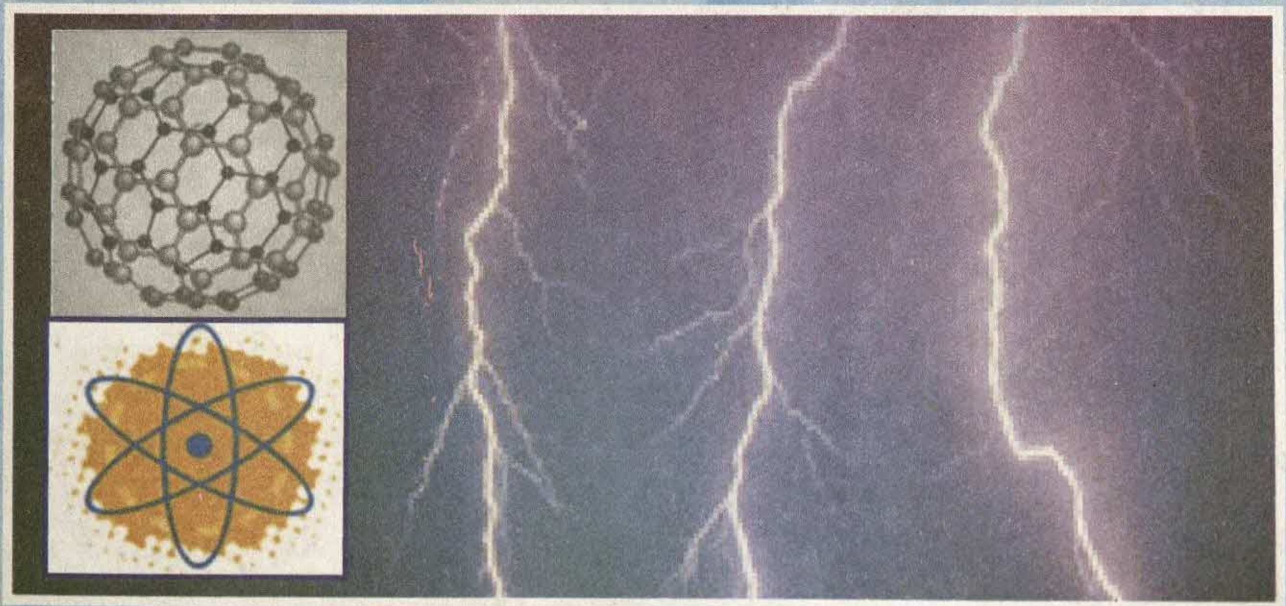

NATIONAL INSTITUTE OF OPEN SCHOOLING

SENIOR SECONDARY COURSE

PHYSICS

1

(CORE MODULES)

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FOREWORD

Dear Learner,

We are very happy that you have joined NIOS and decided to become an Open and Distant Learner.

NIOS has brought out its revised Senior Secondary Course material. This course has been designed in a **modular format** in the sense that the content is divided into different modules. These modules are made up of a number of lessons. The modules are self contained and you can pick up any of the modules first that interests you. However, we would like you to proceed as the modules have been arranged because there are references and cross references to other lessons.

One of the main features of this course is the division of the modules into **core** and **optional** modules. The core modules are compulsory for all learners, but you can choose any **one** of the two optional modules. For example, in Geography, you can pick up either Local Area Planning **or** Tourism Geography; in Business Studies, you can study either Wage Employment in Business **or** Self Employment in Business. Each subject offers two optional modules. The idea is to allow you to study what interests you more even in one particular subject. This is something unique to the NIOS courses of study. You will not find such **choice** elsewhere.

This course is revised on the lines of the guidelines contained in the National Curriculum Framework 2005. We have tried to make the material as **activity based** as possible. We believe that you learn more when you do something with your own hands rather than just passive reading. We have made efforts to keep the **language of the study material simple** to facilitate you to understand the content easily.

The examples that we have chosen and used in the course material are from daily life to enable you to relate easily what is new to what you already know. Through the Self Learning Materials, we are trying to help you to construct your own knowledge so that you learn by understanding and not just by memorising everything.

Another important feature of this learning material is integration of Adolescence Education issues with the learning content. Realising that development of life skills like self awareness, critical thinking, negotiation and communication skills is important, we have used different opportunities to build desirable skills in the lessons.

The study materials developed by NIOS are **self-learning materials**. You are supposed to read and work on your own. Unlike a textbook, you do not need a teacher to tell you what to do.

With our good wishes, start studying, do what you are told to do, attempt all activities, answer the intext questions, check your answers from the answers given, learn each topic well and be a **successful self learner**.

Chairman, NIOS

A Word With You

Dear Learner,

Welcome!

Keen observation, careful experimentation and single minded devotion have helped successive generations of researchers to accumulate vast treasure of knowledge. As you go to higher classes, you will appreciate that the method of sciences is characterised by objectivity, openness to change, innovation, self-correction and dynamism. It is therefore important in these formative years for you to learn science by doing: develop problem solving and experimenting skills to unfold unknown situations. To encourage this, we have included a number of exercises and activities. These can be performed by using readily available materials to get a feel of the physical principles in operation. This will also provide you an opportunity to reflect on how a scientist works.

Physics has always been an exciting subject. But fundamental discoveries in rapid succession in the early half of the 20th century brought in profound changes in our concepts of space, time, matter and energy. Another phenomenon characteristic of the previous century is the reduction in the time gap between a new discovery and its applications from a decade or so to a few years due to close linking of science and technology. Therefore, future development in knowledge society will heavily depend on the availability of well trained scientific human capital endowed with entrepreneurship capabilities. This should provide you enough motivation to study science, do well and participate in the process of sustainable growth and national development.

The organisation of the course is generic. It is divided into eight core modules spread over 29 lessons. Out of two optional modules, which intend to develop professional competencies, you will be required to opt for any one. You will get an opportunity to work in a physics laboratory and make precise measurements using sensitive instruments. This will also give you an opportunity to understand basic physical principles.

As a self-learner, you would be required to demonstrate the ability, capacity and eagerness of Ekalavya. Your confidence in yourself and genuine interest in learning science should help you develop being an independent learner with drive and initiative. Experience shows that interactive learning is more rewarding. So to ensure your active participation in teaching-learning as also to facilitate self-regulation and pacing, we have given questions in the body of each lesson. You must answer these.

In curriculum design an effort has been made to put thematically coherent topics together for braviety and completeness. Although we have strived hard to lucidly explain various concepts, it is possible that you may still find some concepts/topics difficult to comprehend. You are therefore advised to make a note of your difficulties and discuss them in the counselling sessions as well as amongst peers.

You will find some useful information on the life and works of leading physicists/scientists who have contributed to our vast pool of knowledge. It is sincerely hoped that their lives will inspire you as role models to contribute your best!

Our best wishes are with you.

Curriculum Design and
Course Development Team

A Note From the Director

Dear Learner,

Welcome!

The Academic Department at the National Institute of Open Schooling tries to bring you new programmes in accordance with your needs and requirements. After making a comprehensive study, we found that our curriculum is more functional, related to life situations and simple. The task now was to make it more effective and useful for you. We invited leading educationists of the country and under their guidance, we have been able to revise and update the curriculum in the subject of Physics.

At the same time, we have also removed old, outdated information and added new, relevant things and tried to make the learning material attractive and appealing for you.

I hope you will find the new material interesting and exciting with lots of activities to do. Any suggestions for further improvement are welcome.

Let me wish you all a happy and successful future.

HOW TO USE THE STUD[illegible] MATERIAL

Your learning material has been developed by a team of physics experts in open and distance learning. A consistent format has been developed for self-study. The following points will give you an idea on how to make best use of the print material.

***Title** is an advance organisor and conveys an idea about the contents of the lesson. Reflect on it.*

***Introduction** highlights the contents of the lesson and correlates it with your prior knowledge as well as the natural phenomena in operation in our immediate environment. Read it thoroughly.*

***Objectives** relate the contents to your desired achievements after you have learnt the lesson. Remember these.*

***Content** of the lesson has been divided into sections and sub-sections depending on thematic unity of concepts. Read the text carefully and make notes on the side margin of the page. After completing each section, answer intext questions and solve numerical problems yourself. This will give you an opportunity to check your understanding. You should continue reading a section till such time that you gain mastery over it.*

*At some places you will find some text in **italics and bold.** This indicates that it is important. You must learn them.*

***Solved Examples** will help you to understand the concepts and fix your ideas. In fact, problem solving is an integral part of training in physics. Do them yourself and note the main concept being taught through a particular example.*

***Activities** are simple experiments which you can perform at your home or work place using readily available (low cost) materials. These will help you to understand physics by doing. Do them yourself and correlate your findings with your observations.*

***Intext questions** are based on the concepts discussed in every section. Answer these questions yourself in the space given below the question and then check your answers with the model answers given at the end of the lesson. This will help you to judge your progress. If you are not satisfied with the quality and authenticity of your answers, turn the pages back and study the section again.*

***What you have learnt** is essentially summary of the learning points for quick recapitulation. You may like to add more points in this list.*

***Terminal exercises** in the form of short, long and numerical questions will help you to develop a perspective of the subject, if you answer these meticulously. Discuss your responses with your peers or counsellors.*

***Answers to intext questions** : These will help you to know how correctly you have answered the intext questions.*

***Audio:** For understanding difficult or abstract concepts, audio programmes are available on certain content areas. You may listen to these on FM Gyanvani or may buy the CDs from Priced Publication Unit, NIOS*

***Video:** Video programmes on certain elements related to your subject have been made to clarify certain concepts. You may watch these at your study center or may purchase these CDs from Priced Publication Unit, NIOS.*

www *These are few selected websites that you can access for extended learning.*

Studying at a distance requires self-motivation, self-discipline and self-regulation. Therefore you must develop regular study habit. Drawing a daily schedule will help you in this endeavour. You should earmark a well-ventilated and well-lighted space in your home for your study. However, it should not be noisy or distract your concentration from your work.

Overview of the Learning Material

1

Module - I
Motion, Force and Energy

1. Units, Dimensions and Vectors
2. Motion in a straight line
3. Laws of Motion
4. Motion in a Plane
5. Gravitation
6. Work Energy and Power
7. Motion of Rigid Body

Module - II
Mechanics of Solids and Fluids

8. Elastic Properties of Solids
9. Properties of Fluids

Module - III
Thermal Physics

10. Kinetic Theory of Gases
11. Thermodynamics
12. Heat Transfer and Solar Energy

Module - IV
Oscillations and Waves

13. Simple Harmonic Motion
14. Wave Phenomena

2

Module - V
Electricity and Magnetism

15. Electric Charge and Electric Field
16. Electric potential and Capacitors
17. Electric Current
18. Magnetism and Magnetic Effect of Electric Current
19. Electromagnetic induction and Alternating Current

Module - VI
Optics and Optical Instruments

20. Reflection and Refraction of Light
21. Dispersion and Scattering of Light
22. Wave Phenomena of Light
23. Optical Instruments

Module - VII
Atoms and Nuclei

24. Structure of Atom
25. Dual Nature of Radiation and Matter
26. Nuclei and Radioactivity
27. Nuclear Fission and Fusion

Module - VIII
Semiconductor

28. Semiconductors and Semiconductor Devices
29. Applications of Semiconductor Devices

3

Module - IXA
Electronics and Communications

30. Electronics in Daily Life
31. Communication Systems
32. Communication Technique and Devices
33. Communication Media

Module - IXB
Photography and Audio-Videography

30. Photography Camera
31. Film Exposing and Processing
32. Audio-Video Recording
33. Compact Disc for Audio-Video Recording

CONTENTS

Name of the Lesson *Page No.*

Module - I : Motion, Force and Energy

1. Units, Dimensions and Vectors 1
2. Motion in a straight line 26
3. Laws of Motion 47
4. Motion in a Plane 72
5. Gravitation 92
6. Work Energy and Power 120
7. Motion of Rigid Body 151

Modul - II : Mechanics of Solids and Fluids

8. Elastic Properties of Solids 182
9. Properties of Fluids 199

Module - III : Thermal Physics

10. Kinetic Theory of Gases 237
11. Thermodynamics 257
12. Heat Transfer and Solar Energy 273

Module - IV : Oscillations and Waves

13. Simple Harmonic Motion 291
14. Wave Phenomena 308

T.M.A. I, II, III

MODULE - I

MOTION, FORCE AND ENERGY

1. Units, Dimensions and Vectors
2. Motion in a straight line
3. Laws of Motion
4. Motion in a Plane
5. Gravitation
6. Work Energy and Power
7. Motion of Rigid Body

1

UNITS, DIMENSIONS AND VECTORS

In science, particularly in physics, we try to make measurements as precisely as possible. Several times in the history of science, precise measurements have led to new discoveries or important developments. Obviously, every measurement must be expressed in some units. For example, if you measure the length of your room, it is expressed in suitable units. Similarly, if you measure the interval between two events, it is expressed in some other units. The unit of a physical quantity is derived, by expressing it in base units fixed by international agreement. The idea of base units leads us to the concept of **dimensions**, which as we shall see, has important applications in physics.

You will learn that physical quantities can generally be divided in two groups: **scalars** and **vectors**. Scalars have only magnitudes while vectors have both magnitude and direction. The mathematical operations with vectors are somewhat different from those which you have learnt so far and which apply to scalars. The concepts of vectors and scalars help us in understanding physics of different natural phenomena. You will experience it in this course.

Objectives

After studying this lesson, you should be able to:

- *distinguish between the fundamental and derived quantities and give their SI units;*
- *write the dimensions of various physical quantities;*
- *apply dimensional analysis to check the correctness of an equation and determine the dimensional nature of 'unknown' quantities;*
- *differentiate between scalar and vector quantities and give examples of each;*
- *add and subtract two vectors and resolve a vector into its components; and*
- *calculate the product of two vectors.*

1.1 Unit of Measurement

The laws of physics are expressed in terms of physical quantities such as distance, speed, time, force, volume, electric current, etc. For measurement, each physical quantity is assigned a unit. For example, time could be measured in minutes, hours or days. But for the purpose of useful communication among different people, this unit must be compared with a standard unit acceptable to all. As another example, when we say that the distance between Mumbai and Kolkata is nearly 2000 kilometres, we have for comparison a basic unit in mind, called a kilometre. Some other units that you may be familiar with are a kilogram for mass and a second for time. It is essential that all agree on the standard units, so that when we say 100 kilometres, or 10 kilograms, or 10 hours, others understand what we mean by them. In science, international agreement on the basic units is absolutely essential; otherwise scientists in one part of the world would not understand the results of an investigation conducted in another part.

Suppose you undertake an investigation on the solubility of a chemical in water. You weigh the chemical in tolas and measure the volume of water in cupfuls. You communicate the results of your investigation to a scientist friend in Japan. Would your friend understand your results?

It is very unlikely that your friend would understand your results because he/she may not be familiar with tola and the cup used in your measerments, as they are not standard units. Do you now realize the need for agreed standards and units?

Remember that in science, the results of an investigation are considered established only if they can be reproduced by investigations conducted elsewhere under identical conditions.

Measurements in Indian Traditions

Practices of systematic measurement are very old in India. The following quote from Manusmriti amply illustrates this point :

"The king should examine the weights and balance every six months to ensure true measurements and to mark them with royal stamp." – Manusmriti, 8th Chapter, sloka–403.

In **Harappan Era,** signs of systematic use of measurement are found in abundance : the equally wide roads, bricks having dimensions in the ratio 4 : 2 : 1, Ivory scale in Lothal with smallest division of 1.70 mm, Hexahedral weights of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200 and 500 units (1 unit = 20 g)

In Mauriyan Period, the following units of length were prevalent
8 Parmanu = 1 Rajahkan
8 Rajahkan = 1 Liksha
8 Liksha = 1 Yookamadhya
8 Yookamadhya = 1 Yavamadhya
8 Yavamadhya = 1 Angul
8 Angul = 1 Dhanurmushthi

In **Mughal Period**, Shershah and Akbar tried to re-establish uniformity of weights and measures. Akbar introduced gaz of 41 digits for measuring length. For measuring area of land, bigha was the unit. 1 bigha was 60 gaz × 60 gaz.

Units of mass and volume were also well obtained in Ayurveda.

1.1.1 The SI Units

With the need of agreed units in mind, the 14th General Conference on Weights and Measures held in 1971, adopted seven **base or fundamental units**. These units form the SI system. The name SI is abbreviation for **Système International d'Unités** for the International System of units. The system is popularly known as the metric system. The SI units along with their symbols are given in Table 1.1.

Table 1.1 : Base SI Units

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Luminous Intensity	candela	cd
Amount of Substance	mole	mol

The mile, yard and foot as units of length are still used for some purposes in India as well in some other countries. **However, in scientific work we always use SI units.**

As may be noted, the SI system is a metric system. It is quite easy to handle because the smaller and larger units of the base units are always submultiples or multiples of ten. These multiples or submultiples are given special names. These are listed in Table 1.2.

Table 1.2 : Prefixes for powers of ten

Power of ten	Prefix	Symbol	Example	
10^{-18}	atto	a	attometre	(am)
10^{-15}	femto	f	femtometre	(fm)
10^{-12}	pico	p	picofarad	(pF)
10^{-9}	nano	n	nanometre	(nm)
10^{-6}	micro	μ	micron	(μm)
10^{-3}	milli	m	milligram	(mg)
10^{-2}	centi	c	centimetre	(cm)
10^{-1}	deci	d	decimetre	(dm)
10^{1}	deca	da	decagram	(dag)
10^{2}	hecto	h	hectometre	(hm)
10^{3}	kilo	k	kilogram	(kg)
10^{6}	mega	M	megawatt	(MW)
10^{9}	giga	G	gigahertz	(GHz)
10^{12}	tera	T	terahertz	(THz)
10^{15}	peta	P	peta kilogram	(Pkg)
10^{18}	exa	E	exa kilogram	(Ekg)

Table 1.3 : Order of magnitude of some masses

Mass	kg
Electron	10^{-30}
Proton	10^{-27}
Amino acid	10^{-25}
Hemoglobin	10^{-22}
Flu virus	10^{-19}
Giant amoeba	10^{-8}
Raindrop	10^{-6}
Ant	10^{-2}
Human being	10^{2}
Saturn 5 rocket	10^{6}
Pyramid	10^{10}
Earth	10^{24}
Sun	10^{30}
Milky Way galaxy	10^{41}
Universe	10^{52}

Just to get an idea of the masses and sizes of various objects in the universe, see Table 1.3 and 1.4. Similarly, Table 1.5 gives you an idea of the time scales involved in the universe.

1.1.2 Standards of Mass, Length and Time

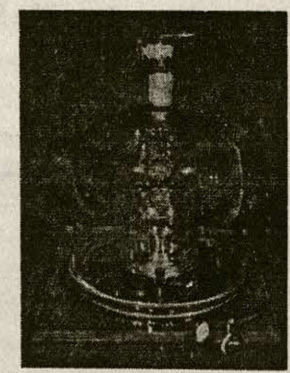


Fig. 1.1 : Prototype of kilogram

Once we have chosen to use the SI system of units, we must decide on the set of standards against which these units will be measured. We define here standards of mass, length and time.

(i) Mass : The SI unit of mass is **kilogram**. The standard kilogram was established in 1887. **It is the mass of a particular cylinder made of platinum-iridium alloy,** which is an unusually stable alloy. The standard is kept in the International Bureau of Weights and Measures in Paris, France. The prototype kilograms made of the same alloy have been distributed to all countries the world over. For India, the national prototype is the kilogram no. 57. This is maintained by the National Physical Laboratory, New Delhi (Fig. 1.1).

(ii) **Length :** The SI unit of length is metre. It is defined in terms of a natural phenomenon: **One metre is defined as the distance travelled by light in vacuum in a time interval of 1/299792458 second.**

This definition of metre is based on the adoption of the speed of light in vacuum as 299792458 ms^{-1}

(iii) **Time :** One second is defined as **the time required for a Cesium - 133 (^{133}Cs) atom to undergo 9192631770 vibrations between two hyperfine levels of its ground state.**

This definition of a second has helped in the development of a device called atomic clock (Fig. 1.2). The cesium clock maintained by the National Physical Laboratory

Table 1.4 : Order of magnitude of some lengths

Length	m
Radius of proton	10^{-15}
Radius of atom	10^{-16}
Radius of virus	10^{-7}
Radius of giant amoeba	10^{-4}
Radius of walnut	10^{-2}
Height of human being	10^{0}
Height of highest mountain	10^{4}
Radius of earth	10^{7}
Radius of sun	10^{9}
Earth-sun distance	10^{11}
Radius of solar system	10^{13}
Distance to nearest star	10^{16}
Radius of Milky Way galaxy	10^{21}
Radius of visible universe	10^{26}

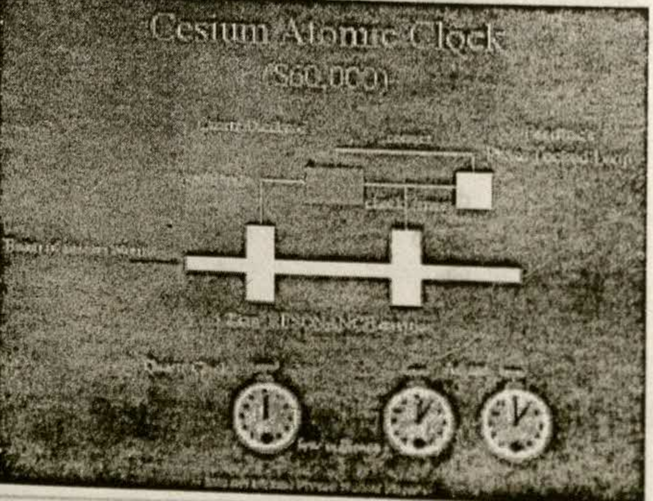


Fig. 1.2 : Atomic Clock

(NPL) in India has an uncertainty of ± 1×10^{-12} s, which corresponds to an accuracy of one picosecond in a time interval of one second.

As of now, clock with an uncertainty of 5 parts in 10^{15} have been developed. This means hat if this clock runs for 10^{15} seconds, it will gain or lose less than 5 seconds. You can convert 10^{15}s to years and get the astonishing result that this clock could run for 6 million years and lose or gain less than a second. This is not all. Researches are being conducted oday to improve upon this accuracy constantly. Ultimately, we expect to have a clock which would run for 10^{18} second before it could gain or lose a second. To give you an idea of this technological achievement, if this clock were started at the time of the birth of the universe, an event called the Big Bang, it would have lost or gained only two seconds till now.

Role of Precise Measurements in New Discoveries

A classical example of the fact that precise measurements may lead to new discoveries are the experiments conducted by Lord Rayleigh to determine density of nitrogen.

In one experiment, he passed the air bubbles through liquid ammonia over red hot copper contained in a tube and measured the density of pure nitrogen so obtained. In another experiment, he passed air directly over red hot copper and measured the density of pure nitrogen. The density of nitrogen obtained in second experiment was found to be 0.1% higher than that obtained in the first case. The experiment suggested that air has some other gas heavier than nitrogen present in it. Later he discovered this gas – Argon, and got Nobel Prize for this discovery.

Another example is the failed experiment of Michelson and Morley. Using Michelson's interferometer, they were expecting a shift of 0.4 fringe width in the interference pattern obtained by the superposition of light waves travelling in the direction of motion of the earth and those travelling in a transverse direction. The instrument was hundred times more sensitive to defect the shift than the expected shift. Thus they were expecting to measure the speed of earth with respect to ether and conclusively prove that ether existed. But when they detected no shift, the world of science entered into long discussions to explain the negative results. This led to the concepts of length contraction, time dilation etc and ultimately to the theory of relativity.

Several discoveries in nuclear physics became possible due to the new technique of spectroscopy which enabled detection, with precision, of the traces of new atoms formed in a reaction.

Table 1.5 : Order of magnitude of some time intervals

Interval	s
Time for light to cross nucleus	10^{-23}
Period of visible light	10^{-15}
Period of microwaves	10^{-10}
Half-life of muon	10^{-6}
Period of highest audible sound	10^{-4}
Period of human heartbeat	10^{0}
Half-life of free neutron	10^{3}
Period of the Earth's rotation (day)	10^{5}
Period of revolution of the Earth (year)	10^{7}
Lifetime of human beings	10^{9}
Half-life of plutonium-239	10^{12}
Lifetime of a mountain range	10^{15}
Age of the Earth	10^{17}
Age of the universe	10^{18}

1.1.3. Derived Units

We have so far defined three basic units for the measurement of mass, length and time. For many quantities, we need units which we get by combining the basic units. These units are called derived units. For example, combination of the units of length and time gives us the derived unit of speed or velocity, m s^{-1}. Another example is the interaction of the unit of length with itself. We get derived units of area and volume as m^2 and m^3, respectively.

Now are would like you to list all the physical quantities that you are familiar with and the units in which they are expressed.

Some derived units have been given special names. Examples of most common of such units are given in Table 1.6.

Table 1.6 : Examples of derived units with special names

Quantity	Name	Symbol	Unit Symbol
Force	newton	N	$kg\ m\ s^{-2}$
Pressure	pascal	Pa	$N\ m^{-2}$
Energy/work	joule	J	N m
Power	watt	W	$J\ s^{-1}$

One of the advantages of the SI system of units is that they form a coherent set in the sense that the product or division of the SI units gives a unit which is also the SI unit of some other derived quantity. For example, product of the SI units of force and length gives directly the SI unit of work, namely, newton-metre (Nm) which has been given a special name joule. **Some care should be exercised in the order in which the units are written.** For example, Nm should be written in this order. If by mistake we write it as mN, it becomes millinewton, which is something entirely different.

Remember that in physics, a quantity must be written with correct units. Otherwise, it is meaningless and, therefore, of no significance.

Example 1.1 : Anand, Rina and Kaif were asked by their teacher to measure the volume of water in a beaker.

Anand wrote : 200; Rina wrote : 200 mL; Kaif wrote : 200 Lm

Which one of these answers is correct?

Solution : The first one has no units. Therefore, we do not know what it means. The third is also not correct because there is no unit like Lm. The second one is the only correct answer. It denotes millilitre.

Note that the mass of a book, for example, can be expressed in kg or g. **You should not use gm for gram because the correct symbol is g and not gm.**

Nomenclature and Symbols

(i) Symbols for units should not contain a full stop and should remain the same in the plural. For example, the length of a pencil should be expressed as 7cm and not 7cm. or 7cms.

(ii) Double prefixes should be avoided when single prefixes are available, e.g., for nanosecond, we should write ns and not mμs; for pico farad we write pF and not μμf.

(iii) When a prefix is placed before the symbol of a unit, the combination of prefix and symbol should be considered as one symbol, which can be raised to a positive

Notes

or a negative power without using brackets, e.g., μs^{-1}, cm^2, mA^2.

$\mu s^{-1} = (10^{-6}s)^{-1}$ (and not $10^{-6}s^{-1}$)

(iv) Do not write cm/s/s for cm s^{-2}. Similarly 1 poise = 1 g $s^{-1}cm^{-1}$ and not 1 g/s/cm.

(v) When writing a unit in full in a sentence, the word should be spelt with the letter in lower case and not capital, e.g., 6 hertz and not 6 Hertz.

(vi) For convenience in reading of large numbers, the digits should be written in groups of three starting from the right but no comma should be used: 1 532; 1 568 320.

Albert Abraham Michelson
(1852-1931)

German-American Physicst, inventor and experimenter devised Michelson's interferometer with the help of which, in association with Morley, he tried to detect the motion of earth with respect to ether but failed. However, the failed experiment stirred the scientific world to reconsider all old theories and led to a new world of physics.

He developed a technique for increasing the resolving power of telescopes by adding external mirrors. Through his stellar interferometer along with 100″ Hookes telescope, he made some precise measurements about stars.

Now, it is time to check your progress. Solve the following questions. In case you have any problem, check answers given at the end of the lesson.

Intext Questions 1.1

1. The mass of the sun is 2×10^{30} kg. The mass of a proton is 2×10^{-27} kg. If the sun was made only of protons, calculate the number of protons in the sun?

   ....

2. Earlier the wavelength of light was expressed in angstroms. One angstrom equals 10^{-8} cm. Now the wavelength is expressed in nanometers. How many angstroms make one nanometre?

   ....

3. A radio station operates at a frequency of 1370 kHz. Express this frequency in GHz.

   ....

4. How many decimetres are there in a decametre? How many MW are there in one GW?

   ....

1.2 Dimensions of Physical Quantities

Most physical quantities you would come across in this course can be expressed in terms of five basic dimensions : mass (M), length (L), time (T), electrical current (I) and temperature (θ). Since all quantities in mechanics can be expressed in terms of mass, length and time, it is sufficient for our present purpose to deal with only these three dimensions. Following examples show how dimensions of the physical quantities are combinations of the powers of M, L and T :

(i) Volume requires 3 measurements in length. So it has 3 dimensions in length (L^3).

(ii) Density is mass divided by volume. Its dimensional formula is ML^{-3}.

(iii) Speed is distance travelled in unit time or length divided by time. Its dimensional formula is LT^{-1}.

(iv) Acceleration is change in velocity per unit time, i.e., length per unit time per unit time. Its dimensionsal formula is LT^{-2}.

(v) Force is mass multiplied by acceleration. Its dimensions are given by the formula MLT^{-2}.

Similar considerations enable us to write dimensions of other physical quantities.

Note that numbers associated with physical quantities have no significance in dimensional considerations. Thus if dimension of x is L, then dimension of $3x$ will also be L.

Write down the dimensions of momentum, which is product of mass and velocity and work which is product of force and displacement.

Remember that dimensions are not the same as the units. For example, speed can be measured in ms^{-1} or kilometre per hour, but its dimensions are always given by length divided by time, or simply LT^{-1}.

Dimensional analysis is the process of checking the dimensions of a quantity, or a combination of quantities. One of the important principles of dimensional analysis is that **each physical quantity on the two side of an equation must have the same dimensions.** Thus if $x = p + q$, then p and q will have the same dimensions as x. This helps us in checking the accuracy of equations, or getting the dimensions of a quantity using an equation. The following examples illustrate the use of dimensional analysis.

Example 1.2 : You know that the kinetic energy of a particle of mass m is $\frac{1}{2}mv^2$ while its potential energy is mgh, where v is the velocity of the particle, h is its height from the ground and g is the acceleration due to gravity. Since the two expressions represent the same physical quantity i.e, energy, their dimensions must be the same. Let us prove this by actually writing the dimensions of the two expressions.

Solution : The dimensions of $\frac{1}{2}mv^2$ are $M.(LT^{-1})^2$, or ML^2T^{-2}. (Remember that the numerical factors have no dimensions.) The dimensions of mgh are $M.LT^{-2}.L$, or ML^2T^{-2}. Clearly, the two expressions are the same and hence represent the same physical quantity.

Let us take another example to find an expression for a physical quantity in terms of other quantities.

Example 1.3 : Experience tells us that the distance covered by a car, say x, starting from rest and having uniform acceleration depends on time t and acceleration a. Let us use dimensional analysis to find expression for the distance covered.

Solution : Suppose x depends on the mth power of t and nth power of a. Then we may write

$$x \propto t^m . a^n$$

Expressing the two sides now in terms of dimensions, we get

$$L^1 \propto T^m (LT^{-2})^n,$$

or,

$$L^1 \propto T^{m-2n} L^n.$$

Comparing the powers of L and T on both sides, you will easily get $n = 1$, and $m = 2$. Hence, we have

$$x \propto t^2 a^1, \text{ or } x \propto at^2.$$

This is as far as we can go with dimensional analysis. It does not help us in getting the numerical factors, since they have no dimensions. To get the numerical factors, we have to get input from experiment or theory. In this particular case, of course, we know that the complete relation is $x = (1/2)at^2$.

Besides numerical factors, other quantities which do not have dimensions are angles and arguments of trigonometric functions (sine, cosine, etc), exponential and logarithmic functions. In $\sin x$, x is said to be the argument of sine function. In e^x, x is said to be the argument of the exponential function.

Now take a pause and attempt the following questions to check your progress.

Intext Questions 1.2

1. Experiments with a simple pendulum show that its time period depends on its length (l) and the acceleration due to gravity (g). Use dimensional analysis to obtain the dependence of the time period on l and g .

....

2. Consider a particle moving in a circular orbit of radius r with velocity υ and acceleration a towards the centre of the orbit. Using dimensional analysis, show that $a \propto \upsilon^2/r$.

....

3. You are given an equation: $m\upsilon = Ft$, where m is mass, υ is speed, F is force and t is time. Check the equation for dimensional correctness.

....

1.3 Vectors and Scalars

1.3.1 Scalar and Vector Quantities

In physics we classify physical quantities in two categories. In one case, we need only to state their magnitude with proper units and that gives their complete description. Take, for example, mass. If we say that the mass of a ball is 50 g, we do not have to add anything to the description of mass. Similarly, the statement that the density of water is 1000 kg m^{-3} is a complete description of density. Such quantities are called scalars. **A scalar quantity has only magnitude; no direction.**

On the other hand, there are quantities which require both magnitude and direction for their complete description. A simple example is velocity. The statement that the velocity of a train is 100 km h^{-1} does not make much sense unless we also tell the direction in which the train is moving. Force is another such quantity. We must specify not only the magnitude of the force but also the direction in which the force is applied. Such quantities are called vectors. **A vector quantity has both magnitude and direction.**

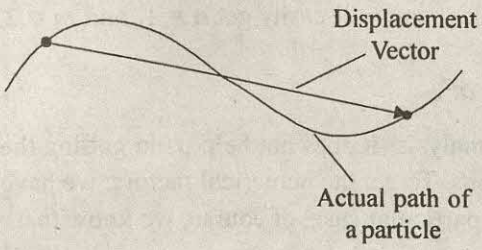


Fig.1.3 : Displacement vector

Some examples of vector quantities which you come across in mechanics are: displacement (Fig. 1.3), acceleration, momentum, angular momentum and torque etc,

What about energy? Is it a scalar or a vector?

To get the answer, think if there is a direction associated with energy. If not, it is a scalar.

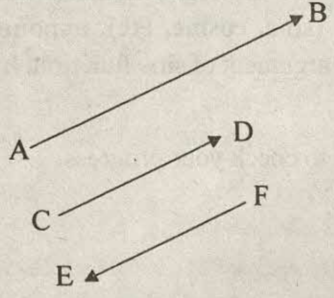


Fig. 1.4 : Directions and magnitudes of vectors

1.3.2 Representation of Vectors

A vector is represented by a line with an arrow indicating its direction. Take vector AB in Fig. 1.4. The length of the line represents its magnitude on some scale. The arrow indicates its direction. Vector CD is a vector in the same direction but its magnitude is smaller. Vector EF is a vector whose magnitude is the same as that of vector CD, but its direction is different. In any vector, the initial point, (point A in **AB**), is called the **tail** of the vector and the final point, (point B in **AB**) with the arrow mark is called its **tip** (or **head**).

A vector is written with an arrow over the letter representing the vector, for example, $\vec{A}$. The magnitude of vector $\vec{A}$ is simply A or $|\vec{A}|$. In print, a vector is indicated by a bold letter as **A**.

Two vectors are said to be equal if their magnitudes are equal and they point in the same direction. This means that all vectors which are parallel to each other have the same magnitude and point in the same direction are equal. Three vectors **A, B** and **C** shown in Fig. 1.5 are equal. We say **A = B = C**. But **D** is not equal to **A**.

A vector (here **D**) which has the same magnitude as A but has opposite direction, is called **negative of A,** or –**A**. Thus, **D** = –**A**.

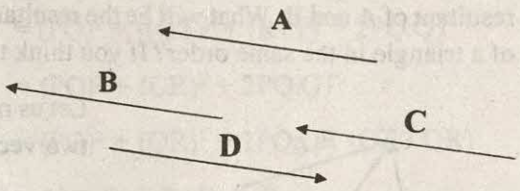


Fig. 1.5 : Three vectors are equal but fourth vector D is not equal.

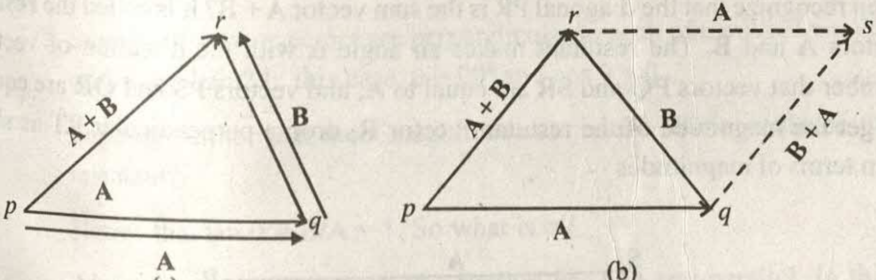
For respresenting a physical vector quantitatively, we have to invariably choose a proportionality scale. For instance, the vector displacement between Delhi and Agra, which is about 300 km, is represented by choosing a scale 100 km = 1 cm, say. Similarly, we can represent a force of 30 N by a vector of length 3cm by choosing a scale 10N = 1cm.

From the above we can say that if we translate a vector parallel to itself, it remains unchanged. This important result is used in addition of vectors. Let us sec how.

1.3.3 Addition of Vectors

You should remember that only **vectors of the same kind can be added**. For example, two forces or two velocities can be added. But a force and a velocity cannot be added.

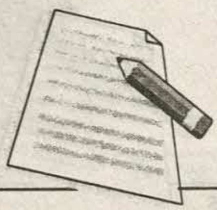
Suppose we wish to add vectors **A** and **B**. First redraw vector **A** [Fig. 1.6 (a)]. For this draw a line (say *pq*) parallel to vector **A.** The length of the line i.e. *pq* should be equal to the magnitude of the vector. Next draw vector **B** such that its tail coincides with the tip of vector **A**. For this, draw a line *qr* from the tip of **A** (i.e., from the point *q*) parallel to the direction of vector **B.** The sum of two vectors then is the vector from the tail of **A** to the tip of **B,** i.e. the resultant will be represented in magnitude and direction by line *pr.* You can now easily prove that **vector addition is commutative. That is, A + B = B + A,** as shown in Fig. 1.6 (b). In Fig. 1.6(b) we observe that *pqr is* a triangle and its two sides *pq* and *qr* respectively represent the vectors **A** and **B** in magnitude and direction, and the third side *pr*, of the triangle represents the resultant vector with its direction from *p* to *r*. This gives us a rule for finding the resultant of two vectors :



(a) (b)

Fig. 1.6 : Addition of vectors A and B

If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, the resultant is represented by the third side of the triangle taken in the opposite order. This is called triangle law of vectors.



The sum of two or more vectors is called the **resultant** vector. In Fig. 1.6(b), **pr** is the resultant of **A** and **B**. What will be the resultant of three forces acting along the three sides of a triangle in the same order? If you think that it is zero, you are right.

Let us now learn to calculate resultant of more than two vectors.

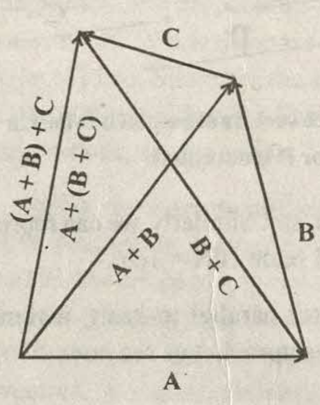


Fig.1.7 : Addition of three vectors in two different orders

The resultant of more than two vectors, say **A**, **B** and **C**, can be found in the same manner as the sum of two vectors. First we obtain the sum of **A** and **B**, and then add the resultant of the two vectors, (**A** + **B**), to **C**. Alternatively, you could add **B** and **C**, and then add **A** to (**B** + **C**) (Fig. 1.7). In both cases you get the same vector. Thus, vector addition is associative. That is, **A** + (**B** + **C**) = (**A** + **B**) + **C**.

If you add more than three vectors, you will discover that **the resultant vector is the vector from the tail of the first vector to the tip of the last vector.**

Many a time, the point of application of vectors is the same. In such situations, it is more convenient to use parallelogram law of vector addition. Let us now learn about it.

1.3.4 Parallelogram Law of Vector Addition

Let **A** and **B** be the two vectors and let θ be the angle between them as shown in Fig. 1.8. To calculate the vector sum, we complete the parallelogram. Here side PQ represents vector **A**, side PS represents **B** and the diagonal PR represents the resultant vector **R**. Can you recognize that the diagonal PR is the sum vector **A** + **B**? It is called the **resultant** of vectors **A** and **B**. The resultant makes an angle α with the direction of vector **A**. Remember that vectors **PQ** and **SR** are equal to **A**, and vectors **PS** and **QR** are equal, to **B**. To get the magnitude of the resultant vector **R**, drop a perpendicular RT as shown. Then in terms of magnitudes

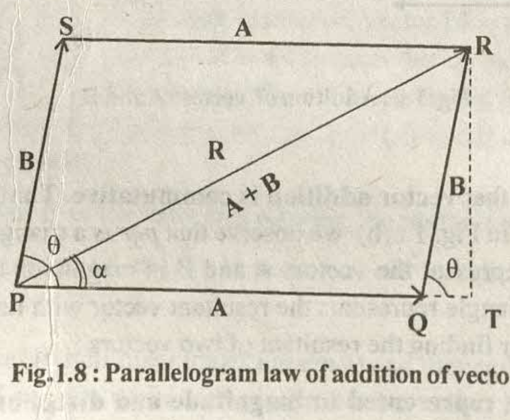


Fig. 1.8 : Parallelogram law of addition of vectors

$$(PR)^2 = (PT)^2 + (RT)^2$$

$$= (PQ + QT)^2 + (RT)^2$$

$$= (PQ)^2 + (QT)^2 + 2PQ.QT + (RT)^2$$

$$= (PQ)^2 + [(QT)^2 + (RT)^2] + 2PQ.QT \qquad (1.1)$$

$$= (PQ)^2 + (QR)^2 + 2PQ.QT$$

$$= (PQ)^2 + (QR)^2 + 2PQ.QR\ (QT / QR)$$

$$R^2 = A^2 + B^2 + 2AB.\cos\theta$$

Therefore, the magnitude of **R** is

$$|R| = \sqrt{A^2 + B^2 + 2AB.\cos\theta} \qquad (1.2)$$

For the direction of the vector **R**, we observe that

$$\tan\alpha = \frac{RT}{PT} = \frac{RT}{PQ + QT} = \frac{B\sin\theta}{A + B\cos\theta} \qquad (1.3)$$

So, the direction of the resultant can be expressed in terms of the angle it makes with base vector.

Special Cases

Now, let us consider as to what would be the resultant of two vectors when they are parallel?

To find answer to this question, note that the angle between the two parallel vectors is zero and the resultant is equal to the sum of their magnitudes and in the direction of these vectors.

Suppose that two vectors are perpendicular to each other. What would be the magnitude of the resultant? In this case, $\theta = 90^\circ$ and $\cos\theta = 0$.

Suppose further that their magnitudes are equal. What would be the direction of the resultant?

Notice that $\tan\alpha = B/A = 1$. So what is α?

Also note that when $\theta = \pi$, the vectors become anti-parallel. In this case $\alpha = 0$. The resultant vector will be along **A** or **B**, depending upon which of these vectors has larger magnitude.

Example 1.4 A cart is being pulled by Ahmed north-ward with a force of magnitude 70 N. Hamid is pulling the same cart in the south-west direction with a force of magnitude 50 N. Calculate the magnitude and direction of the resulting force on the cart.

Solution :

Here, magnitude of first force, say, A = 70 N.

The magnitude of the second force, say, B = 50 N.

Angle θ between the two forces = 135 degrees.

So, the magnitude of the resultant is given by Eqn. (1.2) :

$$R = \sqrt{(70)^2 + (50)^2 + 2 \times 70 \times 50 \times \cos(135)}$$

$$= \sqrt{4900 + 2500 - 7000 \times \sin 45}$$

$$= 49.5 \text{ N}$$

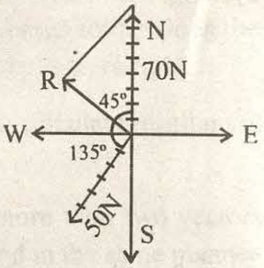


Fig. 1.9: **Resultant of forces inclined at an angle**

The magnitude of **R** = 49.5 N.

The direction is given by Eqn. (1.3):

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \frac{50 \times \sin(135)}{70 + 50\cos(135)} = \frac{50 \times \cos 45}{70 - 50\sin 45} = 1.00$$

Therefore, $\alpha = 45.0°$ (from the tables). Thus **R** makes an angle of 45° with 70 N force. That is, **R** is in North-west direction as shown in Fig. 1.9.

1.3.5 Subtraction of Vectors

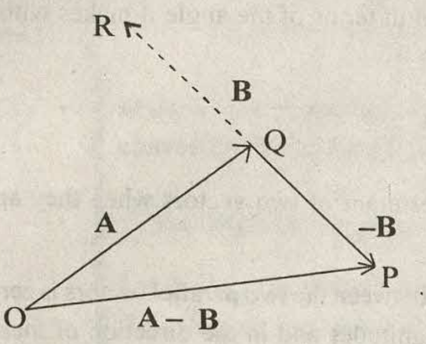
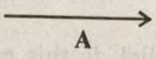
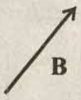


Fig. 1.10 : Subtraction of vector B from vector A

How do we subtract one vector from another? If you recall that the difference of two vectors, **A – B**, is actually equal to **A + (–B)**, then you can adopt the same method as for addition of two vectors. It is explained in Fig. 1.10. Draw vector **–B** from the tip of **A**. Join the tail of **A** with the tip of **–B**. The resulting vector is the difference **(A – B)**.

You may now like to check your progress.

Intext Questions 1.3

Given vectors  and 

1. Make diagrams to show how to find the following vectors:
   (a) **B – A**, (b) **A + 2B**, (c) **A – 2B** and (d) **B – 2A**.

   ....

2. Two vectors **A** and **B** of magnitudes 10 units and 12 units are anti-parallel. Determine **A + B** and **A – B**.

   ....

3. Two vectors **A** and **B** of magnitudes A = 30 units and B = 60 units respectively are inclined to each other at angle of 60 degrees. Find the resultant vector.

   ....

1.4 Multiplication of Vectors

1.4.1 Multiplication of a Vector by a Scalar

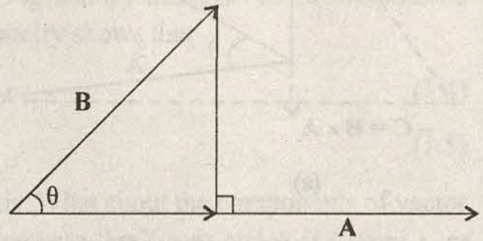


Fig. 1.11: Projection of B on A

If we multiply a vector **A** by a scalar *k*, the product is a vector whose magnitude is the absolute value of *k* times the magnitude of **A**. This means that the magnitude of the resultant vector is *k* |**A**|. The direction of the new vector remains unchanged if *k* is positive. If *k* is negative, the direction of the new vector is opposite to its original direction. For example, vector 3**A** is thrice the magnitude of vector **A**, and it is in the same direction as **A**. But vector –3**A** is in a direction opposite to vector **A**, although its magnitude is thrice that of vector **A**.

1.4.2 Scalar Product of Vectors

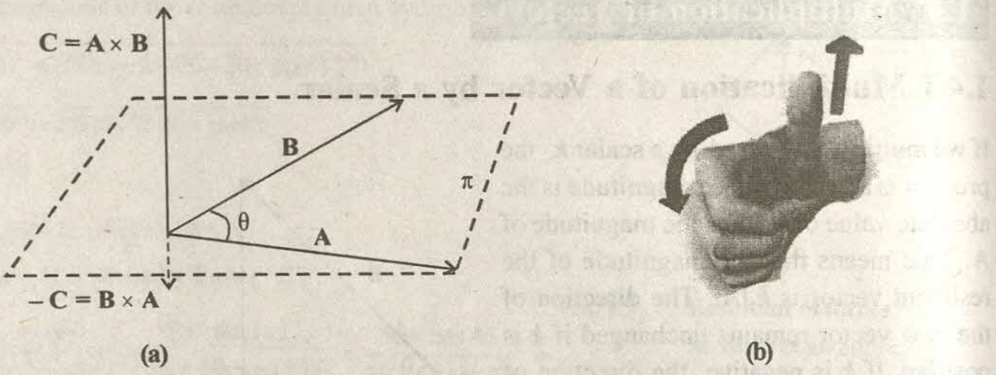
The **scalar product** of two vectors **A** and **B** is written as **A.B** and is equal to AB cosθ, where θ is the angle between the vectors. If you look carefully at Fig. 1.11, you would notice that B cosθ is the projection of vector **B** along vector **A**. Therefore, the scalar product of A and B is the product of magnitude of A with the length of the projection of B along A. Another thing to note is that even if we take the angle between the two vectors as 360 – θ, it does not matter because the cosine of both angles is the same. Since a dot between the two vectors indicates the scalar product, it is also called the **dot product. Remember that the scalar product of two vectors is a scalar quantity.**

A familiar example of the scalar product is the work done when a force **F** acts on a body moving at an angle to the direction of the force. If **d** is the displacement of the body and θ is the angle between **F** and **d**, then the work done by the force is $Fd\cos\theta$.

Since dot product is a scalar, it is commutative: **A.B** = **B.A** = ABcosθ. It is also distributive: **A.(B + C) = A.B + A.C.**

1.4.3 Vector Product of Vectors

Suppose we have two vectors **A** and **B** inclined at an angle θ. We can draw a plane which contains these two vectors. Let that plane be called Ω ((Fig. 1.12 a) which is perpendicular to the plane of paper here. Then the vector product of these vectors, written as **A × B**, is a vector, say **C**, whose magnitude is AB sinθ and whose direction is perpendicular to the plane Ω. The direction of the vector **C** can be found by **right-hand rule** (Fig. 1.12 b). Imagine the fingers of your right hand curling from **A** to **B** along the smaller angle between them. Then the direction of the thumb gives the direction of the product vector **C**. If you follow this rule, you can easily see that direction of vector **B × A** is opposite to that of the vector **A × B**. This means that **the vector product is not commutative**. Since a cross is inserted between the two vectors to indicate their vector product, the vector product is also called the cross product.



(a) (b)

Fig.1.12 (a) : Vector product of Vectors; (b) Direction of the product vector C = A × B is given by the right hand rule. If the right hand is held so that the curling fingers point from A to B through the smaller angle between the two, then the thumb streetched at right angles to fingers will point in the direction of C.

A familiar example of vector product is the angular momentum possessed by a rotating body.

To check your progress, try the following questions.

Intext Questions 1.4

1. Suppose vector **A** is parallel to vector **B**. What is their vector product? What will be the vector product if **B** is anti-parallel to **A**?

....

2. Suppose we have a vector **A** and a vector $\mathbf{C} = \frac{1}{2}\mathbf{B}$. How is the direction of vector **A** × **B** related to the direction of vector **A** × **C**.

....

3. Suppose vectors **A** and **B** are rotated in the plane which contains them. What happens to the direction of vector **C** = **A** × **B**.

....

4. Suppose you were free to rotate vectors **A** and **B** through arbitrary amounts keeping them confined to the same plane. Can you make vector **C** = **A** × **B** to point in exactly opposite direction?

....

5. If vector **A** is along the *x*-axis and vector **B** is along the *y*-axis, what is the direction of vector **C** = **A** × **B**? What happens to **C** if **A** is along the *y*-axis and **B** is along the *x*-axis?

....

6. **A** and **B** are two mutually perpendicular vectors. Calculate (a) **A . B** and (b) **A × B**.

....

1.5 Resolution of Vectors

Resolution of vectors is converse of addition of vectors. Here we calculate components of a given vector along any set of coordinate axes. Suppose we have vector **A** as shown in Fig. 1.13 and we need to find its components along *x* and *y*-axes. Let these components be called A_x and A_y respectively. Simple trigonometry shows that

$$A_x = A\cos\theta \qquad (1.4)$$

and

$$A_y = A\sin\theta, \qquad (1.5)$$

where θ is the angle that **A** makes with the *x* - axis. What about the components of vector **A** along X and Y-axes (Fig. 1.13)? If the angle between the X-axis and **A** is ϕ, then

$$A_X = A\cos\phi$$

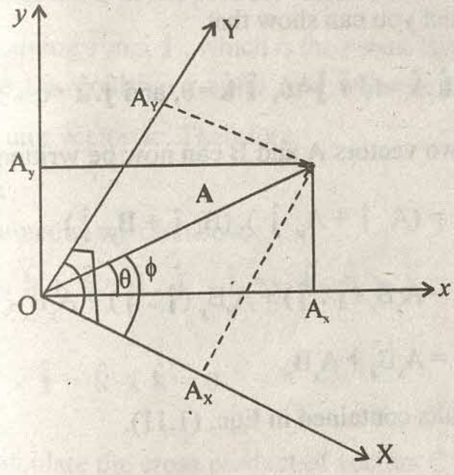


Fig. 1.13 : Resolution of vector A along two sets of coordinates *(x, y)* and (X, Y)

and

$$A_Y = A\sin\phi.$$

It must now be clear that the components of a vector are not fixed quantities; they depend on the particular set of axes along which components are required. Note also that the magnitude of vector **A** and its direction in terms of its components are given by

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{A_X^2 + A_Y^2} \qquad (1.6)$$

and

$$\tan\theta = A_y / A_x, \qquad \tan\phi = A_Y / A_X. \qquad (1.7)$$

So, if we are given the components of a vector, we can combine them as in these equations to get the vector.

1.6 Unit Vector

At this stage we introduce the concept of a **unit vector**. As the name suggests, a unit vector has unitary magnitude and has a specified direction. It has no units and no dimensions. As an example, we can write vector **A** as A $\hat{\mathbf{n}}$ where a cap on **n** (i.e. $\hat{\mathbf{n}}$) denotes a unit vector in the direction of **A**. Notice that a unit vector has been introduced to take care of the direction of the vector; the magnitude has been taken care of by A. Of particular importance are the unit vectors along coordinate axes. Unit vector along *x*-axis is denoted by $\hat{\mathbf{i}}$, along *y*-axis by $\hat{\mathbf{j}}$ and along *z*-axis by $\hat{\mathbf{k}}$. Using this notation, vector **A**, whose components along *x* and *y* axes are respectively A_x and A_y, can be written as

$$\mathbf{A} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}}\ . \tag{1.8}$$

Another vector **B** can similarly be written as

$$\mathbf{B} = B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}}\ . \tag{1.9}$$

The sum of these two vectors can now be written as

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\,\hat{\mathbf{i}} + (A_y + B_y)\,\hat{\mathbf{j}} \tag{1.10}$$

By the rules of scalar product you can show that

$$\hat{\mathbf{i}}.\,\hat{\mathbf{i}}=1,\ \hat{\mathbf{j}}.\hat{\mathbf{j}}=1,\ \hat{\mathbf{k}}.\hat{\mathbf{k}}=1,\ \hat{\mathbf{i}}.\hat{\mathbf{j}}=0,\ \hat{\mathbf{i}}.\hat{\mathbf{k}}=0, \text{ and } \hat{\mathbf{j}}.\hat{\mathbf{k}}=0 \tag{1.11}$$

The dot product between two vectors **A** and **B** can now be written as

$$\begin{aligned}\mathbf{A}\,.\,\mathbf{B} &= (A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}}\,).\,(B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}}\,)\\ &= A_xB_x\,(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}) + A_xB_y\,(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}) + A_yB_x\,(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}) + A_yB_y\,(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}})\\ &= A_xB_x + A_yB_y,\end{aligned} \tag{1.12}$$

Here, we have used the results contained in Eqn. (1.11).

Example 1.4 On a coordinate system (showing all the four quadrants) show the following vectors:

$\mathbf{A} = 4\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$, $\mathbf{B} = 0\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$, $\mathbf{C} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$,

$\mathbf{D} = 6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$.

Find their magnitudes and directions.

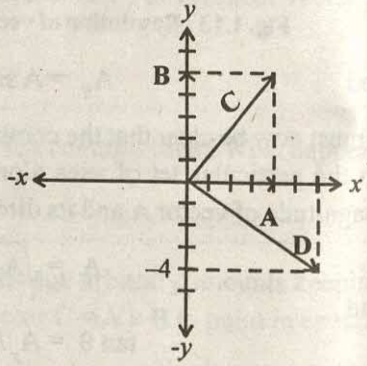


Fig. 1.14

Solution : The vectors are given in component form. The factor multiplying $\hat{\mathbf{i}}$ is the *x* component and the factor multiplying $\hat{\mathbf{j}}$ is the *y* component. All the vectors are shown on the coordinate grid (Fig. 1.14).

The components of **A** are $A_x = 4$, $A_y = 0$. So, the magnitude of **A** = 4. Its direction is $\tan^{-1}\left(\dfrac{A_y}{A_x}\right)$ in accordance with Eqn. (1.7). This quantity is zero, since $A_y = 0$. This makes it to be along the *x*-axis, as it is. Vector **B** has *x*-component = 0, so it lies along the *y*-axis and its magnitude is 5.

Let us consider vector **C**. Here, $C_x = 4$ and $C_y = 5$. Therefore, the magnitude of **C** is $C = \sqrt{4^2+5^2} = \sqrt{41}$. The angle that it makes with the x-axis is $\tan^{-1}(C_y/C_x) = 51.3$ degrees. Similarly, the magnitude of **D** is $D = \sqrt{60}$. Its direction is $\tan^{-1}(D_y/D_x) = \tan^{-1}(0.666) = -33.7°$ (in the fourth quadrant).

Example 1.5 Calculate the product **C . D** for the vectors given in Example 1.4.

Solution : The dot product of **C** with **D** can be found using Eqn. (1.12):

$$\mathbf{C}\cdot\mathbf{D} = C_xD_x + C_yD_y = 4\times6 + 5\times(-4) = 24 - 20 = 4.$$

The cross product of two vectors can also be written in terms of the unit vectors. For this we first need the cross product of unit vectors. For this remember that the angle between the unit vectors is a right angle. Consider, for example, $\hat{\mathbf{i}} \times \hat{\mathbf{j}}$. Sine of the angle between them is one. The magnitude of the product vector is also 1. Its direction is perpendicular to the *xy* - plane containing $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, which is the *z*-axis. By the right hand rule, we also find that this must be the positive *z*-axis. And what is the unit vector in the positive z - direction. The unit vector $\hat{\mathbf{k}}$. Therefore,

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}. \qquad (1.13)$$

Using similar arguments, we can show,

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}},\ \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}},\ \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}},\ \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}},\ \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}, \qquad (1.14)$$

and

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0. \qquad (1.15)$$

Example 1.6 Calculate the cross product of vectors **C** and **D** given in Example (1.4).

Solution : We have

$$\mathbf{C} \times \mathbf{D} = (4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}) \times (6\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})$$

$$= 24\,(\hat{\mathbf{i}} \times \hat{\mathbf{i}}) - 16\,(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) + 30\,(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) - 20\,(\hat{\mathbf{j}} \times \hat{\mathbf{j}})$$

Using the results contained in Eqns. (1.13 – 1.15), we can write

$$\mathbf{C} \times \mathbf{D} = -16\,\hat{\mathbf{k}} - 30\,\hat{\mathbf{k}} = -46\,\hat{\mathbf{k}}$$

So, the cross product of **C** and **D** is a vector of magnitude 46 and in the negative *z* direction. Since **C** and **D** are in the *xy*-plane, it is obvious that the cross product must be perpendicular to this plane, that is, it must be in the *z*-direction.

Intext Questions 1.5

1. A vector **A** makes an angle of 60 degrees with the *x*-axis of the *xy*-system of coordinates. If its magnitude is 50 units, find its components in *x*, *y* directions. If another vector **B** of the same magnitude makes an angle of 30 degrees with the X-axis of the XY-system of coordinates. Find its components now. Are they same as before?

....

2. Two vectors **A** and **B** are given respectively as $3\,\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ and $-2\,\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$. Sketch them on the coordinate grid. Find their magnitudes and the angles that they make with the *x*-axis (see Fig. 1.14).

....

3. Calculate the dot and cross product of the vectors given in the above question.

....

You now know that each term in an equation must have the same dimensions. Having learnt vectors, we must now add this: **For an equation to be correct, each term in it must have the same character: either all of them be vectors or all of them be scalars.**

What You Have Learnt

- Every physical quantity must be measured in some unit and also expressed in this unit. The SI system has been accepted and followed universally for scientific reporting.
- Base SI units for mass, length and time are respectively kg, m and s. In addition to base units, there are derived units.
- Every physical quantity has dimensions. Dimensional analysis is a useful tool for checking correctness of equations.
- In physics, we deal generally with two kinds of quantities, scalars and vectors. A scalar has only magnitude. A vector has both direction and magnitude.
- Vectors are added according to the parallelogram rule.
- The scalar product of two vectors is a scalar.
- The vector product of two vectors is a vector perpendicular to the plane containing the two vectors.
- Vectors can be resolved into components along a specified set of coordinates axes.

Terminal Exercise

1. A unit used for measuring very large distances is called a light year. It is the distance covered by light in one year. Express light year in metres. Take speed of light as 3×10^8 m s^{-1}.

2. Meteors are small pieces of rock which enter the earth's atmosphere occasionally at very high speeds. Because of friction caused by the atmosphere, they become very hot and emit radiations for a very short time before they get completely burnt. The streak of light that is seen as a result is called a 'shooting star'. The speed of a meteor is 51 kms^{-1} In comparison, speed of sound in air at about 20°C is 340 ms^{-1} Find the ratio of magnitudes of the two speeds.

3. The distance covered by a particle in time *t* while starting with the initial velocity *u* and moving with a uniform acceleration *a* is given by $s = ut + (1/2)at^2$. Check the

correctness of the expression using dimensional analysis.

4. Newton's law of gravitation states that the magnitude of force between two particles of mass m_1 and m_2 separated by a distance r is given by

$$F = G\,\frac{m_1 m_2}{r^2}$$

where G is the universal constant of gravitation. Find the dimensions of G.

5. Hamida is pushing a table in a certain direction with a force of magnitude 10 N. At the same time her, classmate Lila is pushing the same table with a force of magnitude 8 N in a direction making an angle of 60° to the direction in which Hamida is pushing. Calculate the magnitude of the resultant force on the table and its direction.

6. A physical quantity is obtained as a dot product of two vector quantities. Is it a scalar or a vector? What is the nature of a physical quantity obtained as cross product of two vectors?

7. John wants to pull a cart applying a force parallel to the ground. His friend Ramu suggests that it would be easier to pull the cart by applying a force at an angle of 30 degrees to the ground. Who is correct and why?

8. Two vectors are given by $5\,\hat{i} - 3\,\hat{j}$ and $3\,\hat{i} - 5\,\hat{j}$. Calculate their scalar and vector products.

Answers to Intext Questions

1.1

1. Mass of the sun = 2×10^{30} kg

   Mass of a proton = 2×10^{-27} kg

   (No of protons in the sun = $\dfrac{2\times10^{30}\,kg}{2\times10^{-27}\,kg} = 10^{57}$.

2. 1 angstrom = 10^{-8} cm = 10^{-10} m

   1 nanometer (nm) = 10^{-9} m

   ∴ 1 nm/1 angstrom = 10^{-9} m /10^{-10} m = 10 so 1 nm = 10 Å

3. 1370 kHz = 1370×10^3 Hz = (1370×10^3)/10^9 GHz = 1.370×10^{-3} GHz

4. 1 decameter (dam) = 10 m

   1 decimeter (dm) = 10^{-1} m

   ∴ 1 dam = 100 dm

   1 MW = 10^6 W

   1 GW = 10^9 W

   ∴ 1 GW = 10^3 MW

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1.2

1. Dimension of length = L

   Dimension of time = T

   Dimensions of $g = LT^{-2}$

   Let time period t be proportional to l^{α} and g^{β}

   Then, writing dimensions on both sides $T = L^{\alpha} (LT^{-2})^{\beta} = L^{\alpha+\beta} T^{-2\beta}$

   Equating powers of L and T,

   $\alpha + \beta = 0,\ 2\beta = -1 \Rightarrow \beta = -1/2$ and $\alpha = 1/2$

   So, $t\ \alpha \sqrt{\dfrac{l}{g}}$.

2. Dimension of $a = LT^{-2}$

   Dimension of $\upsilon = LT^{-1}$

   Dimension of $r = L$

   Let a be proportional to υ^{α} and r^{β}

Then dimensionally,

$LT^{-2} = (LT^{-1})^{\alpha}\ L^{\beta} = L^{\alpha+\beta}\ T^{-\alpha}$

Equating powers of L and T,

$\alpha + \beta = 1,\ \alpha = 2, \Rightarrow \alpha = -1$

So, $\alpha \propto \upsilon^2/r$

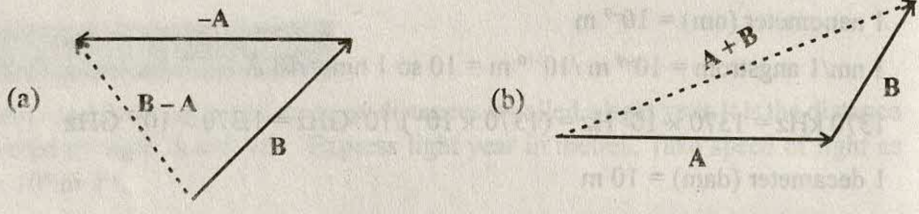
3. Dimensions of $m\upsilon = MLT^{-1}$

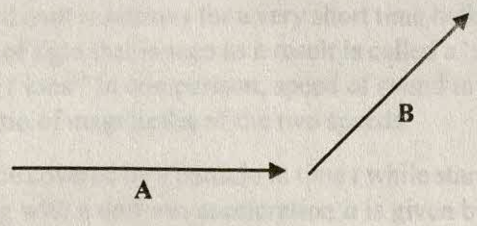
   Dimensions of $Ft = MLT^{-2}\ T^{1} = MLT^{-1}$

   Dimensions of both the sides are the same, therefore, the equation is dimensionally correct.

1.3

1. Suppose





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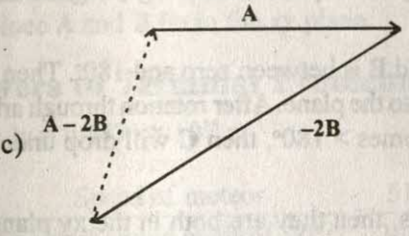

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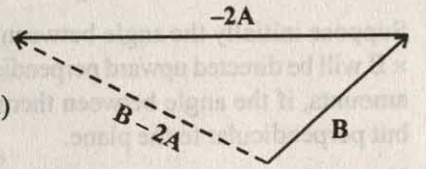
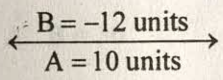


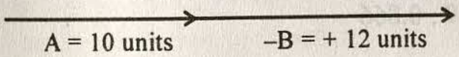
2.



$\mathbf{A} + \mathbf{B} = 10 + (-12)$

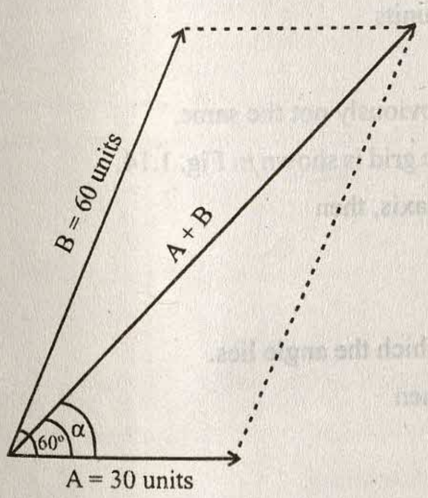
$= -2$ units

also



$\mathbf{A} - \mathbf{B} = 22$ units

3.



$|\mathbf{A} + \mathbf{B}| = 77$ units

1.4

1. If **A** and **B** are parallel, the angle θ between them is zero. So, their cross product

   $\mathbf{A} \times \mathbf{B} = AB \sin\theta = 0.$

   If they are antiparallel then the angle between them is 180°. Therefore,

   $\mathbf{A} \times \mathbf{B} = AB \sin\theta = 0$, because $\sin 180° = 0$.

2. If magnitude of **B** is halved, but it remains in the same plane as before, then the direction of the vector product **C** = **A** × **B** remains unchanged. Its magnitude may change.

3. Since vectors **A** and **B** rotate without change in the plane containing them, the direction of **C** = **A** × **B** will not change.

4. Suppose initially the angle between **A** and **B** is between zero and 180°. Then **C** = **A** × **B** will be directed upward perpendicular to the plane. After rotation through arbitrary amounts, if the angle between them becomes > 180°, then **C** will drop underneath but perpendicular to the plane.

5. If **A** is along x-axis and **B** is along y-axis, then they are both in the xy plane. The vector product **C** = **A** × **B** will be along z-direction. If **A** is along y-axis and **B** is along x-axis, then **C** is along the negative z-axis.

6. (a) $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos\theta = 0$ when $\theta = 90°$

   (b) $\mathbf{A} \times \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \sin\theta = |\mathbf{A}|\,|\mathbf{B}|$ as $\sin\theta = 1$ at $\theta = 90°$

1.5

1. When **A** makes an angle of 60° with the x-axis:

$$A_x = A\cos 60 = 50 \,.\, ½ = 25 \text{ units}$$

$$A_y = A\sin 60 = 50.\sqrt{3}/2 = 50 \,.\, 0.866$$

$$= 43.3 \text{ units}$$

When **A** makes an angle of 30° with the x-axis

$$A_x = 50\cos 30 = 50 \,.\, 0.866 = 43.3 \text{ units}$$

$$A_y = 50\sin 30 = 50 \,.\, ½ = 25 \text{ units}$$

The components in the two cases are obviously not the same.

2. The position of vectors on the coordinate grid is shown in Fig. 1.14.

Suppose **A** makes an angle θ with the x-axis, then

$$\tan\theta = -4/3 \Rightarrow \theta = \tan^{-1}(-4/3)$$

$$= -53° \, 6' \text{ or } 306° \, 54'$$

after taking account of the quadrant in which the angle lies.

If B makes an angle ϕ with the x-axis, then

$$\tan\phi = 6/{-2} = -3 \Rightarrow \phi = \tan^{-1}(-3)$$

$$= 108° \, 24'$$

3. The dot product of **A** and **B**:

$$\mathbf{A} \cdot \mathbf{B} = (3\hat{i} - 4\hat{j}).(-2\hat{i} + 6\hat{j})$$

$$= -6(\hat{i}.\hat{i}) - 24(\hat{j}.\hat{j}) = -30$$

because $\hat{i}.\hat{j} = \hat{j}.\hat{i} = 0$, and $\hat{i}.\hat{i} = \hat{j}.\hat{j} = 1$

The cross product of **A** and **B**:

$$\mathbf{A} \times \mathbf{B} = (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 6\hat{j})$$

$$= 18(\hat{i} \times \hat{j}) + 8(\hat{j} \times \hat{i}) = 18\,\hat{k} - 8\,\hat{k} = 10\,\hat{k}$$

on using Eqs.(1.14) and (1.15). So, the cross product is in the direction of z-axis, since **A** and **B** lie in the *xy* plane.

Answers to Terminal Problems

1. 1 ly = 9.4673×10^{15} m.

2. $\dfrac{\text{Speed of meteor}}{\text{Speed of sound in air of } 20°\text{C}} = \dfrac{51}{340} = \dfrac{3}{20}$

5. 15.84 N and $\alpha = \tan^{-1}\left(\dfrac{1}{2}\right)$

8. **A . B** = 30

$\mathbf{A} \times \mathbf{B} = \left(5\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}}\right) \times \left(3\,\hat{\mathbf{i}} - 5\,\hat{\mathbf{j}}\right)$ is a single vector **C** such that |**C**| = 16 units along negative *z*-direction.

2

MOTION IN A STRAIGHT LINE

We see a number of things moving around us. Humans, animals, vehicles can be seen moving on land. Fish, frogs and other aquatic animals move in water. Birds and aeroplanes move in air. Though we do not feel it, the earth on which we live also revolves around the sun as well as its own axis. It is, therefore, quite apparent that we live in a world that is very much in constant motion. Therefore to understand the physical world around us, the study of motion is essential. Motion can be in a straight line(1D), in a plane(2D) or in space(3D). If the motion of the object is only in one direction, it is said to be the motion in a straight line. For example, motion of a car on a straight road, motion of a train on straight rails, motion of a freely falling body, motion of a lift, and motion of an athlete running on a straight track, etc.

In this lesson you will learn about motion in a straight line. In the following lessons, you will study the laws of motion, motion in plane and other types of motion.

Objectives

After studying this lesson, you should be able to,

- *distinguish between distance and displacement, and speed and velocity;*
- *explain the terms instantaneous velocity, relative velocity and average velocity;*
- *define acceleration and instantaneous acceleration;*
- *interpret position - time and velocity - time graphs for uniform as well as non-uniform motion;*
- *derive equations of motion with constant acceleration; and*
- *describe motion under gravity; and*
- *solve numericals based on equations of motion.*

2.1 Speed and Velocity

We know that the total length of the path covered by a body is the **distance** travelled by it. But the difference between the initial and final position vectors of a body is called its **displacement**. Basically, **displacement is the shortest distance between the two positions and has a certain direction**. Thus, the displacement is a vector quantity but distance is a scalar. You might have also learnt that the rate of change of distance with time is called **speed** but the rate of change of displacement is known as **velocity**. Unlike speed, velocity is a vector quantity. For 1-D motion, the directional aspect of the vector is taken care of by putting + and – signs and we do not have to use vector notation for displacement, velocity and acceleration for motion in one dimension.

2.1.1 Average Velocity

When an object travels a certain distance with different velocities, its motion is specified by its average velocity. The **average velocity** of an object is defined as the displacement per unit time. Let x_1 and x_2 be its positions at instants t_1 and t_2, respectively. Then mathematically we can express average velocity as

$$\bar{v} = \frac{\text{displacement}}{\text{time taken}}$$

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad (2.1)$$

where $x_2 - x_1$ signifies change in position (denoted by Δx) and $t_2 - t_1$ is the corresponding change in time (denoted by Δt). Here the bar over the symbol for velocity ($\bar{v}$) is standard notation used to indicate an average quantity. Average velocity can be represented as v_{av} also. The average speed of an object is obtained by dividing the total distance travelled by the total time taken:

$$\text{Average speed} = \frac{\text{total distancetravelled}}{\text{total time taken}} \qquad (2.2)$$

If the motion is in the same direction along a straight line, the average speed is the same as the magnitude of the average velocity. However, this is always not the case (see example 2.2).

Following examples will help you in understanding the difference between average speed and average velocity.

Example 2.1 : The position of an object moving along the x-axis is defined as $x = 20t^2$, where t is the time measured in seconds and position is expressed in metres. Calculate the average velocity of the object over the time interval from 3s to 4s.

Solution : Given,

$$x = 20t^2$$

Note that x and t are measured in metres and seconds. It means that the constant of proportionality (20) has dimensions ms^{-2}.

We know that the average velocity is given by the relation

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1}$$

At $t_1 = 3s$,

$$x_1 = 20 \times (3)^2$$

$$= 20 \times 9 = 180 \text{ m}$$

Similarly, for $t_2 = 4$s

$$x_2 = 20 \times (4)^2$$

$$= 20 \times 16 = 320 \text{ m}$$

$$\therefore \quad \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(320 - 180)\text{ m}}{(4 - 3)\text{ s}} = \frac{140 \text{ m}}{1 \text{ s}} = 140 \text{ ms}^{-1}$$

Hence, average velocity = 140 ms^{-1}.

Example 2.2 : A person runs on a 300m circular track and comes back to the starting point in 200s. Calculate the average speed and average velocity.

Solution : Given,

Total length of the track = 300m.

Time taken to cover this length = 200s

Hence,

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{time taken}}$$

$$= \frac{300}{200} \text{ ms}^{-1} = 1.5 \text{ ms}^{-1}$$

As the person comes back to the same point, the displacement is zero. Therefore, the average velocity is also zero.

Note that in the above example, the average speed is not equal to the magnitude of the average velocity. Do you know the reason?

2.1.2 Relative Velocity

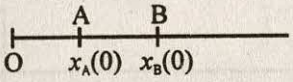
When we say that a bullock cart is moving at 10km h^{-1} due south, it means that the cart travels a distance of 10km in 1h in southward direction from its starting position. Thus it is implied that the referred velocity is with respect to some reference point. In fact, the velocity of a body is always specified with respect to some other body. Since all bodies are in motion, we can say that every velocity is relative in nature.

The relative velocity of an object with respect to another object is the rate at which it changes its position relative to the object / point taken as reference. For example, if v_A and v_B are the velocities of the two objects along a straight line, the relative velocity of B with respect to A will be $v_B - v_A$.

The rate of change of the relative position of an object with respect to the other object is known as the ***relative velocity*** *of that object with respect to the other.*

Importance of Relative Velocity

The position and hence velocity of a body is specified in relation with some other body. If the reference body is at rest, the motion of the body can be described easily .You will learn the equations of kinematics in this lesson. But what happens, if the reference body is also moving? Such a motion is seen to be of the two body system by a stationary observer. However, it can be simplified by invoking the concept of relative motion.



Let the initial positions of two bodies A and B be $x_A(0)$ and $x_B(0)$. If body A moves along positive x-direction with velocity υ_A and body B with velocity υ_B, then the positions of points A and B after t seconds will be given by

$$x_A(t) = x_A(0) + \upsilon_A t$$

$$x_B(t) = x_B(0) + \upsilon_B t$$

Therefore, the relative separation of B from A will be

$$x_{BA}(t) = x_B(t) - x_A(t) = x_B(0) - x_A(0) + (\upsilon_B - \upsilon_A)\, t$$

$$= x_{BA}(0) + \upsilon_{BA} t$$

where $\upsilon_{BA} = (\upsilon_B - \upsilon_A)$ is called the relative velocity of B with respect to A. Thus by applying the concept of relative velocity, a two body problem can be reduced to a single body problem.

Example 2.3 : A train A is moving on a straight track (or railway line) from North to South with a speed of 60km h^{-1}. Another train B is moving from South to North with a speed of 70km h^{-1}. What is the velocity of B relative to the train A?

Solution : Considering the direction from South to North as positive, we have

velocity (υ_B) of train B $= +$ 70km h^{-1}

and, velocity (υ_A) of train A $= -$ 60km h^{-1}

Hence, the velocity of train B relative to train A

$$= \upsilon_B - \upsilon_A$$

$$= 70 - (-60) = 130\text{km h}^{-1}.$$

In the above example, you have seen that the relative velocity of one train with respect to the other is equal to the sum of their respective velocities. This is why a train moving in a direction opposite to that of the train in which you are travelling appears to be travelling very fast. But, if the other train were moving in the same direction as your train, it would appear to be very slow.

2.1.3 Acceleration

While travelling in a bus or a car, you might have noticed that sometimes it speeds up and sometimes it slows down. That is, its velocity changes with time. Just as the velocity is defined as the time rate of change of displacement, ***the acceleration is defined as time rate of change of velocity.*** Acceleration is a vector quantity and its SI unit is m s^{-2}. In one dimension, there is no need to use vector notation for acceleration as explained in the case of velocity. The average acceleration of an object is given by,

$$\text{Average acceleration } (\bar{a}) = \frac{\text{Final velocity - Initial velocity}}{\text{Time taken for change in velocity}}$$

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad (2.3)$$

In one dimensional motion, when the acceleration is in the same direction as the motion or velocity (normally taken to be in the positive direction), the acceleration is positive. But the acceleration may be in the opposite direction of the motion also. Then the acceleration is taken as negative and is often called deceleration or ***retardation***. So we can say that an increase in the rate of change of velocity is ***acceleration***, whereas the decrease in the rate of change of velocity is ***retardation***.

Example 2.4 : The velocity of a car moving towards the East increases from 0 to 12ms^{-1} in 3.0 s. Calculate its average acceleration.

Solution : Given,

$$v_1 = 0 \text{ m s}^{-1}$$

$$v_2 = 12 \text{ m s}^{-1}$$

$$t = 3.0 \text{ s}$$

$$a = \frac{(12.0\text{m s}^{-1})}{3.0\text{s}}$$

$$= 4.0 \text{ m s}^{-2}$$

Intext Questions 2.1

1. Is it possible for a moving body to have non-zero average speed but zero average velocity during any given interval of time? If so, explain.

....

2. A lady drove to the market at a speed of 8 km h^{-1}. Finding market closed, she came back home at a speed of 10 km h^{-1}. If the market is 2km away from her home, calculate the average velocity and average speed.

....

3. Can a moving body have zero relative velocity with respect to another body? Give an example.

....

4. A person strolls inside a train with a velocity of 1.0 m s^{-1} in the direction of motion of the train. If the train is moving with a velocity of 3.0 m s^{-1}, calculate his

(a) velocity as seen by passengers in the compartment, and (b) velocity with respect to a person sitting on the platform.

....

2.2 Position - Time Graph

If you roll a ball on the ground, you will notice that at different times, the ball is found at different positions. The different positions and corresponding times can be plotted on a graph giving us a certain curve. Such a curve is known as position-time curve. Generally, the time is represented along x-axis whereas the position of the body is represented along y-axis.

Let us plot the position - time graph for a body at rest at a distance of 20m from the origin. What will be its position after 1s, 2s, 3s, 4s and 5s? You will find that the graph is a straight line parallel to the time axis, as shown in Fig. 2.1

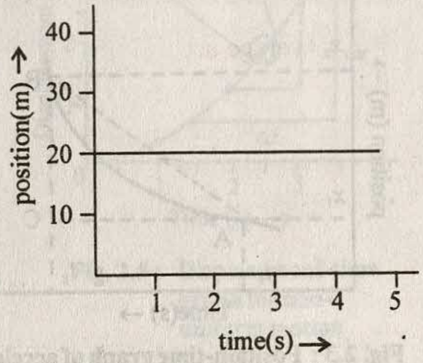


Fig. 2.1 : Position-time graph for a body at rest

2.2.1 Position-Time Graph for Uniform Motion

Now, let us consider a case where an object covers equal distances in equal intervals of time. For example, if the object covers a distance of 10m in each second for 5 seconds, the positions of the object at different times will be as shown in the following table.

Time (t) in s	1	2	3	4	5
Position (x) in m	10	20	30	40	50

In order to plot this data, take time along x-axis assuming 1cm as 1s, and position along y-axis with a scale of 1cm to be equal to 10m. The position-time graph will be as shown in Fig.2.2

The graph is a straight line inclined with the x-axis. ***A motion in which the velocity of the moving object is constant is known as uniform motion.*** Its position-time graph is a straight line inclined to the time axis.

In other words, we can say that when a moving object covers equal distances in equal intervals of time, it is in ***uniform motion.***

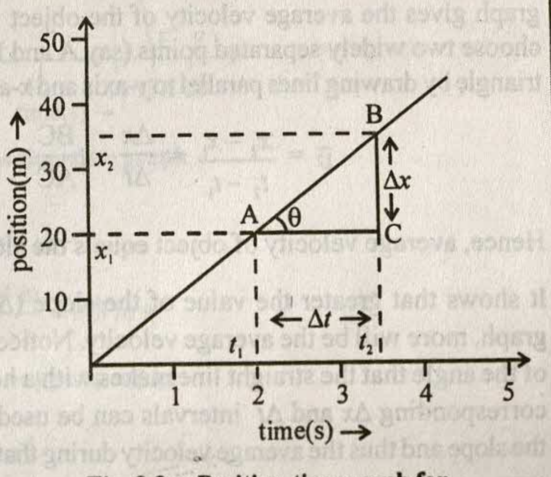


Fig. 2.2 : Position-time graph for uniform motion

2.2.2 Position-Time Graph for Non-Uniform Motion

Let us now take an example of a train which starts from a station, speeds up and moves with uniform velocity for certain duration and then slows down before steaming in the next station. In this case you will find that the ***distances covered in equal intervals of time are not equal. Such a motion is said to be non-uniform motion.*** If the distances covered in successive intervals are increasing, the motion is said to be accelerated motion. The position-time graph for such an object is as shown in Fig.2.3.

Note that the position-time graph of accelerated motion is a continuous curve. Hence, the velocity of the body changes continuously. In such a situation, it is more appropriate to define average velocity of the body over an extremely small interval of time or instantaneous velocity. Let us learn to do so now.

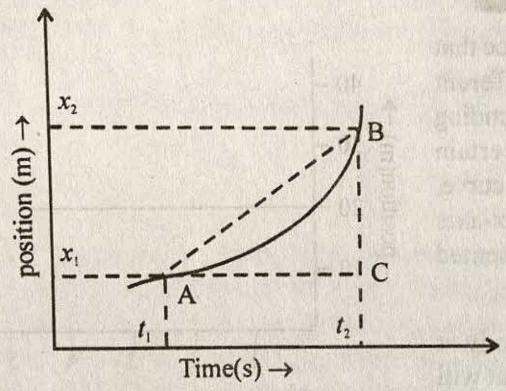


Fig. 2.3 : Position-time graph of accelerated motion as a continuous curve

2.2.3 Interpretation of Position - Time Graph

As you have seen, the position - time graphs of different moving objects can have different shapes. If it is a straight line parallel to the time axis, you can say that the body is at rest (Fig. 2.1). And the straight line inclined to the time axis shows that the motion is uniform (Fig.2.2). A continuous curve implies continuously changing velocity.

(a) Velocity from position - time graph : The slope of the straight line of position - time graph gives the average velocity of the object in motion. For determining the slope, we choose two widely separated points (say A and B) on the straight line (Fig.2.2) and form a triangle by drawing lines parallel to y-axis and x-axis. Thus, the average velocity of the object

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{BC}{AC} \qquad (2.4)$$

Hence, average velocity of object equals the slope of the straight line AB.

It shows that greater the value of the slope ($\Delta x/\Delta t$) of the straight line position - time graph, more will be the average velocity. Notice that the slope is also equal to the tangent of the angle that the straight line makes with a horizontal line, i.e., $\tan \theta = \Delta x/\Delta t$. Any two corresponding Δx and Δt intervals can be used to determine the slope and thus the average velocity during that time internal.

Example 2.5 : The position - time graphs of two bodies A and B are shown in Fig. 2.4. Which of these has greater velocity?

Solution : Body A has greater slope and hence greater velocity.

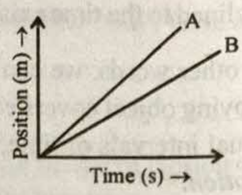


Fig. 2.4 : Position-time graph of

(b) Instantaneous velocity : As you have learnt, a body having uniform motion along a straight line has the same velocity at every instant. But in the case of non-uniform motion, the position - time graph is a curved line, as shown in Fig.2.5. As a result, the slope or the average velocity varies, depending on the size of the time intervals selected. The velocity of the particle at any instant of time or at some point of its path is called its instantaneous velocity.

Note that the average velocity over a time interval Δt is given by $\bar{v} = \dfrac{\Delta x}{\Delta t}$. As Δt is made smaller and smaller the average velocity approaches instantaneous velocity.

In the limit $\Delta t \rightarrow 0$, the slope $(\Delta x/\Delta t)$ of a line tangent to the curve at that point gives the instantaneous velocity. However, for uniform motion, the average and instantaneous velocities are the same.

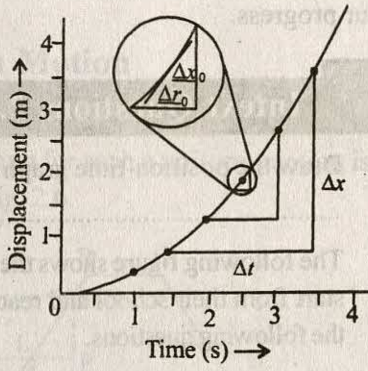


Fig. 2.5 : Displacement-time graph for non-uniform motion

Example 2.6 : The position - time graph for the motion of an object for 20 seconds is shown in Fig. 2.6. What distances and with what speeds does it travel in time intervals (i) 0 s to 5 s, (ii) 5 s to 10 s, (iii) 10 s to 15 s and (iv) 15 s to 17.5 s? Calculate the average speed for this total journey.

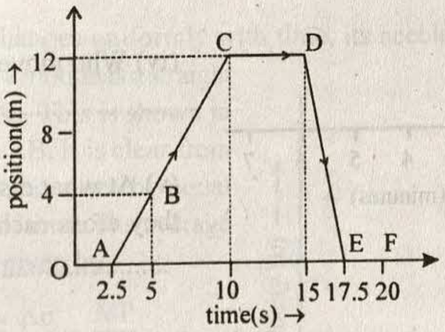


Fig. 2.6: Position-time graph

Solution :

i) During 0 s to 5 s, distance travelled = 4 m

$$\therefore \text{speed} = \frac{\text{Distance}}{\text{Time}} = \frac{4\ \text{m}}{(5-0)\,s} = \frac{4\ \text{m}}{5\,s} = 0.8\ \text{m s}^{-1}$$

ii) During 5 s to 10 s, distance travelled = 12 – 4 = 8 m

$$\therefore \text{speed} = \frac{(12-4)\ \text{m}}{(10-5)\ \text{s}} = \frac{8\ \text{m}}{5\ \text{s}} = 1.6\ \text{m s}^{-1}$$

iii) During 10 s to 15 s, distance travelled = 12 – 12 = 0 m

$$\therefore \text{speed} = \frac{\text{Distance}}{\text{Time}} = \frac{0}{5} = 0$$

iv) During 15 s to 17.5 s, distance travelled = 12 m

Notes

$$\therefore \text{ Speed} = \frac{12 \text{ m}}{2.5 \text{ s}} = 4.8 \text{ m s}^{-1}$$

Now we would like you to pause for a while and solve the following questions to check your progress.

Intext Questions 2.2

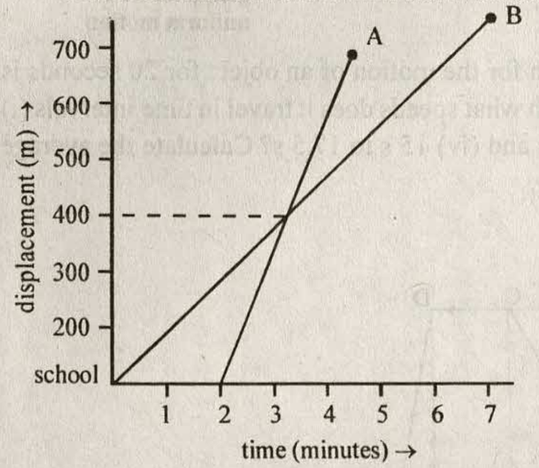
1. Draw the position-time graph for a motion with zero acceleration.

....

2. The following figure shows the displacement - time graph for two students A and B who start from their school and reach their homes. Look at the graphs carefully and answer the following questions.

(i) Do they both leave school at the same time?

....



(ii) Who stays farther from the school?

....

(iii) Do they both reach their respective houses at the same time?

....

(iv) Who moves faster?

....

(v) At what distance from the school do they cross each other?

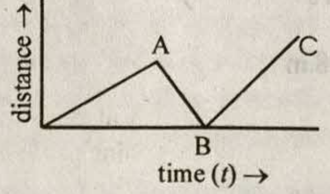
....

3. Under what conditions is average velocity of a body equal to its instantaneous velocity?

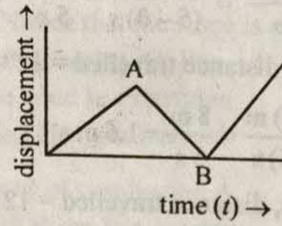
....

4. Which of the following graphs is not possible? Give reason for your answer?

....



(a)



(b)

2.3 Velocity - Time Graph

Just like the position-time graph, we can plot velocity-time graph. While plotting a velocity-time graph, generally the time is taken along the *x*-axis and the velocity along the *y*-axis.

2.3.1 Velocity-Time Graph for Uniform Motion

As you know, in uniform motion the velocity of the body remains constant, i.e., there is no change in the velocity with time. The velocity-time graph for such a uniform motion is a straight line parallel to the time axis, as shown in the Fig. 2.6.

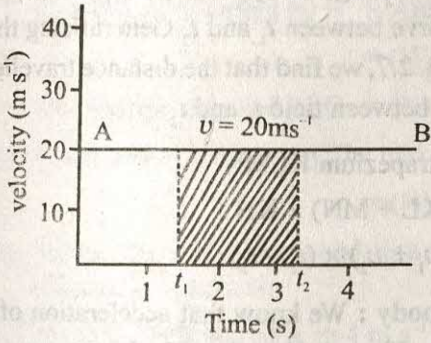


Fig. 2.6 : Velocity-time graph for uniform motion

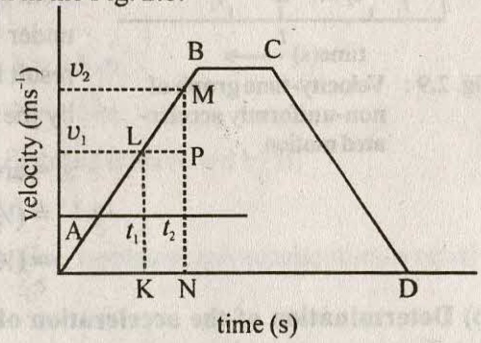


Fig. 2.7 : Velocity-time graph for motion with three different stages of constant acceleration

2.3.2 Velocity-Time Graph for Non-Uniform Motion

If the velocity of a body changes uniformly with time, its acceleration is constant. The velocity-time graph for such a motion is a straight line inclined to the time axis. This is shown in Fig. 2.7 by the straight line AB. It is clear from the graph that the velocity increases by equal amounts in equal intervals of time. The average acceleration of the body is given by

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} = \frac{MP}{LP}$$

= slope of the straight line

Since the slope of the straight line is constant, the average acceleration of the body is constant. However, it is also possible that the rate of variation in the velocity is not constant. Such a motion is called non-uniformly accelerated motion. In such a situation, the slope of the velocity-time graph will vary at every instant, as shown in Fig.2.8. It can be seen that θ_A, θ_B and θ_C are different at points A, B and C.

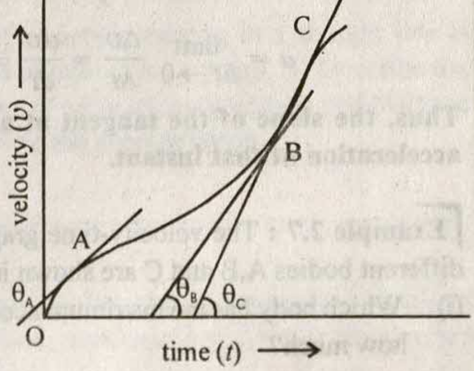


Fig. 2.8 : Velocity-time graph for a motion with varying acceleration

2.3.3 Interpretation of Velocity-Time Graph

Using $v - t$ graph of the motion of a body, we can determine the distance travelled by it and the acceleration of the body at different instants. Let us see how we can do so.

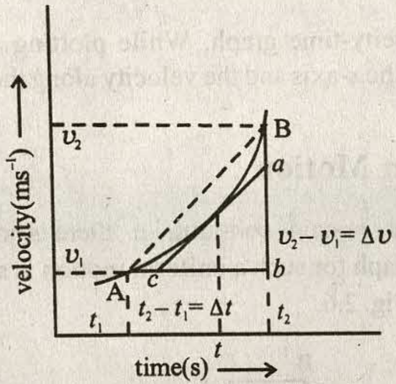


Fig. 2.9 : Velocity-time graph of non-uniformly accelerated motion

(a) Determination of the distance travelled by the body : Let us again consider the velocity-time graph shown in Fig. 2.7. The portion AB shows the motion with constant acceleration, whereas the portion CD shows the constantly retarded motion. The portion BC represents uniform motion (i.e., motion with zero acceleration).

For uniform motion, the distance travelled by the body from time t_1 to t_2 is given by $s = v(t_2 - t_1)$ = the area under the curve between t_1 and t_2 Generalising this result for Fig. 2.7, we find that the distance travelled by the body between time t_1 and t_2

s = area of trapezium KLMN

$= (½) \times (\text{KL} + \text{MN}) \times \text{KN}$

$= (½) \times (v_1 + v_2) \times (t_2 - t_1)$

(b) Determination of the acceleration of the body : We know that acceleration of a body is the rate of change of its velocity with time. If you look at the velocity-time graph given in the Fig.2.9, you will note that the average acceleration is represented by the slope of the chord AB, which is given by

$$\text{average acceleration } (\bar{a}) = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}.$$

If the time interval Δt is made smaller and smaller, the average acceleration becomes instantaneous acceleration. Thus, instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \text{slope of the tangent at } (t = t) = \frac{ab}{bc}$$

Thus, the slope of the tangent at a point on the velocity-time graph gives the acceleration at that instant.

Example 2.7 : The velocity-time graphs for three different bodies A,B and C are shown in Fig. 2.9(a).

(i) Which body has the maximum acceleration and how much?

(ii) Calculate the distances travelled by these bodies in first 3s.

(iii) Which of these three bodies covers the maximum distance at the end of their journey?

(iv) What are the velocities at t = 2s?

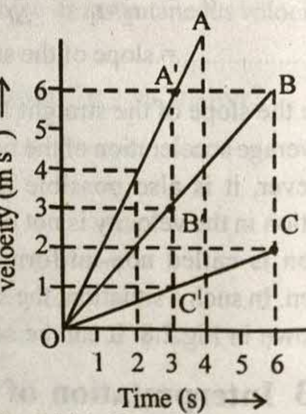


Fig. 2.9(a) : Velocity-time graph of uniformly accelerated motion of three different bodies

Solution :

(i) As the slope of the v-t graph for body A is maximum, its acceleration is maximum:

$$a = \frac{\Delta \upsilon}{\Delta t} = \frac{6-0}{3-0} = \frac{6}{3} = 2 \text{ ms}^{-2}.$$

(ii) The distance travelled by a body is equal to the area of the υ-t graph.

$\therefore$ In first 3s,

the distance travelled by A = Area OA′L

$= (½) \times 6 \times 3 = 9$m.

the distance travelled by B = Area OB′L

$= (½) \times 3 \times 3 = 4.5$ m.

the distance travelled by C $= (½) \times 1 \times 3 = 1.5$ m.

(iii) At the end of the journey, the maximum distance is travelled by B.

$= (½) \times 6 \times 6 = 18$ m.

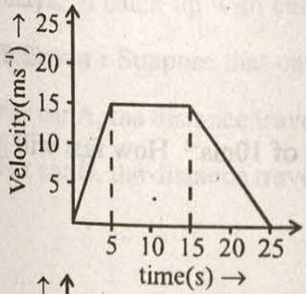
(iv) Since υ-t graph for each body is a straight line, instantaneous acceleration is equal to average acceleration.

At 2s, the velocity of A = 4 m s^{-1}

the velocity of B = 2 m s^{-1}

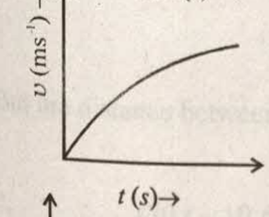
the velocity of C = 0.80 m s^{-1} (approx.)

Intext Questions 2.3



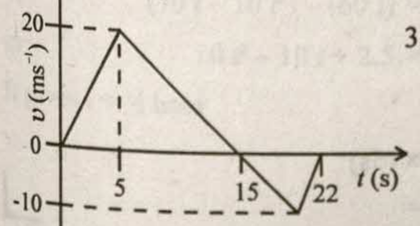
1. The motion of a particle moving in a straight line is depicted in the adjoining $v - t$ graph. (i) Describe the motion in terms of velocity, acceleration and distance travelled (ii) Find the average speed.

....



2. What type of motion does the adjoining graph represent - uniform motion, accelerated motion or decelerated motion? Explain.

....



3. Using the adjoining υ-t graph, calculate the (i) average velocity, and (ii) average speed of the particle for the time interval 0 – 22 seconds. The particle is moving in a straight line all the time.

....

2.4 Equations of Motion

As you now know, for describing the motion of an object, we use physical quantities like distance, velocity and acceleration. In the case of constant acceleration, the velocity acquired and the distance travelled in a given time can be calculated by using one or more of three equations. These equations, generally known as equations of motion for constant acceleration or kinematical equations, are easy to use and find many applications.

2.4.1 Equation of Uniform Motion

In order to derive these equations, let us take initial time to be zero i.e. $t_1 = 0$. We can then assume $t_2 = t$ to be the elapsed time. The initial position (x_1) and initial velocity (υ_1) of an object will now be represented by x_0 and υ_0 and at time t they will be called x and υ (rather than x_2 and υ_2). According to Eqn. (2.1), the average velocity during the time t will be

$$\bar{\upsilon} = \frac{x - x_0}{t}. \qquad (2.4)$$

2.4.2 First Equation of Uniformly Accelerated Motion

The first equation of uniformly accelerated motion helps in determining the velocity of an object after a certain time when the acceleration is given. As you know, by definition

$$\text{Acceleration } (a) = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}$$

If at $t_1 = 0$, $\upsilon_1 = \upsilon_0$ and at $t_2 = t$, $\upsilon_2 = \upsilon$. Then

$$a = \frac{\upsilon - \upsilon_0}{t} \qquad (2.5)$$

$$\Rightarrow \qquad \upsilon = \upsilon_0 + at \qquad (2.6)$$

Example 2.8 : A car starting from rest has an acceleration of 10ms^{-2}. How fast will it be going after 5s?

Solution : Given,

Initial velocity $\upsilon_0 = 0$

Acceleration $a = 10 \text{ ms}^{-2}$

Time $t = 5\text{s}$

Using first equation of motion

$$\upsilon = \upsilon_0 + at$$

we find that for $t = 5$s, the velocity is given by

$$\upsilon = 0 + (10 \text{ ms}^{-2}) \times (5\text{s})$$

$$= 50 \text{ ms}^{-1}$$

2.4.2 Second Equation of Uniformly Accelerated Motion

Second equation of motion is used to calculate the position of an object after time t when it is undergoing constant acceleration a.

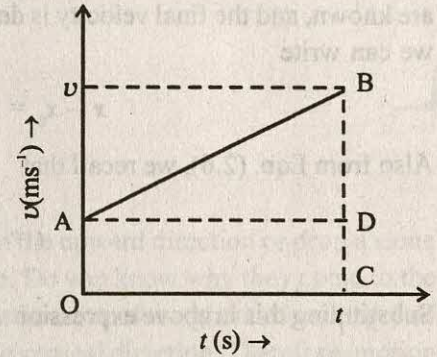


Fig. 2.10 : υ–t graph for uniformly accelerated motion

Suppose that at $t = 0$, $x_1 = x_0$; $\upsilon_1 = \upsilon_0$ and at $t = t$, $x_2 = x$; $\upsilon_2 = \upsilon$.

The distance travelled = area under $\upsilon - t$ graph

= Area of trapezium OABC

$$= \frac{1}{2}(\mathrm{CB} + \mathrm{OA}) \times \mathrm{OC}$$

$$x - x_0 = \tfrac{1}{2}(\upsilon + \upsilon_0)\, t$$

Since $\upsilon = \upsilon_0 + at$, we can write

$$x - x_0 = \tfrac{1}{2}(\upsilon_0 + at + \upsilon_0)t$$

$$= \upsilon_0 t + \tfrac{1}{2} at^2$$

or

$$x = x_0 + \upsilon_0 t + \tfrac{1}{2} at^2 \qquad (2.7)$$

Example 2.9 : A car A is travelling on a straight road with a uniform speed of 60 km h^{-1}. Car B is following it with uniform velocity of 70 km h^{-1}. When the distance between them is 2.5 km, the car B is given a decceleration of 20 km h^{-1}. At what distance and time will the car B catch up with car A?

Solution : Suppose that car B catches up with car A at a distance x after time t.

For car A, the distance travelled in t time, $x = 60 \times t$.

For car B, the distance travelled in t time is given by

$$x' = x_0 + \upsilon_0 t + \tfrac{1}{2} at^2$$

$$= 0 + 70 \times t + \tfrac{1}{2}(-20) \times t^2$$

$$x' = 70\,t - 10\,t^2$$

But the distance between two cars is

$$x' - x = 2.5$$

$$\therefore \quad (70\,t - 10\,t^2) - (60\,t) = 2.5$$

or

$$10\,t^2 - 10\,t + 2.5 = 0$$

It gives $t = \tfrac{1}{2}$ hour

$$\therefore \quad x = 70t - 10t^2$$

$$= 70 \times \tfrac{1}{2} - 10 \times (\tfrac{1}{2})^2$$

$$= 35 - 2.5 = 32.5 \text{ km.}$$

2.4.4 Third Equation of Uniformly Accelerated Motion

The third equation is used in a situation when the acceleration, position and initial velocity are known, and the final velocity is desired but the time *t* is not known. From Eqn. (2.7.), we can write

$$x - x_0 = \tfrac{1}{2}(v + v_0)\,t.$$

Also from Eqn. (2.6), we recall that

$$t = \frac{v - v_0}{a}$$

Substituting this in above expression we get

$$x - x_0 = \tfrac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right)$$

$$\Rightarrow \qquad 2a\,(x - x_0) = v^2 - v^0$$

$$\Rightarrow \qquad v^2 = v_0^2 + 2a\,(x - x_0) \qquad (2.8)$$

Thus, the three equations for constant acceleration are

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

and $$v^2 = v_0^2 + 2a\,(x - x_0)$$

Example 2.10 : A motorcyclist moves along a straight road with a constant acceleration of 4m s^{-2}. If initially she was at a position of 5m and had a velocity of 3m s^{-1}, calculate

(i) the position and velocity at time t = 2s, and

(ii) the position of the motorcyclist when its velocity is 5ms^{-1}.

Solution : We are given

$$x_0 = 5\text{m},\ v_0 = 3\text{m s}^{-1},\ a = 4\text{ ms}^{-2}.$$

(i) Using Eqn. (2.7)

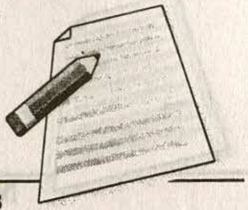
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$= 5 + 3 \times 2 + \tfrac{1}{2} \times 4 \times (2)^2 = 19\text{m}$$

From Eqn. (2.6)

$$v = v_0 + at$$
$$= 3 + 4 \times 2 = 11\text{ms}^{-1}$$

Velocity, $v = 11\text{ms}^{-1}$.

(ii) Using equation



$$v^2 = v_0^2 + 2a\,(x - x_0)$$

$$(5)^2 = (3)^2 + 2 \times 4 \times (x - 5)$$

$$\Rightarrow \qquad x = 7\text{m}$$

Hence position of the motor cyclist (x) = 7m.

2.5 Motion under gravity

You must have noted that when we throw a body in the upward direction or drop a stone from a certain height, they come down to the earth. Do you know why they come to the earth and what type of path they follow? It happens because of the gravitational force of the earth on them. The gravitational force acts in the vertical direction. Therefore, motion under gravity is along a straight line. It is a one dimensional motion. ***The free fall of a body towards the earth is one of the most common examples of motion with constant acceleration.*** In the absence of air resistance, it is found that all bodies, irrespective of their size or weight, fall with the same acceleration. Though the acceleration due to gravity varies with altitude, for small distances compared to the earth's radius, it may be taken constant throughout the fall. For our practical use, the effect of air resistance is neglected.

The acceleration of a freely falling body due to gravity is denoted by g. At or near the earth's surface, its magnitude is approximately 9.8 ms^{-2}. More precise values, and its variation with height and latitude will be discussed in detail in lesson 5 of this book.

Galileo Galilei (1564 – 1642)

He was born at Pisa in Italy in 1564. He enunciated the laws of falling bodies. He devised a telescope and used it for astronomical observations. His major works are : Dialogues about the Two great Systems of the World and Conversations concerning Two New Sciences. He supported the idea that the earth revolves around the sun.

Example 2.11 : A stone is dropped from a height of 50m and it falls freely. Calculate the (i) distance travelled in 2 s, (ii) velocity of the stone when it reaches the ground, and (iii) velocity at 3 s i.e., 3 s after the start.

Solution : Given

Height h = 50 m and Initial velocity v_0 = 0

Consider, initial position (y_0) to be zero and the origin at the starting point. Thus, the y-axis (vertical axis) below it will be negative. Since acceleration is downward in the negative y-direction, the value of $a = -g = -9.8\ \text{ms}^{-2}$.

(i) From Eqn. (2.7), we recall that

$$y = y_0 + v_0 t + \tfrac{1}{2} at^2$$

For the given data, we get

$$y = 0 + 0 - \tfrac{1}{2}gt^2 = -\tfrac{1}{2} \times 9.8 \times (2)^2$$

$$= -19.6\text{m}.$$

The negative sign shows that the distance is below the starting point in downward direction.

(ii) At the ground $y = -50$m,

Using equation (2.8),

$$\upsilon^2 = \upsilon_0^2 + 2a\,(y - y_0)$$

$$= 0 + 2\,(-9.8)\,(-50 - 0)$$

$$\upsilon = 9.9 \text{ ms}^{-1}$$

(iii) Using $\upsilon = \upsilon_0 + at$, at $t = 3$s, we get

$$\therefore \quad \upsilon = 0 + (-9.8) \times 3$$

$$\upsilon = -29.4 \text{ ms}^{-1}$$

This shows that the velocity of the stone at $t = 3$ s is 29.4 m s^{-1} and it is in downward direction.

Note : *It is important to mention here that in kinematic equations, we use certain sign convention according to which quantities directed upwards and rightwards are taken as positive and those downwards and leftward are taken as negative.*

Take a pause and solve the following questions.

Intext Questions 2.4

1. A body starting from rest covers a distance of 40 m in 4s with constant acceleration along a straight line. Compute its final velocity and the time required to cover half of the total distance.

   ....

2. A car moves along a straight road with constant aceleration of 5 ms^{-2}. Initially at 5m, its velocity was 3 ms^{-1} Compute its position and velocity at $t = 2$ s.

   ....

3. With what velocity should a body be thrown vertically upward so that it reaches a height of 25 m? For how long will it be in the air?

   ....

4. A ball is thrown upward in the air. Is its acceleration greater while it is being thrown or after it is thrown?

   ....

What You Have Learnt

- The ratio of the displacement of an object to the time interval is known as average velocity.
- The total distance travelled divided by the time taken is average speed.
- The rate of change of the relative position of an object with respect to another object is known as the relative velocity of that object with respect to the other.
- The change in the velocity in unit time is called acceleration.
- The position-time graph for a body at rest is a straight line parallel to the time axis.
- The position-time graph for a uniform motion is a straight line inclined to the time axis.
- A body covering equal distance in equal intervals of time, however small, is said to be in uniform motion.
- The velocity of a particle at any one instant of time or at any one point of its path is called its instantaneous velocity.
- The slope of the position-time graph gives the average velocity.
- The velocity-time graph for a body moving with constant acceleration is a straight line inclined to the time axis.
- The area under the velocity-time graph gives the displacement of the body.
- The average acceleration of the body can be computed by the slope of velocity-time graph.
- The motion of a body can be described by following three equations :

(i) $v = v_0 + at$

(ii) $x = x_0 + v_0 t + \frac{1}{2} at^2$

(iii) $v^2 = v_0^2 + 2a.(x - x_0)$

Terminal Exercise

1. Distinguish between average speed and average velocity.
2. A car C moving with a speed of 65 km h^{-1} on a straight road is ahead of motorcycle M moving with the speed of 80 km h^{-1} in the same direction. What is the velocity of M relative to A?
3. How long does a car take to travel 30m, if it accelerates from rest at a rate of 2.0 m s^2?
4. A motorcyclist covers half of the distance between two places at a speed of 30 km h^{-1} and the second half at the speed of 60 kmh^{-1}. Compute the average speed of the motorcycle.

5. A duck, flying directly south for the winter, flies with a constant velocity of 20 km h^{-1} to a distance of 25 km. How long does it take for the duck to fly this distance?

6. Bangalore is 1200km from New Delhi by air (straight line distance) and 1500 km by train. If it takes 2h by air and 20h by train, calculate the ratio of the average speeds.

7. A car accelerates along a straight road from rest to 50 kmh^{-1} in 5.0 s. What is the magnitude of its average acceleration?

8. A body with an initial velocity of 2.0 ms^{-1} is accelerated at 8.0 ms^{-2} for 3 seconds. (i) How far does the body travel during the period of acceleration? (ii) How far would the body travel if it were initially at rest?

9. A ball is released from rest from the top of a cliff. Taking the top of the cliff as the reference (zero) level and upwards as the positive direction, draw (i) the displacement-time graph, (ii) distance-time graph (iii) velocity-time graph, (iv) speed-time graph.

10. A ball thrown vertically upwards with a velocity v_0 from the top of the cliff of height h, falls to the beach below. Taking beach as the reference (zero) level, upward as the positive direction, draw the motion graphs. i.e., the graphs between (i) distance-time, (ii) velocity-time, (iii) displacement-time, (iv) speed - time graphs.

11. A body is thrown vertically upward, with a velocity of 10m/s. What will be the value of the velocity and acceleration of the body at the highest point?

12. Two objects of different masses, one of 10g and other of 100g are dropped from the same height. Will they reach the ground at the same time? Explain your answer.

13. What happens to the uniform motion of a body when it is given an acceleration at right angle to its motion?

14. What does the slope of velocity-time graph at any instant represent?

Answers to Intext Questions

2.1

1. Yes. When body returns to its initial postion its velocity is zero but speed is non-zero.

2. Average speed $= \dfrac{2+2}{\dfrac{2}{8}+\dfrac{2}{10}} = \dfrac{4}{9} \times 20 = 8.89$ km h^{-1}, average velocity = 0

3. Yes, two cars moving with same velocity in the same direction, will have zero relative velocity with respect to each other.

4. (a) 1 m s^{-1}

   (b) 2 m s^{-1}

2.2

1. See Fig. 2.2.

2. (i) A, (ii) B covers more distance, (iii) B, (iv) A, (v) When they are 3km from the

starting point of B.

3. In the uniform motion.

4. (a) is wrong, because the distance covered cannot decrease with time or become zero.

2.3

1. (i) (a) The body starts with a zero velocity.

(b) Motion of the body between start and 5th seconds is uniformly accelerated. It has been represented by the line OA.

$$a = \frac{15-0}{5-0} = 3 \text{ m s}^{-2}$$

(c) Motion of the body between 5th and 10th second is a uniform motion (represented by AB). $a = \frac{15-15}{15-5} = \frac{0}{10} = 0 \text{ m s}^{-2}$.

(d) Motion between 15th and 25th second is uniformly retarded. (represented by the line BC). $a = \frac{0-15}{25-15} = -1.5 \text{ m s}^{-2}$.

(ii) (a) Average speed $= \frac{\text{Distance covered}}{\text{time taken}} = \frac{\text{Area of OA BC}}{(25-0)}$

$$= \frac{\left(\frac{1}{2}\times 15\times 5\right) + (15\times 10) + \left(\frac{1}{2}\times 15\times 10\right)}{25} = \frac{525}{50} = 10.5 \text{ m s}^{-1}.$$

(b) Deccelerated Velocity decreases with time.

(c) Total distance covered $= \left(\frac{20\times 15}{2}\right)\text{m} + \left(\frac{10\times 7}{2}\right)\text{m} = 185 \text{ m}.$

$\therefore$ average speed $= \left(\frac{185}{22}\right)\text{ms}^{-1} = 8.4 \text{ ms}^{-1}.$

Total displacement $= \left(\frac{20\times 15}{2}\right)\text{m} - \left(\frac{10\times 7}{2}\right)\text{m} = 115 \text{ m}.$

$\therefore$ average velocity $= \frac{115}{22}\text{ms}^{-1} = 5.22 \text{ m s}^{-1}.$

2.4

1. Using $x = x_0 + \upsilon_0 t + ½ at^2$

$$40 = \frac{1}{2} \times a \times 16$$

$$\Rightarrow a = 5 \text{ ms}^{-2}$$

Next using $\upsilon^2 = \upsilon_0^2 + 2a(x - x_0)$

$\upsilon = 20 \text{ m s}^{-1}$,

$$20 = 0 + \frac{1}{2} \times 5 \times t^2 \Rightarrow t = 2\sqrt{2} \text{ s}$$

2. Using Eqn.(2.9), $x = 21$m, and using Eqn.(2.6), $\upsilon = 13 \text{ m s}^{-1}$.
3. At maximum height $\upsilon = 0$, using Eqn. (2.10), $\upsilon_0 = 7\sqrt{10} \text{ ms}^{-1} = 22.6 \text{ m s}^{-1}$.

   The body will be in the air for the twice of the time it takes to reach the maximum height.
4. The acceleration of the ball is greater while it is thrown.

Answers to Terminal Exercises

2. 15 km h^{-1}
3. 5.47 s
4. 40 ms^{-1}
5. 1.25 h
6. 8 : 1
7. 2.8 m s^{-2} (or 3000 km h^{-2})
8. (i) 42 m (ii) 36 m
11. O and 9.8 m s^{-2}.

3

LAWS OF MOTION

In the previous lesson you learnt to describe the motion of an object in terms of its displacement, velocity and acceleration. But an important question is : what makes an object to move? Or what causes a ball rolling along the ground to come to a stop? From our everyday experience we know that we need to push or pull an object if we wish to change its position in a room. Similarly, a football has to be kicked in order to send it over a large distance. A cricket ball has to be hit hard by a batter to send it across the boundary for a six. You will agree that muscular activity is involved in all these actions and its effect is quite visible.

There are, however, many situations where the cause behind an action is not visible. For example, what makes rain drops to fall to the ground? What makes the earth to go around the sun? In this lesson you will learn the basic laws of motion and discover that force causes motion. The concept of force developed in this lesson will be useful in different branches of physics. Newton showed that force and motion are intimately connected. The laws of motion are fundamental and enable us to understand everyday phenomena.

Objectives

After studying this lesson, you should be able to :

- *explain the significance of inertia;*
- *state Newton's laws of motion and illustrate them with examples;*
- *explain the law of conservation of momentum and illustrate it with examples;*
- *define coefficient of friction and distinguish between static friction, kinetic friction and rolling friction;*
- *suggest different methods of reducing friction and highlight the role of friction in every-day life; and*
- *analyse a given situation and apply Newton's laws of motion using free body diagrams.*

3.1 Concepts of Force and Inertia

We all know that stationary objects remain wherever they are placed. These objects cannot move on their own from one place to another place unless forced to change their state of rest. Similarly, an object moving with constant velocity has to be forced to change its state of motion. **The property of an object by which it resists a change in its state of rest or of uniform motion in a straight line is called inertia. Mass of a body is a measure of its inertia.**

In a way, inertia is a fantastic property. If it were not present, your books or classnotes could mingle with those of your younger brother or sister. Your wardrobe could move to your friend's house creating chaos in life. You must however recall that the state of rest or of uniform motion of an object are not absolute. In the previous lesson you have learnt that an object at rest with respect to one observer may appear to be in motion with respect to some other observer. Observations show that **the change in velocity of an object can only be brought, if a net force acts on it.**

You are very familiar with the term force. We use it in so many situations in our everyday life. We are exerting force when we are pulling, pushing, kicking, hitting etc. Though a force is not visible, its effect can be seen or experienced. Forces are known to have different kinds of effects :

(a) **They may change the shape and the size of an object.** A balloon changes shape depending on the magnitude of force acting on it.

(b) **Forces also influence the motion of an object.** A force can set an object into motion or it can bring a moving object to rest. A force can also change the direction or speed of motion.

(c) **Forces can rotate a body about an axis.** You will learn about it in lesson seven.

3.1.1 Force and Motion

Force is a vector quantity. For this reason, when several forces act on a body simultaneously, a net equivalent force can be calculated by vector addition, as discussed in lesson 1.

Motion of a body is characterised by its displacement, velocity etc. We come across many situations where the velocity of an object is either continuously increasing or decreasing. For example, in the case of a body falling freely, the velocity of the body increases continuously, till it hits the ground. Similarly, in the case of a ball rolling on a horizontal surface, the velocity of the ball decreases continuously and ultimately becomes zero.

From experience we know that a net non-zero force is required to change the state of a body. ***For a body in motion, the velocity will change depending on the direction of the force acting on it.*** If a net force acts on a body in motion, its velocity will increase in magnitude, if the direction of the force and velocity are same. If the direction of net force acting on the body is opposite to the direction of motion, the magnitude of velocity will decrease. However, if a net force acts on a body in a direction perpendicular to its velocity, the magnitude of velocity of the body remains constant (see Sec 4.3). Such a force changes only the direction of velocity of the body. We may therfore conclude that ***velocity of a body changes as long as a net force is acting on it.***

3.1.2 First Law of Motion

When we roll a marble on a smooth floor, it stops after some time. It is obvious that its velocity decreases and ultimately it becomes zero. However, if we want it to move continuously with the same velocity, a force will have to be constantly applied on it.

We also see that in order to move a trolley at constant velocity, it has to be continuously pushed or pulled. Is there any net force acting on the marble or trolley in the situations mentioned here?

Motion and Inertia

Galileo carried out experiments to prove that in the absence of any external force, a body would continue to be in its state of rest or of uniform motion in a straight line. He observed that a body is accelerated while moving down an inclined plane (Fig. 3.1 a) and is retarded while moving up an inclined plane (Fig. 3.1 b). He argued that if the plane is neither inclined upwards nor downwards (i.e. if it is a horizontal plane surface), the motion of the body will neither be accelerated not retarded. That is, on a horizontal plane surface, a body will move with a uniform speed/velocity (if there is no external force).

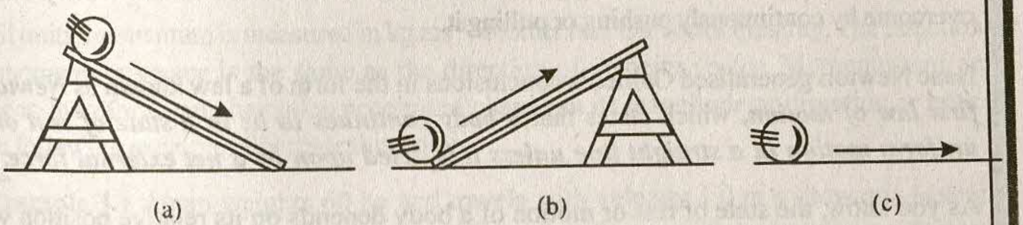


Fig. 3.1 : Motion of a body on inclined and horizontal planes

In another thought experiment, he considered two inclined planes facing each other, as shown in Fig. 3.2. The inclination of the plane PQ is same in all the three cases, whereas the inclination of the plane RS in Fig. 3.2 (a) is more than that in (b) and (c). The plane PQRS is very smooth and the ball is of marble. When the ball is allowed to roll down the plane PQ, it rises to nearly the same height on the face RS. As the inclination of the plane RS decreases, the balls moves a longer distance to rise to the same height on the inclined plane (Fig. 3.2b). When the plane RS becomes horizontal, the ball keeps moving to attain the same height as on the plane PQ, i.e. on a horizontal plane, the ball will keep moving if there is no friction between the plane and the ball.

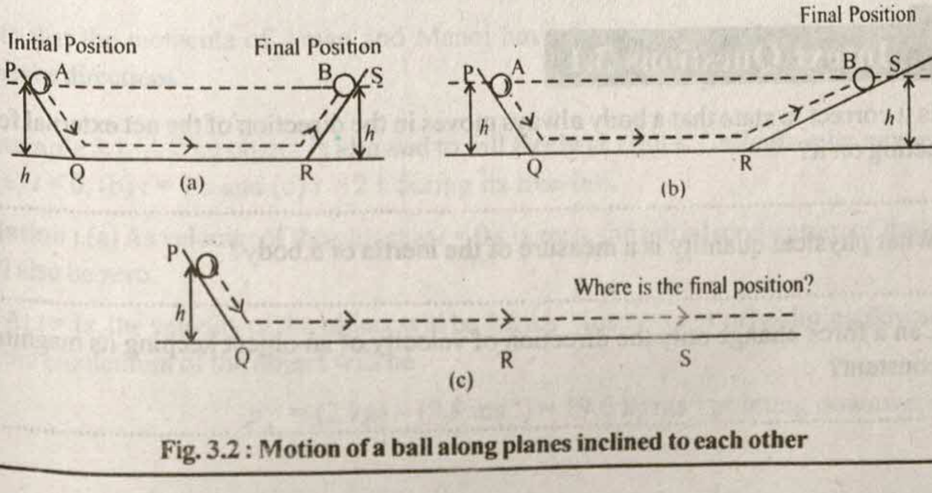


Fig. 3.2 : Motion of a ball along planes inclined to each other

Sir Issac Newton

(1642–1727)

Newton was born at Wollsthorpe in England in 1642. He studied at Trinity College, Cambridge and became the most profound scientist. The observation of an apple falling towards the ground helped him to formulate the basic law of gravitation. He enunciated the laws of motion and the law of gravitation. Newton was a genius and contributed significantly in all fields of science, including mathematics. His contributions are of a classical nature and form the basis of the modern science. He wrote his book **"Principia"** in Latin and his book on optics was written in English.

You may logically ask : Why is it necessary to apply a force continuously to the trolley to keep it moving uniformly? We know that a forward force on the cart is needed for balancing out the force of friction on the cart. That is, the force of friction on the trolley can be overcome by continuously pushing or pulling it.

Isaac Newton generalised Galileo's conclusions in the form of a law known as ***Newton's first law of motion,*** which states that ***a body continues to be in a state of rest or of uniform motion in a straight line unless it is acted upon by a net external force.***

As you know, the state of rest or motion of a body depends on its relative position with respect to an observer. A person in a running car is at rest with respect to another person in the same car. But the same person is in motion with respect to a person standing on the road..For this reason, it is necessary to record measurements of changes in position, velocity, acceleration and force with respect to a chosen frame of reference.

A reference frame relative to which a body in translatory motion has constant velocity, if no net external force acts on it, is known as an ***inertial frame of reference.*** This nomenclature follows from the property of inertia of bodies due to which they tend to preserve their state (of rest or of uniform linear motion). A reference frame fixed to the earth (for all practical purposes) is considered an inertial frame of reference.

Now you may like to take a break and answer the following questions.

Intext Questions 3.1

1. Is it correct to state that a body always moves in the direction of the net external force acting on it?

   ....

2. What physical quantity is a measure of the inertia of a body?

   ....

3. Can a force change only the direction of velocity of an object keeping its magnitude constant?

   ....

4. State the different types of changes which a force can bring in a body when applied on it.

....

3.2 Concept of Momentum

You must have seen that a fielder finds it difficult to stop a cricket ball moving with a large velocity although its mass is small. Similarly, it is difficult to stop a truck moving with a small velocity because its mass is large. These examples suggest that both, mass and velocity of a body, are important, when we study the effect of force on the motion of the body.

***The product of mass m of a body and its velocity* v *is called its linear momentum* p. Mathmatically, we write**

$$\mathbf{p} = m\boldsymbol{v}$$

In SI units, momentum is measured in kg ms^{-1}. Momentum is a vector quantity. The direction of momentum vector is the same as the direction of velocity vector. Momentum of an object, therefore, can change on account of change in its magnitude or direction or both. The following examples illustrate this point.

Example 3.1 Aman weights 60 kg and travels with velocity 1.0 m s^{-1} towards Manoj who weights 40 kg, and is moving with 1.5 m s^{-1} towards Aman. Calculate their momenta.

Solution : For Aman

$$\begin{aligned} \text{momentum} &= \text{mass} \times \text{velocity} \\ &= (60\text{ kg}) \times (1.0\text{ m s}^{-1}) \\ &= 60\text{ kgms}^{-1} \end{aligned}$$

For Manoj

$$\begin{aligned} \text{momentum} &= 40\text{ kg} \times (-1.5\text{ ms}^{-1}) \\ &= -60\text{ kgms}^{-1} \end{aligned}$$

Note that the momenta of Aman and Manoj have the same magnitude but they are in opposite directions.

Example 3.2 A 2 kg object is allowed to fall freely at $t = 0$ s. Callculate its momentum at (a) $t = 0$, (b) $t = 1$ s and (c) $t = 2$ s during its free-fall.

Solution : (a) As velocity of the object at $t = 0$ s is zero, the initial momentum of the object will also be zero.

(b) At $t = 1$s, the velocity of the object will be 9.8 ms^{-1} [use $v = v_0 + at$] pointing downward. So the momentum of the object will be

$$p_1 = (2\text{ kg}) \times (9.8\text{ ms}^{-1}) = 19.6\text{ kgms}^{-1} \text{ pointing downward.}$$

(c) At $t = 2$ s, the velocity of the object will be 19.6 m s^{-1} pointing downward. So the momentum of the object will now be

$$p_2 = (2\text{ kg}) \times (19.6\text{ ms}^{-1}) = 39.2\text{ kgms}^{-1}\text{ pointing downward.}$$

Thus, we see that the momentum of a freely-falling body increases continuously in magnitude and points in the same direction. Now think what causes the momentum of a freely-falling body to change in magnitude?

Example 3.3 A rubber ball of mass 0.2 kg strikes a rigid wall with a speed of 10 ms^{-1} and rebounds along the original path with the same speed. Calculate the change in momentum of the ball.

Solution : Here the momentum of the ball has the same magnitude before and after the impact but there is a reversal in its direction. In each case the magnitude of momentum is (0.2 kg)×(10 ms^{-1}) i.e. 2 kgms^{-1}.

If we choose initial momentum vector to be along $+x$ axis, the final momentum vector will be along $-x$ axis. So $p_i = 2$ kgms^{-1}, $p_f = -2$ kgms^{-1}. Therefore, the change in momentum of the ball, $p_f - p_i = (-2\text{ kg ms}^{-1}) - (2\text{ kg ms}^{-1}) = -4\text{ kg ms}^{-1}$.

Here negative sign shows that the momentum of the ball changes by 4 kgms^{-1} in the direction of $-x$ axis. What causes this change in momentum of the ball?

In actual practice, a rubber ball rebounds from a rigid wall with a speed less than its speed before the impact. In such a case also, the magnitude of the momentum will change.

3.3 Second Law of Motion

You now know that a body moving at constant velocity will have constant momentum. Newton's first law of motion suggests that no net external force acts on such a body.

In Example 3.2 we have seen that the momentum of a ball falling freely under gravity increases with time. Since such a body falls under the action of gravitational force acting on it, there appears to be a connection between change in momentum of an object, net force acting on it and the time for which it is acting. ***Newton's second law of motion*** gives a quantitative relation between these three physical quantities. It states that ***the rate of change of momentum of a body is directly proportional to the net force acting on the body. Change in momentum of the body takes place in the direction of net external force acting on the body.***

This means that if $\Delta\mathbf{p}$ is the change in momentum of a body in time Δt due to a net external force $\mathbf{F}$, we can write

$$\mathbf{F} \propto \frac{\Delta\mathbf{p}}{\Delta t}$$

or

$$\mathbf{F} = k\frac{\Delta\mathbf{p}}{\Delta t}$$

where k is constant of proportionality.

By expressing momentum as a product of mass and velocity, we can rewrite this result as

$$\mathbf{F} = k\,m\left(\frac{\Delta v}{\Delta t}\right)$$

$$\mathbf{F} = k\,m\,\mathbf{a} \qquad \left(\text{as } \frac{\Delta v}{\Delta t} = \mathbf{a}\right) \tag{3.1}$$

The value of the constant k depends upon the units of m and $\mathbf{a}$. If these units are chosen such that when the magnitude of $m = 1$ unit and $a = 1$ unit, the magnitude of $\mathbf{F}$ is also be 1 unit. Then, we can write

$$1 = k\,.\,1.\,1$$

i.e.,
$$k = 1$$

Using this result in Eqn. (3.1), we get

$$\mathbf{F} = m\,\mathbf{a} \tag{3.2}$$

In SI units, $m = 1$ kg, $a = 1$ m s^{-2}. Then magnitude of external force

$$\begin{aligned} F &= 1\text{ kg} \times 1\text{ ms}^{-2} = 1\text{ kg ms}^{-2} \\ &= 1\text{ unit of force} \end{aligned} \tag{3.3}$$

This unit of force (i.e., 1 kg m s^{-2}) is called one *newton.*

Note that the second law of motion gives us a unit for measuring force. The SI unit of force i.e., a newton may thus, be defined as the force which will produce an acceleration of 1 ms^{-2} in a mass of 1 kg.

Example 3.3 A ball of mass 0.4 kg starts rolling on the ground at 20 ms^{-1} and comes to a stop after 10s. Calculate the force which stops the ball, assuming it to be constant in magnitude throughout.

Solution : Given $m = 0.4$ kg, initial velocity $u = 20$ ms^{-1}, final velocity $v = 0$ m s^{-1} and $t = 10$s. So

$$|\mathbf{F}| = m|\mathbf{a}| = \frac{m(v-u)}{t} = \frac{0.4\text{ kg}\,(-20\text{ ms}^{-1})}{10\text{ s}}$$

$$= -0.8\text{ kg m s}^{-2} = -0.8\text{ N}$$

Here negative sign shows that force on the ball is in a direction opposite to that of its motion.

Example 3.4 A constant force of magnitude 50 N is applied to a body of 10 kg moving initially with a speed of 10 m s^{-1}. How long will it take the body to stop if the force acts in a direction opposite to its motion.

Solution : Given $m = 10$ kg, $F = -50$ N, $\upsilon_0 = 10$ ms^{-1} and $\upsilon = 0$. We have to calculate t. Since

$$F = ma$$

we can write

$$F = m\left(\frac{\upsilon - \upsilon_0}{t}\right)$$

$$\therefore \qquad -50 \text{ N} = 10 \text{ kg}\left(\frac{0 - 10 \text{ m s}^{-1}}{t}\right)$$

or $$t = \frac{-100 \text{ kgms}^{-1}}{-50 \text{ N}} = \frac{100 \text{ kg ms}^{-1}}{50 \text{ kg m s}^{-2}} = 2 \text{ s.}$$

It is important to note here that Newton's second law of motion, as stated here is applicable to bodies having constant mass. Will this law hold for bodies whose mass changes with time, as in a rocket?

Intext Questions 3.2

1. Two objects of different masses have the same momentum. Which of them is moving faster?

   ....

2. A boy throws up a ball with a velocity υ_0. If the ball returns to the thrower with the same velocity, will there be any change in

   (a) momentum of the ball?..

   (b) magnitude of the momentum of the ball?..

3. When a ball falls from a height, its momentum increases. What causes increase in its momentum?

   ....

4. In which case will there be larger change in momentum of the object?

   (a) A 150 N force acts for 0.1 s on a 2 kg object initially at rest.

   (b) A 150 N force acts for 0.2 s on a 2 kg. object initially at rest.

   ....

5. An object is moving at a constant speed in a circular path. Does the object have constant momentum? Give reason for your answer.

   ....

3.4 Forces in Pairs

It is the gravitational pull of the earth, which allows an object to accelerate towards the earth. Does the object also pull the earth? Similarly when we push an almirah, does the almirah also push us? If so, why don't we move in the direction of that force? These situations compel us to ask whether a single force such as a push or a pull exists? It has been observed that actions of two bodies on each other are always mutual. Here, by action and reaction we mean 'forces of interaction'. So, whenever two bodies interact, they exert force on each other. One of them is called 'action' and the other is called 'reaction'. Thus, we can say that forces always exist in pairs.

3.4.1 Third Law of Motion

On the basis of his study of interactions between bodies, Newton formulated ***third law of motion: To every action, there is an equal and opposite reaction.***

Here by 'action' and 'reaction' we mean force. Thus, when a book placed on a table exerts some force on the table, the latter, also exerts a force of equal magnitude on the book in the upward direction,.as shown in Fig. 3.3. Do the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown here cancel out? It is important to note that $\mathbf{F}_1$ and $\mathbf{F}_2$ are acting on different bodies and therefore, they do not cancel out.

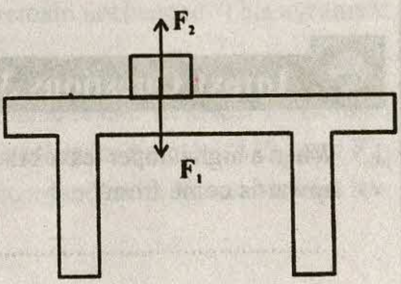


Fig 3.3 : A book placed on a table exerts a force F_1 (equal to its weight mg) on the table, while the table exerts a force F_2 on the book.

The action and reaction in a given situation appear as a pair of forces. Any one of them cannot exist without the other.

If one goes by the literal meaning of words, reaction always follows an action, whereas action and reaction introduced in Newton's third law exist simultaneously. For this reason, it is better to state Newton's third law as ***when two objects interact, the force exerted by one object on the other is equal in magnitude and opposite in direction to the force exerted by the latter object on the former.***

Vectorially, if $\mathbf{F}_{12}$ is the force which object 1 experiences due to object 2 and $\mathbf{F}_{21}$ is the force which object 2 experiences due to object 1, then according to Newton's third law of motion, we can write

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \qquad (3.4)$$

3.4.2 Impulse

The effect of force applied for a short duration is called impulse. Impulse is defined as the product of force (F) and the time duration (Δt) for which the force is applied.

i.e.,

$$\text{Impulse} = \mathbf{F}.\Delta t$$

If the initial and final velocities of body acted upon by a force **F** are **u** and **v** respectively then we can write

$$\text{Impulse} = \frac{m\boldsymbol{v} - m\,\mathbf{u}}{\Delta t}\,.\,\Delta t$$
$$= m\boldsymbol{v} - m\,u$$
$$= \mathbf{p}_f - \mathbf{p}_i$$
$$= \Delta\mathbf{p}$$

That is, impulse is equal to change in linear momentum.

Impulse in a vector quantity and its SI unit is kgms^{-1} (or N s).

Intext Questions 3.3

1. When a high jumper leaves the ground, where does the force which throws the jumper upwards come from?

   ....

2. Identify the action - reaction forces in each of the following situations:

   (a) A man kicks a football..

   (b) Earth pulls the moon...

   (c) A ball hits a wall..

3. "A person exerts a large force on an almirah to push it forward but he is not pushed backward because the almirah exerts a small force on him". Is the argument given here correct? Explain.

   ....

3.5 Conservation of Momentum

It has been experimentally shown that if two bodies interact, the vector sum of their momenta remains unchanged, provided the force of mutual interaction is the only force acting on them. The same has been found to be true for more than two bodies interacting with each other. Generally, a number of bodies interacting with each other are said to be forming a system. If the bodies in a system do not interact with bodies outside the system, the system is said to be a closed system or an isolated system. ***In an isolated system, the vector sum of the momenta of bodies remains constant.*** This is called the ***law of conservation of momentum.***

Here, it follows that it is the total momentum of the bodies in an isolated system remains unchanged but the momentum of individual bodies may change, in magnitude alone or direction alone or both. You may now logically ask : What causes the momentum of individual bodies in an isolated system to change? It is due to mutual interactions and their strengths.

Conservation of linear momentum is applicable in a wide range of phenomena such as collisions, explosions, nuclear reactions, radioactive decay etc.

3.5.1 Conservation of Momentum as a Consequence of Newton's Laws

According to Newton's second law of motion, Eqn. (3.1), the change in momentum $\Delta \mathbf{p}$ of a body, when a force **F** acts on it for time Δt, is

$$\Delta \mathbf{p} = \mathbf{F}\,\Delta t$$

This result implies that if no force acts on the body, the change in momentum of the body will be zero. That is, the momentum of the body will remain unchanged. This agrument can be extended to a system of bodies as well.

Newton's third law can also be used to arrive at the same result. Consider an isolated system of two bodies A and B which interact with each other for time Δt. If $\mathbf{F}_{AB}$ and $\mathbf{F}_{BA}$ are the forces which they exert on each other, then in accordance with Newton's third law

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

or $$\frac{\Delta \mathbf{p}_A}{\Delta t} = -\frac{\Delta \mathbf{p}_B}{\Delta t}$$

or $$\Delta \mathbf{p}_A + \Delta \mathbf{p}_B = 0 \text{ or}$$

or $$\Delta \mathbf{p}_{total} = 0$$

or $$\mathbf{p}_{total} = \text{constant}$$

That is, there is no change in the momentum of the system. In other words, the momentum of the system is conserved.

3.5.2 A Few Illustrations of Conservation of Momentum

a) Recoil of a gun : When a bullet is fired from a gun, the gun recoils. The velocity υ_2 of the recoil of the gun can be found by using the law of conservation of momentum. Let *m* be the mass of the bullet being fired from a gun of mass M. If υ_1 is the velocity of the bullet, then momentum will be said to be conserved if the velocity υ_2 of the gun is given by

$$m\boldsymbol{\upsilon}_1 + M\boldsymbol{\upsilon}_2 = 0$$

or $$m\boldsymbol{\upsilon}_1 = -M\boldsymbol{\upsilon}_2$$

or $$\boldsymbol{\upsilon}_2 = -\frac{m}{M}\boldsymbol{\upsilon}_1 \tag{3.5}$$

Here, negative sign shows that $\boldsymbol{\upsilon}_2$ is in a direction opposite to $\boldsymbol{\upsilon}_1$. Since $m \ll M$, the recoil velocity of the gun will be considerably smaller than the velocity of the bullet.

b) Collision : In a collision, we may regard the colliding bodies as forming a system. In the absence of any external force on the colliding bodies, such as the force of friction, the system can be considered to be an isolated system. The forces of interaction between the colliding bodies will not change the total momentum of the colliding bodies.

Collision of the striker with a coin of carrom or collision between the billiared balls may be quite instructive for the study of collision between elastic bodies.

Example 3.5 : Two trolleys, each of mass m, coupled together are moving with initial velocity v. They collide with three identical stationary trolleys coupled together and continue moving in the same direction. What will be the velocity of the trolleys after the impact?

Solution : Let v' be the velocity of the trolleys after the impact.

Momentum before collision = $2\,mv$

Momentum after collision = $5\,mv'$

In accordance with the law of conservation of momentum, we can write

$$2mv = 5\,mv'$$

or

$$v' = \frac{2}{5}v$$

c) Explosion of a bomb : A bomb explodes into fragments with the release of huge energy. Consider a bomb at rest initially which explodes into two fragments A and B. As the momentum of the bomb was zero before the explosion, the total momentum of the two fragments formed will also be zero after the explosion. For this reason, the two fragments will fly off in opposite directions. If the masses of the two fragments are equal, the velocities of the two fragments will also be equal in magnitude.

d) Rocket propulsion : Flight of a rocket is an important practical application of conservation of momentum. A rocket consists of a shell with a fuel tank, which can be considered as one body. The shell is provided with a nozzle through which high pressure gases are made to escape. On firing the rocket, the combustion of the fuel produces gases at very high pressure and temperature. Due to their high pressure, these gases escape from the nozzle at a high velocity and provide thrust to the rocket to go upward due to the conservation of momentum of the system. If M is the mass of the rocket and m is the mass of gas escaping per second with a velocity $\mathbf{v}$, the change in momentum of the gas in t second = $m\,vt$.

If the increase in velocity of the rocket in t second is V, the increase in its momentum = MV. According to the principle of conservation of momentum,

$$mvt + \mathrm{MV} = 0$$

or

$$\frac{\mathbf{V}}{t} = \mathbf{a} = -\frac{mv}{M}$$

i.e., the rocket moves with an acceleration

$$\mathbf{a} = -\frac{mv}{M}$$

3.6 Friction

You may have noticed that when a batter hits a ball to make it roll along the ground, the ball does not continue to move forever. It comes to rest after travelling some distance. Thus, the momentum of the ball, which was imparted to it during initial push, tends to be zero. We know that some force acting on the ball is responsible for this change in its momentum. Such a force, called the ***frictional force***, exists whenever bodies in contact tend to move with respect to each other. It is the force of friction which has to be overcome when we push or pull a body horizontally along the floor to change its position.

Force of friction is a contact force and always acts along the surfaces in a direction opposite to that of the motion of the body. It is commonly known that friction is caused by roughness of the surfaces in contact. For this reason deliberate attempts are made to make the surfaces rough or smooth depending upon the requirement.

Friction opposes the motion of objects, causes wear and tear and is responsible for loss of mechanical energy. But then, it is only due to friction that we are able to walk, drive vehicles and stop moving vehicles. Friction thus plays a dual role in our lives. It is therefore said that friction is a necessary evil.

3.6.1 Static and Kinetic Friction

We all know that certain minimum force is required to move an object over a surface. To illustrate this point, let us consider a block resting on some horizontal surface, as shown in Fig.3.4. Let some external force $\mathbf{F}_{ext}$ be applied on the block. Initially the block does not move. This is possible only if some other force is acting on the block. The force is called the force of ***static friction*** and is represented by symbol f_s. As $\mathbf{F}_{ext}$ is increased, f_s also increases and remains equal to $\mathbf{F}_{ext}$ in magnitude until it reaches a critical value $f_s^{(max)}$. When $\mathbf{F}_{ext}$ is increased further, the block starts to slide and is then subject to ***kinetic friction.*** It is common experience that the force needed to set an object in motion is larger than the force needed to keep it moving at constant velocity. ***For this reason, the maximum value of static friction f_s between a pair of surfaces in contact will be larger than the force of kinetic friction f_k between them.*** Fig. 3.5 shows the variation of the force of friction with the external force.

For a given pair of surfaces in contact, you may like to know the ***factors*** on which $f_s^{(max)}$ and f_k depend? It is an experimental fact that $f_s^{(max)}$ is directly proportional to the normal force $\mathbf{F}_N$. i.e.

$$f_s^{(max)} \alpha \mathbf{F}_N \quad \text{or} \quad f_s^{(max)} = \mu_s \mathbf{F}_N \qquad (3.6)$$

where μ_s is called the ***coefficient of static friction.*** The normal force $\mathbf{F}_N$ of the surface on the block can be found by knowing the force with which the block presses the surface. Refer to Fig. 3.4. The normal force $\mathbf{F}_N$ on the block will be *mg,* where *m* is mass of the block.

Since $f_s = \mathbf{F}_{ext}$ for $f_s \le f_s^{max}$, we can write

$f_s \le \mu_s \mathbf{F}_N$.

It has also been experimentally found that ***maximum force of static friction between a pair of surfaces is independent of the area of contact.***

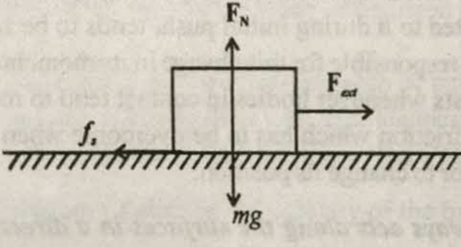


Fig. 3.4 : Forces acting on the block

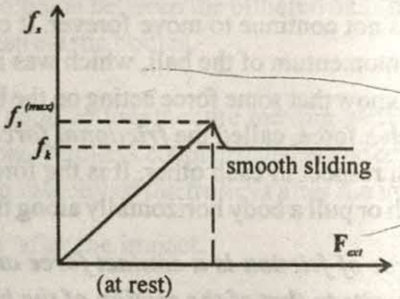


Fig. 3.5 : Variation of force of friction with external force

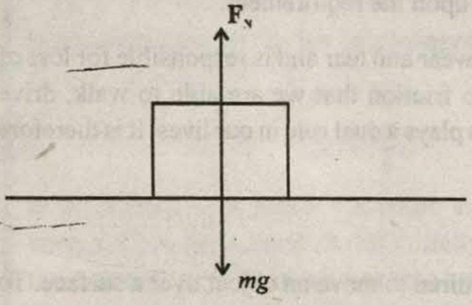


Fig. 3.6 : Normal force on the block

Similarly, we can write

$$f_k = \mu_k F_N$$

where μ_k is the ***coefficient of kinetic friction***. In general, $\mu_s > \mu_k$. Moreover, coefficients μ_s and μ_k are not really constants for any pair of surfaces such as wood on wood or rubber on concrete, etc. ***Values of μ_s and μ_k for a given pair of materials depend on the roughness of surfaces, there cleanliness, temperature, humidity etc.***

Example 3.6 A 2 kg block is resting on a horizontal surface. The coefficient of static friction between the surfaces in contact is 0.25. Calculate the maximum magnitude of force of static friction between the surfaces in contact.

Solution :

Here $m = 2$ kg and $\mu_s = 0.25$. From Eqn. (3.6), we recall that

$$\begin{aligned} f_s^{(max)} &= \mu_s F_N \\ &= \mu_s\, mg \\ &= (0.25)\ (2\ \text{kg})\ (9.8\ \text{ms}^{-2}) \\ &= 4.9\ \text{N}. \end{aligned}$$

Example 3.7 A 5 kg block is resting on a horizontal surface for which $\mu_k = 0.1$. What will be the acceleration of the block if it is pulled by a 10 N force acting on it in the horizontal direction?

Solution :

As $f_k = \mu_k F_N$ and $F_N = mg$, we can write

$$f_k = \mu_k mg$$

$$= (0.1)\,(5\text{ kg})\,(9.8\text{ ms}^{-2})$$

$$= 4.9\text{ kgms}^{-2} = 4.9\text{ N}$$

Net force on the block $= F_{ext} - f_k$

$$= 10\text{ N} - 4.9\text{ N}$$

$$= 5.1\text{ N}$$

Hence,

$$\text{acceleration} = a = \frac{F_{net}}{m} = \frac{5.1\text{ N}}{5\text{kg}} = 1.02\text{ ms}^{-2}$$

So the block will have an acceleration of 1.02 ms^{-2} in the direction of externally applied force.

3.6.2 Rolling Friction

It is a common experience that pushing or pulling objects such as carts on wheels is much easier. The motion of a wheel is different from sliding motion. It is a rolling motion. The friction in the case of rolling motion is known as ***rolling friction***. For the same normal force, rolling friction is much smaller than sliding friction. For example, when steel wheels roll over steel rails, rolling friction is about 1/100th of the sliding friction between steel and steel. Typical values for coefficient of rolling friction μ_r are 0.006 for steel on steel and 0.02 – 0.04 for rubber on concrete.

We would now like you to do a simple activity :

Activity 3.1

Place a heavy book or a pile of books on a table and try to push them with your fingers. Next put three or more pencils below the books and now push them again. In which case do you need less force? What do you conclude from your experience?

3.6.3 Methods of Reducing Friction

Wheel is considered to be greatest invention of mankind for the simple reason that rolling is much easier than sliding. Because of this, ball bearings are used in machines to reduce friction. In a ball-bearing, steel balls are placed between two co-axial cylinders, as shown in Fig.3.6. Generally one of the two cylinders is allowed to turn with respect to the other. Here the rotation of the balls is almost frictionless. Ball-bearings find application in almost all types of vehicles and in electric motors such as electric fans etc.

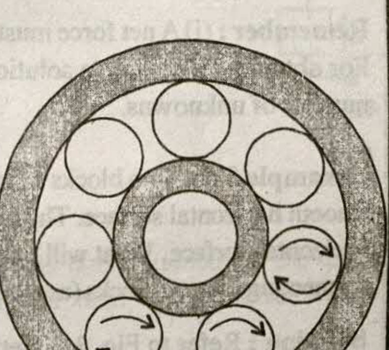


Fig. 3.6 : Balls in the ball-bearing

Use of ***lubricants*** such as grease or oil between the surfaces in contact reduces friction considerably. In heavy machines, oil is made to flow over moving parts. It reduces frictional force between moving parts and also prevents them from getting overheated. In fact, the presence of lubricants changes the nature of friction from dry friction to fluid friction, which is considerably smaller than the former.

Flow of compressed and purified air between the surfaces in contact also reduces friction. It also prevents dust and dirt from getting collected on the moving parts.

Fluid Friction

Bodies moving on or through a liquid or gas also face friction. Shooting stars (meteors) shine because of the heat generated by air-friction. Contrary to solid friction, fluid friction depends upon the shape of the bodies. This is why fishes have a special shape and fast moving aeroplanes and vehicles are also given a fish-like shape, called a stream-line shape. Fluid friction increases rapidly with increase in speed. If a car is run at a high speed, more fuel will have to be burnt to overcome the increased fluid (air) friction. Car manufactures advise us to drive at a speed of 40-45 km h^{-1} for maximum efficiency.

3.7 The Free Body Diagram Technique

Application of Newton's laws to solve problems in mechanics becomes easier by use of the **free body diagram technique**. A diagram which shows all the forces acting on a body in a given situation is called a free body diagram (FBD). The procedure to draw a free body diagram, is described below :

1. Draw a simple, neat diagram of the system as per the given description.
2. Isolate the ***object of interest.*** This object will be called the ***Free Body*** now.
3. Consider all ***external forces*** acting on the free body and mark them by arrows touching the free body with their line of action clearly represented.
4. Now apply Newton's second law $\Sigma \mathbf{F} = m\,\mathbf{a}$

(or $\Sigma F_x = m\,a_x$ and $\Sigma F_y = m\,a_y$)

Remember : (i) A net force must be acting on the object along the direction of motion. (ii) For obtaining a complete solution, you must have as many independent equations as the number of unknowns.

Example 3.8 : Two blocks of masses m_1 and m_2 are connected by a string and placed on a smooth horizontal surface. The block of mass m_2 is pulled by a force **F** acting parallel to the horizontal surface. What will be the acceleration of the blocks and the tension in the string connecting the two blocks (assuming it to be horizontal)?

Solution : Refer to Fig. 3.7. Let **a** be the acceleration of the blocks in the direction of **F** and let the tension in the string be **T**. On applying $\Sigma \mathbf{F} = m\mathbf{a}$ in the component form to the free body diagram of system of two bodies of masses m_1 and m_2, we get

$$N - (m_1 + m_2)\, g = 0$$

and $F = (m_1 + m_2)a$

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2}$$

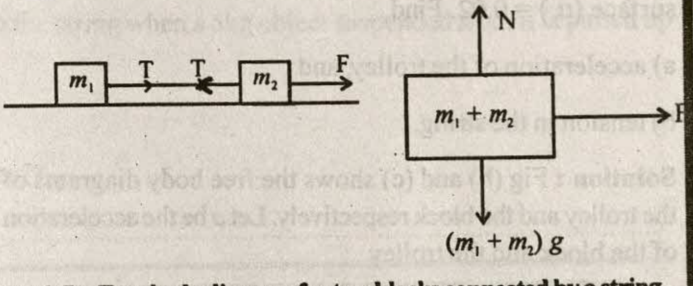


Fig 3.7: Free body diagram for two blocks connected by a string

On applying ΣF = *ma* in the component form to the free body diagram of m_1 we get

$$N_1 - m_1 g = 0 \quad \text{and} \quad T = m_1 a$$

$$\Rightarrow \qquad T = m_1\left(\frac{F}{m_1 + m_2}\right)$$

$$\text{or} \qquad T = \left(\frac{m_1}{m_1 + m_2}\right).F$$

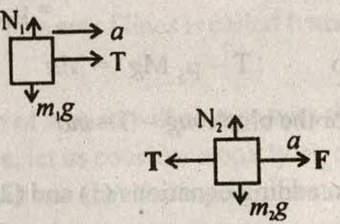


Fig 3.8

Apply ΣF = *ma* once again to the free body diagram of m_2 and see whether you get the same expressions for *a* and T.

Example 3.9 : Two masses m_1 and m_2 $(m_1 > m_2)$ are connected at the two ends of a light inextensible string that passes over a light frictionless fixed pulley. Find the acceleration of the masses and the tension in the string connecting them when the masses are released.

Solution : Let *a* be acceleration of mass m_1 downward. The acceleration of mass m_2 will also be *a* only but upward. (Why?). Let T be the tension in the string connecting the two masses.

On applying ΣF = *ma* to m_1 and m_2 we get

$$m_1 g - T = m_1 a$$

$$T - m_2 g = m_2 a$$

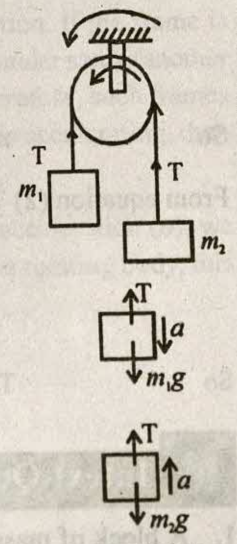


Fig. 3.9

On solving equations (1) and (2) for *a* and T we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right).g \qquad T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)a$$

At this stage you can check the prediction of the results thus obtained for the extreme values of the variables (i.e. m_1 and m_2). Either take $m_1 = m_2$ or $m_1 >> m_2$ and see whether *a* and T take values as expected.

Example 3.10 : A trolley of mass $M = 10$ kg is connected to a block of mass $m = 2$ kg with the help of massless inextensible string passing over a light frictionless pulley as shown in Fig. 3.10 (a). The coefficient of kinetic friction between the trolley and the

Notes

surface (μ_k) = 0.02. Find,

a) acceleration of the trolley, and

b) tension in the string.

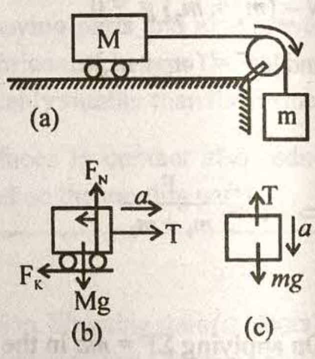


Fig. 3.10

Solution : Fig (b) and (c) shows the free body diagrams of the trolley and the block respectively. Let *a* be the acceleration of the block and the trolley.

For the trolley, $F_N = Mg$ and

$$T - f_k = Ma \text{ where } f_k = \mu_k F_N$$

$$= \mu_k Mg$$

So $$T - \mu_k Mg = Ma \qquad ...(1)$$

For the block $mg - T = ma \qquad ...(2)$

On adding equations (1) and (2) we get $mg - \mu_k Mg = (M + m)\,a$

or $$a = \frac{mg - \mu_k Mg}{M + m} = \frac{(2\text{ kg})(9.8\text{ms}^{-2}) - (0.02)(10\text{ kg})(9.8\text{ ms}^{-2})}{(10\text{ kg} + 2\text{ kg})}$$

$$= \frac{19.6\text{ kg ms}^{-2} - 1.96\text{ kg ms}^{-2}}{12\text{ kg}} = 1.47\text{ ms}^{-2}$$

So $$a = 1.47\text{ ms}^{-2}$$

From equation (2) $T = mg - ma = m\,(g - a)$

$$= 2\text{ kg }(9.8\text{ ms}^{-2} - 1.47\text{ ms}^{-2})$$

$$= 2\text{ kg }(8.33\text{ ms}^{-2})$$

So $$T = 16.66\text{ N}$$

Intext Questions 3.4

1. A block of mass *m* is held on a rough inclined surface of inclination θ. Show in a diagram, various forces acting on the block.

....

2. A force of 100 N acts on two blocks A and B of masses 2 kg and 3 kg respectively, placed in contact on a smooth horizontal surface as shown. What is the magnitude of force which block A exerts on block B?

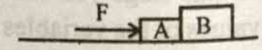


Fig. 3.11

....

3. What will be the tension in the string when a 5kg object suspended from it is pulled up with

(a) a velocity of $2 ms^{-1}$?

(b) an acceleration of $2 ms^{-2}$?

....

3.8 Elementary Ideas of Inertial and Non Inertial Frames

To study motion in one dimension (i.e. in a straight line) a reference point (origin) is enough. But, when it comes to motions in two and three dimensions, we have to use a set of reference lines to specify the position of a point in space. This set of lines is called **frame of reference.**

Every motion is described by an observer. The description of motion will change with the change in the state of motion of the observer. For example, let us consider a box lying on a railway platform. A person standing on the platform will say that the box is at rest. A person in a train moving with a uniform velocity υ will say that the box is moving with velocity $-\upsilon$. But, what will be the description of the box by a person in a train having acceleration (a). He/she will find that the box is moving with an acceleration ($- a$). Obviously, the first law of motion is failing for this observer.

Thus a frame of reference is fixed with the observer to describe motion. If the frame is stationary or moving with a constant velocity with respect to the object under study (another frame of reference), then in this frame law of inertia holds good. Therefore, such frames are called inertial frames. On the other hand, if the observer's frame is accelerating, then we call it non-inertial frame.

For the motion of a body of mass m in a non-inertial frame, having acceleration (a), we may apply second law of motion by involving a psuedo force $m\,a$. In a rotating body, this force is called centrifugal force.

Intext Questions 3.5

1. A glass half filled with water is kept on a horizontal table in a train. Will the free surface of water remain horizontal as the train starts?

....

2. When a car is driven too fast around a curve it skids outwards. How would a passenger sitting inside explain the car's motion? How would an observer standing on a road explain the event?

....

3. A tiny particle of mass 6×10^{-10} kg is in a water suspension in a centrifuge which is being rotated at an angular speed of $2\pi \times 10^{3}$ rad s^{-1}. The particle is at a distance of 4 cm from the axis of rotation. Calculate the net centrifugal force acting on the

particle.

....

4. What must the angular speed of the rotation of earth so that the centrifugal force makes objects fly off its surface? Take $g = 10 \text{ m s}^{-2}$.

....

5. In the reference frame attached to a freely falling body of mass 2 kg, what is the magnitude and direction of inertial force on the body?

....

What You Have Learnt

- The ***inertia*** of a body is its tendency to resist any change in its state of rest or uniform motion.
- ***Newton's first law*** states that a body remains in a state of rest or of uniform motion in a straight line as long as net external force acting on it is zero.
- For a single particle of mass m moving with velocity $\boldsymbol{v}$ we define a vector quantity $\mathbf{p}$ called the linear momentum as $\mathbf{p} = m\,\boldsymbol{v}$.
- ***Newton's second law*** states that the time rate of change of momentum of a body is proportional to the resultant force acting on the body.
- According to Newton's second law, acceleration produced in a body of constant mass is directly proportional to net external force acting on the body : $\mathbf{F} = m\,\mathbf{a}$.
- ***Newton's third law*** states that if two bodies A and B interact with each other, then the force which body A exerts on body B will be equal and opposite to the force which body B exerts on body A.
- According to the law of conservation of momentum, if no net external force acts on a system of particles, the total momentum of the system will remain constant regardless of the nature of forces between them.
- Frictional force is the force which acts on a body when it attempts to slide, or roll along a surface. The force of friction is always parallel to the surfaces in contact and opposite to the direction of motion of the object.
- The maximum force of static friction $f_s^{(\max)}$ between a body and a surface is proportional to the normal force $\mathbf{F}_N$ acting on the body. This maximum force occurs when the body is on the verge of sliding.
- For a body sliding on some surface, the magnitude of the force of kinetic friction f_k is given by $f_k = \mu_k \mathbf{F}_N$ where μ_k is the coefficient of kinetic friction for the surfaces in contact.
- Use of rollers and ball-bearings reduces friction and associated energy losses considerably as rolling friction is much smaller than kinetic friction.

- Newton's laws of motion are applicable only in an inertial frame of reference. An inertial frame is one in which an isolated object has zero acceleration.
- For an object to be in static equilibrium, the vector sum of all the forces acting on it must be zero. This is a necessary and sufficient conditions for point objects only.

Terminal Exercise

1. Which of the following will always be in the direction of net external force acting on the body?

   (a) displacement (b) velocity
   (c) acceleration (d) Change is momentum.

2. When a constant net external force acts on an object, which of the following may not change?

   (a) position (b) speed (c) velocity (d) acceleration

   Justify your answer with an example each.

3. A 0.5 kg ball is dropped from such a height that it takes 4s to reach the ground. Calculate the change in momentum of the ball.

4. In which case will there be larger change in momentum of a 2 kg object:

   (a) When 10 N force acts on it for 1s ?

   (b) When 10 N force acts on it for 1m ?

   Calculate change in momentum in each case.

5. A ball of mass 0.2 kg falls through air with an acceleration of 6 ms^{-2}. Calculate the air drag on the ball.

6. A load of mass 20 kg is lifted with the help of a rope at a constant acceleration. The load covers a height of 5 m in 2 seconds. Calculate the tension in the rope. In a rocket m changes with time. Write down the mathmatical form of Newton's law in this case and interpret it physically.

7. A ball of mass 0.1 kg moving at 10 m s^{-1} is deflected by a wall at the same speed in the direction shown. What is the magnitude of the change in momentum of the ball?

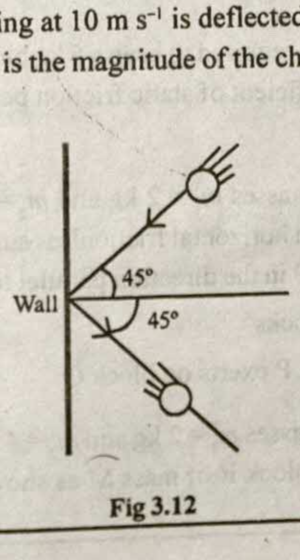


Fig 3.12

8. Find the average recoil force on a machine gun that is firing 150 bullets per minute, each with a speed of 900 m s^{-1}. Mass of each bullet is 12 g.

9. Explain why, when catching a fast moving ball, the hands are drawn back while the ball is being brought to rest.

10. A constant force of magnitude 20 N acts on a body of mass 2 kg, initially at rest, for 2 seconds. What will be the velocity of the body after
(a) 1 second from start? (b) 3 seconds from start?

11. How does a force acting on a block in the direction shown here keep the block from sliding down the vertical wall?

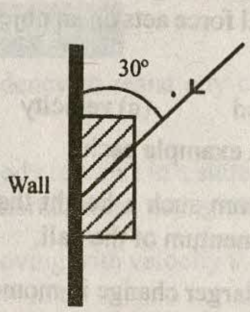


Fig 3.13

12. A 1.2 kg block is resting on a horizontal surface. The coefficient of static friction between the block and the surface is 0.5. What will be the magnitude and direction of the force of friction on the block when the magnitude of the external force acting on the block in the horizontal direction is

(a) 0 N ? (b) 4.9 N ? (c) 9.8 N ?

13. For a block on a surface the maximum force of static friction is 10N. What will be the force of friction on the block when a 5 N external force is applied to it parallel to the surface on which it is resting?

14. What minimum force F is required to keep a 5 kg block at rest on an inclined plane of inclination 30°. The coefficient of static friction between the block and the inclined plane is 0.25.

15. Two blocks P and Q of masses $m_1 = 2$ kg and $m_2 = 3$ kg respectively are placed in contact with each other on horizontal frictionless surface. Some external force F = 10 N is applied to the block P in the direction parallel to the surface. Find the following
(a) acceleration of the blocks
(b) force which the block P exerts on block Q.

16. Two blocks P and Q of masses $m_1 = 2$ kg and $m_2 = 4$ kg are connected to a third block R of mass M as shown

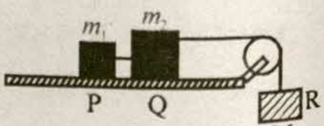


Fig 3.14

in Fig. 3.14 For what maximum value of M will the system be in equilibrium? The frictional force acting on each block is half the force of normal reaction on it.

17. Explain the role of friction in the case of bicycle brakes. What will happen if a few drops of oil are put on the rim?

18. A 2 kg block is pushed up an incline plane of inclination $\theta = 37^0$ imparting it a speed of 20 m s^{-1}. How much distance will the block travel before coming to rest? The coefficient of kinetic friction between the block and the incline plane is $\mu_k = 0.5$.

    Take $g = 10 \text{ m s}^{-2}$ and use $\sin 37^0 = 0.6$, $\cos 37^0 = 0.8$.

Answers to Intext Questions

3.1

1. No. The statement is true only for a body which was at rest before the application of force.
2. Inertial mass
3. Yes, as in uniform circular motion.
4. A force can change motion. It can also deform bodies.

3.2

1. Object of smaller mass
2. (a) Yes (b) No.
3. Momentum of the falling ball increases because gravitational force acts on it in the direction of its motion and hence velocity increases.
4. In case (b) the change in momentum will be larger. It is the F Δt product that gives the change in momentum. $\left(\text{as } F \propto \frac{\Delta p}{\Delta t}\right)$
5. No. Though the speed is constant, the velocity of the object changes due to change in direction. Hence its momentum will not be constant.

3.3

1. The jumper is thrown upwards by the force which the ground exerts on the jumper. This force is the reaction to the force which the jumper exerts on the ground.
2. (a) The force with which a man kicks a football is action and the force which the football exerts on the man will be its reaction.

   (b) The force with which earth pulls the moon is action and the force which the moon exerts on the earth will be its reaction.

   (c) If the force which the ball exerts on the wall is the action then the force which the wall exerts on the ball will be its reaction.

3. No. The arguement is not correct. The almirah moves when the push by the person exceeds the frictional force between the almirah and the floor. He does not get pushed backward due to a large force of friction that he experiences due to the floor. On a slippery surface, he will not be able to push the almirah foward.

3.4

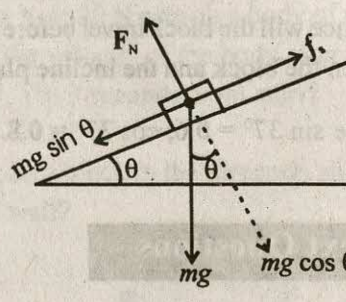


Fig. 3.15

2. 40 N
3. (a) (5×9.8) N

   (b) $F = (5 \times 2)\ N + (5 \times 9.8)\ N = 59\ N$

3.5

(1) When the train starts it has an acceleration, say a. Thus the total force acting on water in the frame of reference attached to the train is

$\mathbf{F} = m\,\mathbf{g} - m\,\mathbf{a}$

where m is the mass of the water and the glass. (Fig. 3.16). The surface of the water takes up a position normal to F as shown.

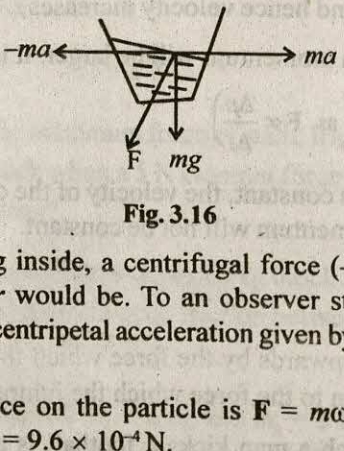


Fig. 3.16

(2) To the passenger sitting inside, a centrifugal force $(-mv^2/r)$ acts on the car. The greater v is the larger r would be. To an observer standing on the road, the car moving in a curve has a centripetal acceleration given by v^2/r. Once again, the greater is v, the larger will be r.

(3) The net centrifugal force on the particle is $\mathbf{F} = m\omega^2 r = (6 \times 10^{-10}\ \text{kg}) \times (2\pi \times 10^3\ \text{rad s}^{-1})^2 \times (0.04\ \text{m}) = 9.6 \times 10^{-4}\ \text{N}$.

(4) For an object to fly off centrifugal force (= centripetal force) should be just more than the weight of a body. If r is the radius of the earth then $\frac{mv^2}{r} = mg$

as $v = r\omega$

$$\frac{r^2\omega^2}{r} = g$$

or, angular speed $\omega = \sqrt{g/r}$

$\therefore$ Any angular speed more than $\sqrt{g/r}$ will make objects fly off.

5. Zero (as it is a case of free fall of a body).

Answers to Terminal Problems

1. (d)

2. (a) if internal forces developed within the material counter bank the external force. A it happens in case of force applied on a wall.

   (b) It force is applied at right angles to the direction of motion of the body, the force changes the direction of motion of body and not to speed.

3. $v = 0 + (-g) \times 4$

   $|v| = 40 \text{ m s}^{-1}$

   $\therefore \Delta P = m\,(v - u) = (0.5 \times 40) = 20 \text{ kg m s}^{-2}$

4. When 10 N force acts for 1s.

5. 0.76 N

7. 250 N.

8. 27 N

10. (a) 10 m s^{-1} (b) 20 m s^{-1}

12. (a) 0 N (b) 4.9 N (c) ~7.5 N

13. 5 N

14. 14.2 N

15. (a) 2 m s^{-2} (b) 6 N

16. 3 kg

18. 20 m

4

MOTION IN A PLANE

In the preceding two lessons you have studied the concepts related to motion in a straight line. Can you describe the motion of objects moving in a plane, i.e, in two dimensions, using the concepts discussed so far. To do so, we have to introduce certain new concepts. An interesting example of motion in two dimensions is the motion of a ball thrown at an angle to the horizontal. This motions is called a *projectile motion.*

In this lesson you will learn to answer questions like : What should be the position and speed of an aircraft so that food or medicine packets dropped from it reach the people affected by floods or an earthquake? How should an athlete throw a discuss or a javelin so that it covers the maximum horizontal distance? How should roads be designed so that cars taking a turn around a curve do not go off the road? What should be the speed of a satellite so that it moves in a circular orbit around the earth? And so on.

Such situations arise in projectile motion and circular motion. We will introduce the concepts of angular speed, centripetal acceleration, and centripetal force to explain this kind of motion.

Objectives

After studying this lesson, you should be able to :

- *explain projectile motion and circular motion and give their examples;*
- *derive expressions for the time of flight, range and maximum height of a projectile;*
- *derive the equation of the trajectory of a projectile;*
- *derive expressions for velocity and acceleration of a particle in circular motion; and*
- *define radial and tangential acceleration.*

4.1 Projectile Motion

The first breakthrough in the description of projectile motion was made by Galileo. He showed that the horizontal and vertical motions of a slow moving projectile are mutually independent. This can be understood by doing the following activity.

Take two cricket balls. Project one of them horizontally from the top of building. At the same time drop the other ball downward from the same height. What will you notice?

You will find that both the balls hit the ground at the same time. This shows that the downward acceleration of a projectile is the same as that of a freely falling body. Moreover, this takes place independent of its horizontal motion. Further, measurement of time and distance will show that the horizontal velocity continues unchanged and takes place independent of the vertical motion.

In other words, the two important properties of a projectile motion are :

(i) a constant horizontal velocity component

(i) a constant vertically downward acceleration component.

The combination of these two motions results in the curved path of the projectile.

Refer to Fig. 4.1. Suppose a boy at A throws a ball with an initial horizontal speed. According to Newton's second law there will be no acceleration in the horizontal direction unless a horizontally directed force acts on the ball. Ignoring friction of air, the only force acting on the ball once it is free from the hand of the boy is the force of gravity.

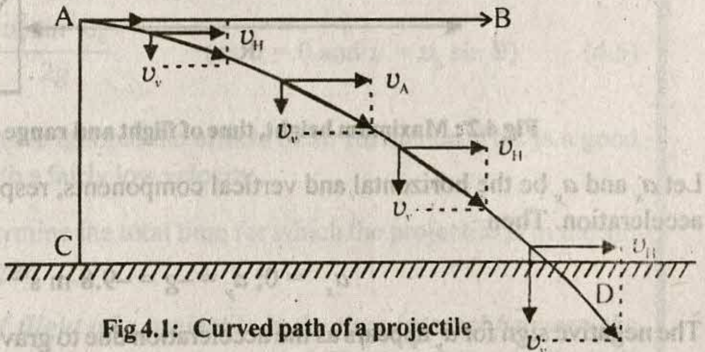


Fig 4.1: Curved path of a projectile

Hence the horizontal speed v_H of the ball does not change. But as the ball moves with this speed to the right, it also falls under the action of gravity as shown by the vector's v_v representing the vertical component of the velocity. Note that $v = \sqrt{v_H^2 + v_v^2}$ and is tangential to the trajectory.

Having defined projectile motion, we would like to determine how high and how far does a projectile go and for how long does it remain in air. These factors are important if we want to launch a projectile to land at a certain target - for instance, a football in the goal, a cricket ball beyond the boundary and relief packets in the reach of people marooned by floods or other natural disasters.

4.1.1 Maximum Height, Time of Flight and Range of a Projectile

Let us analyse projectile motion to determine its maximum height, time of flight and range. In doing so, we will ignore effects such as wind or air resistance. We can characterise the initial velocity of an object in projectile motion by its vertical and horizontal components. Let us take the positive *x*-axis in the horizontal direction and the positive *y*-axis in the vertical direction (Fig. 4.2).

Let us assume that the initial position of the projectile is at the origin O at $t = 0$. As you know, the coordinates of the origin are $x = 0, y = 0$. Now suppose the projectile is launched with an initial velocity υ_0 at an angle θ_0, known as the ***angle of elevation,*** to the x-axis. Its components in the x and y directions are,

$$\upsilon_{ox} = \upsilon_o \cos\theta_0 \tag{4.1 a}$$

and

$$\upsilon_{oy} = \upsilon_o \sin\theta_0 \tag{4.1 b}$$

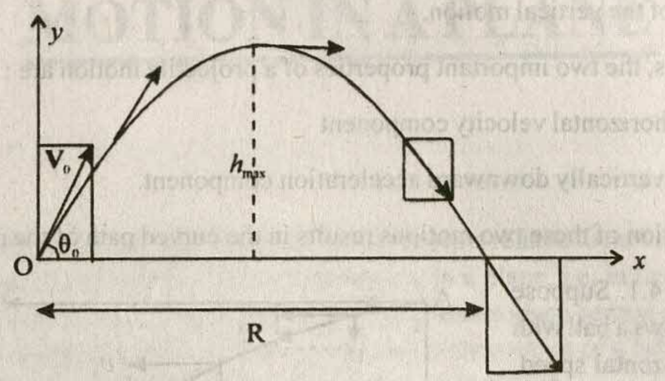


Fig 4.2 : Maximum height, time of flight and range of a projectile

Let a_x and a_y be the horizontal and vertical components, respectively, of the projectile's acceleration. Then

$$a_x = 0; a_y = -g = -9.8 \text{ m s}^{-2} \tag{4.2}$$

The negative sign for a_y appears as the acceleration due to gravity is always in the negative y direction in the chosen coordinate system.

Notice that a_y is constant. Therefore, we can use Eqns. (2.6) and (2.9) to write expressions for the horizontal and vertical components of the projectile's velocity and position at time t. These are given by

Horizontal motion $\upsilon_x = \upsilon_{ox}$, since $a_x = 0$ (4.3a)

$$x = \upsilon_{ox} t = \upsilon_0 \cos\theta_0 t \tag{4.3b}$$

Vertical motion $\upsilon_y = \upsilon_{oy} - g\,t = \upsilon_0 \sin\theta_0 - gt$ (4.3c)

$$y = \upsilon_{oy} t - \tfrac{1}{2} g t^2 = \upsilon_0 \sin\theta_0 t - \tfrac{1}{2} g t^2 \tag{4.3d}$$

The vertical position and velocity components are also related through Eqn. (2.10) as

$$-g\,y = \tfrac{1}{2}(\upsilon_y^2 - \upsilon_{oy}^2) \tag{4.3e}$$

You will note that the horizontal motion, given by Eqns. (4.3a and b), is motion with constant velocity. And the vertical motion, given by Eqns. (4.3c and d), is motion with constant (downward) acceleration. The vector sum of the two respective components would give us the velocity and position of the projectile at any instant of time.

Now, let us make use of these equations to know the maximum height, time of flight and range of a projectile.

(a) Maximum height : As the projectile travels through air, it climbs upto some maximum height (h) and then begins to come down. ***At the instant when the projectile is at the maximum height, the vertical component of its velocity is zero.*** This is the instant when the projectile stops to move upward and does not yet begin to move downward. Thus, putting $v_y = 0$ in Eqns. (4.3c and e), we get

$$0 = v_{oy} - g\,t,$$

Thus the time taken to rise taken to the maximum height is given by

$$t = \frac{v_{oy}}{g} = \frac{v_0 \sin\theta_0}{g} \qquad (4.4)$$

At the maximum height h attained by the projectile, the vertical velocity is zero. Therefore, applying $v^2 - u^2 = 2\,a\,s = 2\,g\,h$, we get the expression for maximum height :

$$h = \frac{v_0^2 \sin^2\theta_0}{2g} \quad (\text{as } v = 0 \text{ and } u = v_0 \sin\theta) \qquad (4.5)$$

Note that in our calculation we have ignored the effects of air resistance. This is a good approximation for a projectile with a fairly low velocity.

Using Eqn.(4.4) we can also determine the total time for which the projectile is in the air. This is termed as the ***time of flight.***

(b) Time of flight : ***The time of flight of a projectile is the time interval between the instant of its launch and the instant when it hits the ground.*** The time t given by Eq.(4.4) is the time for half the flight of the ball. Therefore, the total time of flight is given by

$$T = 2t = \frac{2\,v_0 \sin\theta_0}{g} \qquad (4.6)$$

Finally we calculate the distance travelled horizontally by the projectile. This is also called its ***range.***

(c) Range : The range R of a projectile is calculated simply by multiplying its time of flight and horizontal velocity. Thus using Eqns. (4.3b) and (4.4), we get

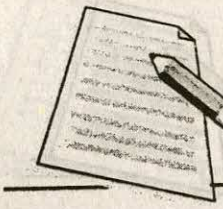
$$R = (v_{ox})\,(2\,t)$$

$$= (v_0 \cos\theta_0)\frac{(2\,v_0 \sin\theta_0)}{g}$$

$$= v_0^2\,\frac{(2\sin\theta_0 \cos\theta_0)}{g}$$

Since $2\sin\theta\cos\theta = \sin 2\theta$, the range R is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \qquad (4.7)$$



From Eqn. (4.7) you can see that the range of a projectile depends on

- its initial speed υ_0, and
- its direction given by θ_0.

Now can you determine the angle at which a disc, a hammer or a javelin should be thrown so that it covers maximum distance horizontally? In other words, let us find out the angle for which the range would be maximum?

Clearly, R will be maximum for any given speed when $\sin 2\theta_0 = 1$ or $2\theta_0 = 90°$.

Thus, for *R to be maximum at a given speed* υ_0, θ_0 should be equal to 45°.

Let us determine these quantities for a particular case.

Example 4.1 : In the centennial (on the occasion of its centenary) Olympics held at Atlanta in 1996, the gold medallist hammer thrower threw the hammer to a distance of 19.6m. Assuming this to be the maximum range, calculate the initial speed with which the hammer was thrown. What was the maximum height of the hammer? How long did it remain in the air? Ignore the height of the thrower's hand above the ground.

Solution : Since we can ignore the height of the thrower's hand above the ground, the launch point and the point of impact can be taken to be at the same height. We take the origin of the coordinate axes at the launch point. Since the distance covered by the hammer is the range, it is equal to the hammer's range for $\theta_0 = 45°$. Thus we have from Eqn.(4.7):

$$R = \frac{\upsilon_0^2}{g}$$

or

$$\upsilon_0 = \sqrt{Rg}$$

It is given that $R = 19.6$ m. Putting $g = 9.8\ \text{ms}^{-2}$ we get

$$\upsilon_0 = \sqrt{(19.6\text{m})\times(9.8\ \text{ms}^{-2})} = 9.8\sqrt{2}\ \text{ms}^{-1} = 14.01\text{ms}^{-1}$$

The maximum height and time of flight are given by Eqns. (4.5) and (4.6), respectively. Putting the value of υ_0 and $\sin\theta_0$ in Eqns. (4.5) and (4.6), we get

$$\text{Maximum height, } h = \frac{\left(9.8\sqrt{2}\right)^2 \text{m}^2\text{s}^{-2} \times \left(\frac{1}{2}\right)^2}{2\times 9.8\text{ms}^{-2}} = 4.9\text{ m}$$

$$\text{Time of flight, } T = \frac{2\times\left(9.8\sqrt{2}\right)\text{m s}^{-1}}{9.8\text{ m s}^{-2}} \times \sqrt{\frac{1}{2}} = 2\text{ s}$$

Now that you have studied some concepts related to projectile motion and their applications, you may like to check your understanding. Solve the following problems.

Intext Questions 4.1

1. Identify examples of projectile motion from among the following situations :

   (a) An archer shoots an arrow at a target

   (b) Rocks are ejected from an exploding volcano

   (c) A truck moves on a mountainous road

   (d) A bomb is released from a bomber plane. [Hint : Remember that at the time of release the bomb shares the horizontal motion of the plane.]

   (e) A boat sails in a river.

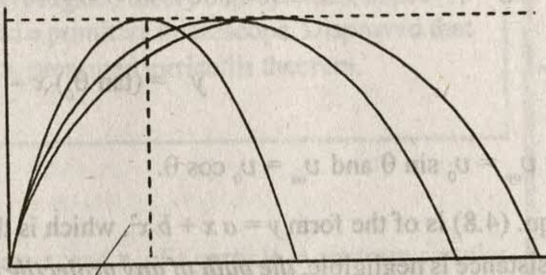


Fig 4.3 : Trajectories of a projectile

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2. Three balls thrown at different angles reach the same maximum height (Fig.4.3):

   (a) Are the vertical components of the initial velocity the same for all the balls? If not, which one has the least vertical component?

   (b) Will they all have the same time of flight?

   (c) Which one has the greatest horizontal velocity component?

....

3. An athelete set the record for the long jump with a jump of 8.90 m. Assume his initial speed on take off to be 9.5 ms^{-1}. How close did he come to the maximum possible range in the absence of air resistance? Take $g = 9.78 \, ms^{-2}$.

....

4.2 The Trajectory of a Projectile

The path followed by a projectile is called its trajectory. Can you recognise the shapes of the trajectories of projectiles shown in Fig. 4.1, 4.2 and 4.3.

Although we have discussed quite a few things about projectile motion, we have still not answered the question: What is the path or trajectory of a projectile? So let us determine the equation for the trajectory of a projectile.

It is easy to determine the equation for the path or trajectory of a projectile. You just have to eliminate *t* from Eqns. (4.3b) and (4.3d) for *x* and *y*. Substituting the value of *t* from Eqn. (4.3b) in Eqn.(4.3d) we get

$$y = \upsilon_{oy}\frac{x}{\upsilon_{ox}} - \frac{1}{2}\frac{g x^2}{\upsilon_{ox}^2} \quad \left(\text{as } t = \frac{x}{\upsilon_{ox}}\right) \qquad (4.8\text{ a})$$

Using Eqns. (4.1 a and b), Eqn (4.8a) becomes

$$y = (\tan\theta_0)\,x - \frac{g}{2(\upsilon_0\cos\theta_0)^2}x^2 \qquad (4.8\text{ b})$$

as $\upsilon_{oy} = \upsilon_0 \sin\theta$ and $\upsilon_{ox} = \upsilon_0\cos\theta$.

Eqn. (4.8) is of the form $y = a\,x + b\,x^2$, which is the equation of a **parabola.** Thus, if air resistance is negligible, ***the path of any projectile launched at an angle to the horizontal is a parabola or a portion of a parabola.*** In Fig 4.3 you can see some trajectories of a projectile at different angles of elevation.

Eqns. (4.5) to (4.7) are often handy for solving problems of projectile motion. For example, these equations are used to calculate the launch speed and the angle of elevation required to hit a target at a known range. However, these equations do not give us complete description of projectile motion, if distance covered are very large. To get a complete description, we must include the rotation of the earth also. This is beyond the scope of this course.

Now, let us summarise the important equations describing projectile motion launched from a point (x_0, y_0) with a velocity υ_0 at an angle of elevation, θ_0.

Equations of Projectile Motion:

$$a_x = 0 \qquad a_y = -g \qquad (4.9\text{ a})$$

$$\upsilon_x = \upsilon_0\cos\theta_0 \qquad \upsilon_y = \upsilon_0\sin\theta - g\,t \qquad (4.9\text{ b})$$

$$x = x_0 + (\upsilon_0\cos\theta_0)t \qquad y = y_0 + (\upsilon_0\sin\theta)\,t - (½)\,g\,t^2 \qquad (4.9\text{ c})$$

Equation of trajectory:

$$y = y_0 + (\tan\theta)\,(x - x_0) - \frac{g}{2(\upsilon_0\cos\theta_0)^2}\,(x - x_0)^2 \qquad (4.9\text{ d})$$

Note that these equations are more general than the ones discussed earlier. The initial coordinates are left unspecified as (x_0, y_0) rather than being placed at (0,0). Can you derive this general equation of the projectile trajectory? Do it before proceeding further?

Thus far you have studied motion of objects in a plane, which can be placed in the category of projectile motion. In projectile motion, the acceleration is constant both in magnitude and direction. There is another kind of two-dimensional motion in which acceleration is constant in magnitude but not in direction. This is uniform circular motion, and you will learn about in the following section.

Evangelista Torricelli
(1608 – 1647)

Italian mathematician and a student of Galelio Galili, he invented mercury barometer, investigated theory of projectiles, improved telescope and invented a primitive microscope. Disproved that nature abhors vacuum, presented torricellis theorem.

4.3 Circular Motion

Look at Fig. 4.4a. It shows the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ of a particle in uniform circular motion at two different instants of time t_1 and t_2, respectively. The word 'uniform' refers to constant speed. We have said that the speed of the particle is constant. What about its velocity? To find out velocity, recall the definition of average velocity and apply it to points P_1 and P_2:

$$\boldsymbol{v}_{av} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{t_2 - t_1} = \frac{\Delta \mathbf{r}}{\Delta t} \tag{4.10 a}$$

The motion of a gramophone record, a grinding wheel at constant speed, the moving hands of an ordinary clock, a vehicle turning around a corner are examples of circular motion. The movement of gears, pulleys and wheels also involve circular motion. The simplest kind of circular motion is uniform circular motion. The most familiar example of uniform circular motion are a point on a rotating fan blade or a grinding wheel moving at constant speed.

One of the example of uniform circular motion is an artificial satellite in circular orbit around the earth. We have been benefitted immensely by the INSAT series of satellites and other artificial satellites. So let us now learn about uniform circular motion.

4.3.1 Uniform Circular Motion

By definition, ***uniform circular motion is motion with constant speed in a circle.***

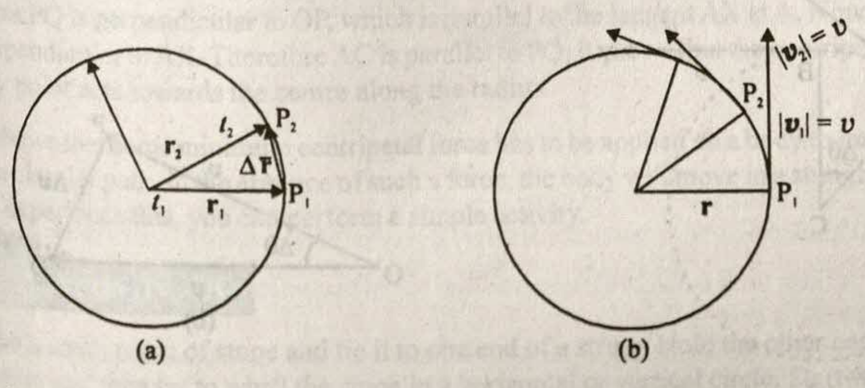


Fig. 4.4 (a): Positions of a particle in uniform circular motion; (b): Uniform circular motion

The vector $\Delta\mathbf{r}$ is shown in Fig. 4.4a. Now suppose you make the time interval Δt smaller and smaller so that it approaches zero. What happens to $\Delta \mathbf{r}$? In particular, what is the direction of $\Delta \mathbf{r}$? It approaches the tangent to the circle at point P_1 as Δt tends to zero. Mathematically, we define the instantaneous velocity at point P_1 as

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

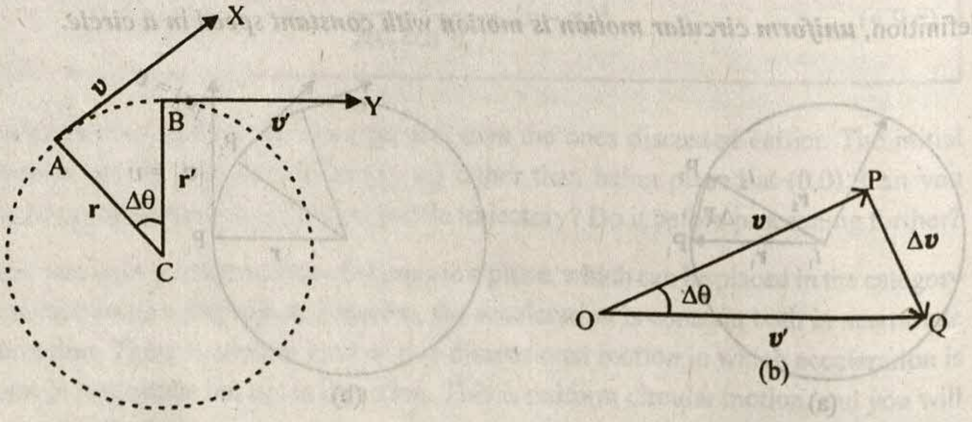
Thus, in uniform circular motion, the velocity vector changes continuously. Can you say why? This is because the direction of velocity is not constant. It goes on changing continuously as the particle travels around the circle (Fig. 4.4b). Because of ***this change in velocity, uniform circular motion is accelerated motion.*** The acceleration of a particle in uniform circular motion is termed as centripetal acceleration. Let us learn about it in some detail.

Centripetal acceleration : Consider a particle of mass m moving with a ***uniform speed*** v in a circle. Suppose at any instant its position is at A and its motion is directed along AX. After a small time Δt, the particle reaches B and its velocity is represented by the tangent at B directed along BY.

Let $\mathbf{r}$ and $\mathbf{r}'$ be the position vectors and $\boldsymbol{v}$ and $\boldsymbol{v}'$; the velocities of the particle at A and B respectively as shown in Fig. 4.5 (a). The change in velocity $\Delta\boldsymbol{v}$ is obtained using the triangle law of vectors. As the path of the particle is circular and velocity is along its tangent, $\boldsymbol{v}$ is perpendicular to $\mathbf{r}$ and $\boldsymbol{v}'$ is perpendicular to $\Delta \mathbf{r}$. As the average acceleration $\left(\mathbf{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}\right)$ is along $\Delta\boldsymbol{v}$, it (i.e., the average acceleration) is perpendicular to $\Delta \mathbf{r}$.

Let the angle between the position vectors $\mathbf{r}$ and $\mathbf{r}'$ be $\Delta\theta$. Then the angle between velocity vectors $\boldsymbol{v}$ and $\boldsymbol{v}'$ will also be $\Delta\theta$ as the velocity vectors are always perpendicular to the position vectors.

To determine the change in velocity $\Delta\boldsymbol{v}$ due to the change in direction, consider a point O outside the circle. Draw a line OP parallel to and equal to AX (or $\boldsymbol{v}$) and a line OQ parallel to and equal to BY (or $\boldsymbol{v}'$). As $|\boldsymbol{v}| = |\boldsymbol{v}'|$, OP = OQ. Join PQ. You get a triangle OPQ (Fig. 4.5b)



(a) (b)

Fig. 4.5

Now in triangle OPQ, sides OP and OQ represent velocity vectors $\boldsymbol{v}$ and $\boldsymbol{v}'$ at A and B respectively. Hence, their difference is represented by the side PQ in magnitude and direction. In other words the change in the velocity equal to PQ in magnitude and direction takes place as the particle moves from A to B in time Δt.

$\therefore$ Acceleration = Rate of change of velocity

$$\mathbf{a} = \frac{\mathbf{PQ}}{\Delta t} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

As Δt is very small AB is also very small and is nearly a straight line. Then Δ ACB and ΔPOQ are isosceles triangles having their included angles equal. The triangles are, therefore, similar and hence,

$$\frac{PQ}{AB} = \frac{OP}{CA}$$

or
$$\frac{\Delta v}{v.\Delta t} = \frac{v}{r}$$

[as magnitudes of velocity vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2 = v$ (say)]

or
$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

But $\frac{\Delta v}{\Delta t}$ is the acceleration of the particle. Hence

$$\textit{Centripetal acceleration, } a = \frac{v^2}{r}$$

Since $v = r\,\omega$, the magnitude of centripetal force in given by

$$F = m\,a = \frac{m\,v^2}{r} = m\,r\omega^2.$$

As Δt is very small, $\Delta\theta$ is also very small and $\angle OPQ = \angle OQP = 1$ right angle.

Thus PQ is perpendicular to OP, which is parallel to the tangent AX at A. Now AC is also perpendicular to AX. Therefore AC is parallel to PQ. It shows that the contripetal force at any point acts towards the centre along the radius.

It shows that some minimum centripetal force has to be applied on a body to make it move in a circular path. In the absence of such a force, the body will move in a straight line path. To experience this, you can perform a simple activity.

Activity 4.1

Take a small piece of stone and tie it to one end of a string. Hold the other end with your fingers and then try to whirl the stone in a horizontal or vertical circle. Start with a small speed of rotation and increase it gradually. What happens when the speed of rotation is low? Do you feel any pull on your fingers when the stone is whirling. What happens to the stone when you leave the end of the string you were holding? How do you explain this?

Activity 4.2

Take an aluminium channel of length one metre and bend it in the form shown in the diagram with a circular loop in the middle. Take help of some technical preson if required.

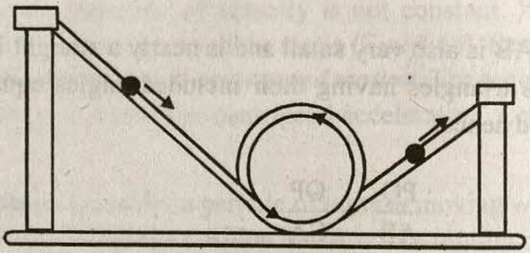


Fig. 4.5: The ball will loop if it starts rolling from a point high enough on the incline

Roll down a glass marble from different heights of the channel on the right hand side, and see whether the marble is able to loop the loop in each case or does it need some minimum height (hence velocity) below which the marble will not be able to complete the loop and fall down. How do you explain it?

Some Applications of Centripetal Force

(i) **Centrifuges :** These are spinning devices used for separating materials having different densities. When a mixture of two materials of different densities placed in a vessel is rotated at high speed, the centripetal force on the heavier material will be more. Therefore, it will move to outermost position in the vessel and hence can be separated. These devises are being used for uranium enrichment. In a chemistry laboratory these are used for chemical analysis.

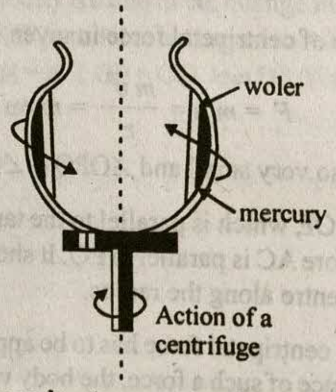


Fig. 4.6: When mercury and water are rotated in a dish, the water stays inside. Centripetal force, like gravitational force, is greater for the more dense substance.

(ii) Mud clings to an automobile tyre until the speed becomes too high and then it flies off tangentially (Fig. 4.7).

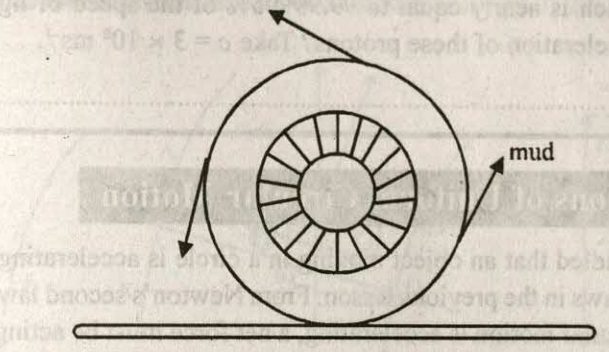


Fig. 4.7: Mud or water on a fast-turning wheel flies off tangentially

(iii) **Planetary motion :** The Earth and the other planets revolving round the sun get necessary centripetal force from the gravitational force between them and the sun.

Example 4.2 : Astronauts experience high acceleration in their flights in space. In the training centres for such situations, they are placed in a closed capsule, which is fixed at the end of a revolving arm of radius 15 m. The capsule is whirled around in a circular path, just like the way we whirl a stone tied to a string in a horizontal circle. If the arm revolves at a rate of 24 revolutions per minute, calculate the centripetal acceleration of the capsule.

Solution : The circumference of the circular path is 2π × (radius) = 2π × 15 m. Since the capsule makes 24 revolutions per minute or 60 s, the time it takes to go once around this circumference is $\frac{60}{24}$ s. Therefore,

$$\text{speed of the capsule, } \upsilon = \frac{2\pi r}{T} = \frac{2\pi \times 15\ \text{m}}{(60/24)\ \text{s}} = 38\ \text{ms}^{-1}$$

The magnitude of the centripetal acceleration

$$a = \frac{\upsilon^2}{r} = \frac{(38\ \text{ms}^{-1})^2}{15\ \text{m}} = 96\ \text{ms}^{-2}$$

Note that centripetal acceleration is about 10 times the acceleration due to gravity.

Intext Questions 4.2

1. In uniform circular motion, (a) Is the speed constant? (b) Is the velocity constant? (c) Is the magnitude of the acceleration constant? (d) Is acceleration constant? Explain.

....

2. An athlete runs around a circular track with a speed of 9.0 ms^{-1} and a centripetal acceleration of 3 ms^{-2}. What is the radius of the track?

....

3. The Fermi lab accelerator is one of the largest particle accelerators. In this accelerator, protons are forced to travel in an evacuated tube in a circular orbit of diameter 2.0 km

at a speed which is nearly equal to 99.99995% of the speed of light. What is the centripetal acceleration of these protons? Take $c = 3 \times 10^8$ ms^{-1}.

....

4.4 Applications of Uniform Circular Motion

So far you have studied that an object moving in a circle is accelerating. You have also studied Newton's laws in the previous lesson. From Newton's second law we can say that as the object in circular motion is accelerating, a net force must be acting on it.

What is the direction and magnitude of this force? This is what you will learn in this section. Then we will apply Newton's laws of motion to uniform circular motion. This helps us to explain why roads are banked, or why pilots feel pressed to their seats when they fly aircrafts in vertical loops.

Let us first determine the force acting on a particle that keeps it in uniform circular motion. Consider a particle moving with constant speed υ in a circle of radius r. From Newton's second law, the net external force acting on a particle is related to its acceleration by

$$\mathbf{F} = -\frac{m\upsilon^2}{r}\,\hat{r}, \; |\mathbf{F}| = \frac{m\upsilon^2}{r} \qquad (4.19)$$

This net external force directed towards the centre of the circle with magnitude given by Eqn. (4.19) is called ***centripetal force. An important thing to understand and remember is that the term 'centripetal force' does not refer to a type of force of interaction like the force of gravitation or electrical force.*** This term only tells us that the net force of a certain magnitude acting on a particle in uniform circular motion is directed towards the centre. It does not tell us how this force is provided.

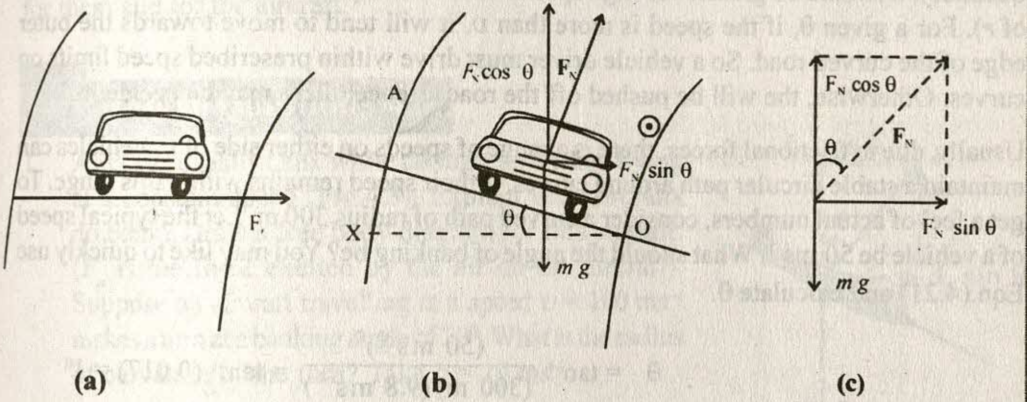
Thus, the force may be provided by the gravitational attraction between two bodies. For example, in the motion of a planet around the sun, the centripetal force is provided by the gravitational force between the two. Similarly, the centripetal force for a car travelling around a bend is provided by the force of friction between the road and the car's tyres and/or by the horizontal component of normal reaction of banked road. You will understand these ideas better when we apply them in certain concrete situations.

4.4.1 Banking of Roads

While riding a bicycle and taking a sharp turn, you may have felt that something is trying to throw you away from your path. Have you ever thought as to why does it happen?

You tend to be thrown out because enough centripetal force has not been provided to keep you in the circular path. Some force is provided by the friction between the tyres and the road, but that may not be sufficient. When you slow down, the needed centripetal force decreases and you manage to complete this turn.

Consider now a car of mass m, travelling with speed υ on a curved section of a highway (Fig.4.6). To keep the car moving uniformly on the circular path, a force must act on it directed towards the centre of the circle and its magnitude must be equal to $m\upsilon^2/r$. Here r is the radius of curvature of the curved section.



(a) (b) (c)

Fig.4.6 : A car taking a turn (a) on a level road; (b) on a banked road; and (c) Forces on the car with F_N resolved into its rectangular components. Generally θ is not as large as shown here in the diagram.

Now if the road is levelled, the force of friction between the road and the tyres provides the necessary centripetal force to keep the car in circular path. This causes a lot of wear and tear in the tyre and may not be enough to give it a safe turn. The roads at curves are, therefore, banked, where banking means the raising of the outer edge of the road above the level of the inner edge (Fig. 4.6). As a matter of fact, roads are designed to minimise reliance on friction. For example, when car tyres are smooth or there is water or snow on roads, the coefficient of friction becomes negligible. Roads are banked at curves so that cars can keep on track even when friction is negligible.

Let us now analyse the free body diagram for the car to obtain an expression for the angle of banking, θ, which is adjusted for the sharpness of the curve and the maximum allowed speed.

Consider the case when there is no frictional force acting between the car tyres and the road. The forces acting on the car are the car's weight *mg* and $\mathbf{F}_N$, the force of normal reaction. The centripetal force is provided by the horizontal component of $\mathbf{F}_N$. Thus, resolving the force $\mathbf{F}_N$ into its horizontal and vertical components, we can write

$$F_N \sin\theta = \frac{m\upsilon^2}{r} \tag{4.20a}$$

Since there is no vertical acceleration, the vertical component of $\mathbf{F}_N$ is equal to the car's weight:

$$F_N \cos\theta = m\,g \tag{4.20b}$$

We have two equations with two unknowns, i.e., F_N and θ. To determine θ, we eliminate F_N. Dividing Eqn. (4.20 a) by Eqn. (4.20 b), we get

$$\tan\theta = \frac{m\upsilon^2/r}{m\,g} = \frac{\upsilon^2}{r\,g}$$

or

$$\theta = \tan^{-1}\frac{\upsilon^2}{rg} \tag{4.21}$$

How do we interpret Eqn. (4.21) for limits on υ and choice of θ? Firstly, Eqn.(4.21) tells us that the angle of banking is independent of the mass of the vehicle. So even large trucks and other heavy vehicles can ply on banked roads.

Secondly, θ should be greater for high speeds and for sharp curves (i.e., for lower values of r). For a given θ, if the speed is more than υ, it will tend to move towards the outer edge of the curved road. So a vehicle driver must drive within prescribed speed limits on curves. Otherwise, the will be pushed off the road. Hence, there may be accidents.

Usually, due to frictional forces, there is a range of speeds on either side of υ. Vehicles can maintain a stable circular path around curves, if their speed remains within this range. To get a feel of actual numbers, consider a curved path of radius 300 m. Let the typical speed of a vehicle be 50 ms^{-1}. What should the angle of banking be? You may like to quickly use Eqn.(4.21) and calculate θ.

$$\theta = \tan^{-1}\frac{(50\ \text{m s}^{-1})^2}{(300\ \text{m})(9.8\ \text{m s}^{-2})} = \tan^{-1}(0.017) = 1^0$$

You may like to consider another application.

4.4.2 Aircrafts in vertical loops

On Republic Day and other shows by the Indian Air Force, you might have seen pilots flying aircrafts in loops (Fig. 4.8a). In such situations, at the bottom of the loop, the pilots feel as if they are being pressed to their seats by a force several times the force of gravity. Let us understand as to why this happens. Fig. 4.8b shows the 'free body' diagram for the pilot of mass m at the bottom of the loop.

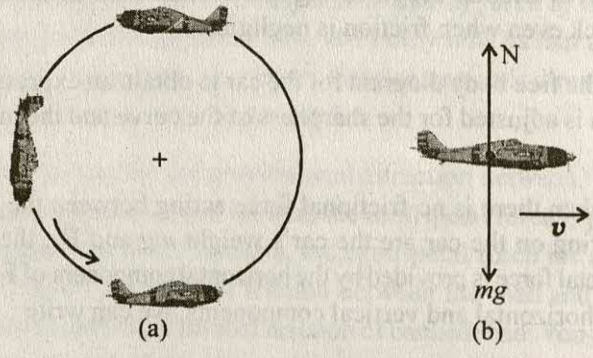


Fig.4.8 : (a) Aircrafts in vertical loops, (b) Free-body diagram for the pilot at the lowest point.

The forces acting on him are mg and the normal force N exerted by the seat. The net vertically upward force is N – mg and this provides the centripetal acceleration:

$$\text{N} - mg = m\,a$$

or

$$\text{N} - mg = m\,\upsilon^2/r$$

or

$$\text{N} = m\,(g + \upsilon^2/r)$$

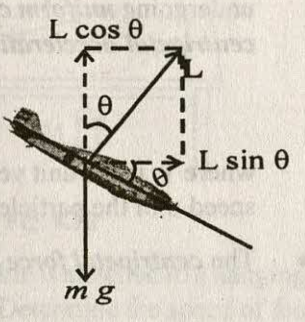
In actual situations, if $\upsilon = 200$ ms^{-1} and $r = 1500$ m, we get

$$\text{N} = m\,g\left[1 + \frac{(200\ \text{m s}^{-1})^2}{(9.8\ \text{m s}^{-2} \times 1500\ \text{m})}\right] = m\,g \times 3.7$$

So the pilots feel as though force of gravity has been magnified by a factor of 3.7. If this force exceeds set limits, pilots may even black out for a while and it could be dangerous for them and for the aircraft.

Intext Questions 4.3

1. Aircrafts usually bank while taking a turn when flying at a constant speed (Fig.4.8). Explain why aircrafts do bank? Draw a free body diagram for this aircraft. (F_a is the force exerted by the air on the aircraft). Suppose an aircraft travelling at a speed $\upsilon = 100\ \text{ms}^{-1}$ makes a turn at a banking angle of 30^0. What is the radius of curvature of the turn? Take $g = 10\ \text{ms}^{-2}$.



2. Calculate the maximum speed of a car which makes a turn of radius 100 m on a horizontal road. The coefficient of friction between the tyres and the road is 0.90. Take $g = 10\ \text{ms}^{-2}$.

....

3. An interesting act performed at variety shows is to swing a bucket of water in a vertical circle such that water does not spill out while the bucket is inverted at the top of the circle. For this trick to be performed sucessfully, the speed of the bucket must be larger than a certain minimum value. Derive an expression for the minimum speed of the bucket at the top of the circle in terms of its radius R. Calculate the speed for R = 1.0 m.

....

What You Have Learnt

- **Projectile motion** is defined as the motion which has constant velocity in a certain direction and constant acceleration in a direction perpendicular to that of velocity:

$$a_x = 0 \qquad a_y = -g$$

$$\upsilon_x = \upsilon_0 \cos\theta \qquad \upsilon_y = \upsilon_0 \sin\theta - g\,t$$

$$x = x_0 + (\upsilon_0 \cos\theta)\,t \qquad y = y_0 + (\upsilon_0 \sin\theta) - \tfrac{1}{2} - g\,t^2$$

- Height $h = \dfrac{\upsilon_0^2 \sin 2\theta}{g}$

- Time of flight $T = \dfrac{2\upsilon_0 \sin\theta}{g}$

- Range of the projectile $R = \dfrac{\upsilon_0^2 \sin 2\theta}{g}$

- Equation of the Trajectory of a projectile $y = (\tan\theta_0)\,x - \dfrac{g}{2(v_0\cos\theta_0)^2}\,x^2$

- **Circular motion** is uniform when the speed of the particle is constant. A particle undergoing ***uniform circular motion*** in a circle of radius r at constant speed v has a ***centripetal acceleration*** given by

$$\mathbf{a}_r = -\frac{v^2}{r}\,\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the unit vector directed from the centre of the circle to the particle. The speed v of the particle is related to its angular speed ω by $v = r\,\omega$.

- The ***centripetal force*** acting on the particle is given by

$$\mathbf{F} = m\,\mathbf{a}_r = \frac{m\,v^2}{r}\,\hat{\mathbf{r}} = m\,r\,\omega^2$$

Terminal Exercise

1. Why does a cyclist bend inward while taking a turn on a circular path?
2. Explain why the outer rail is raised with respect to the inner rail on the curved portion of a railway track?
3. If a particle is having circular motion with constant speed, will its acceleration also be constant?
4. A stone is thrown from the window of a bus moving on horizontal road. What path will the stone follow while reaching the ground; as seen by a observer standing on the road?
5. A string can sustain a maximum force of 100 N without breaking. A mass of 1kg is tied to one end of the piece of string of 1m long and it is rotated in a horizontal plane. Compute the maximum speed with which the body can be rotated without breaking the string?
6. A motorcyclist passes a curve of radius 50 m with a speed of 10 m s^{-1}. What will be the centripetal acceleration when turning the curve?
7. A bullet is fired with an initial velocity 300 ms^{-1} at an angle of 30° with the horizontal. At what distance from the gun will the bullet strike the ground?
8. The length of the second's hand of a clock is 10 cm. What is the speed of the tip of this hand?
9. You must have seen actors in Hindi films jumping over huge gaps on horse backs and motor cycles. In this problem consider a daredevil motor cycle rider trying to cross a gap at a velocity of 100 km h^{-1}. (Fig. 4.9). Let the angle of incline on either side be 45°. Calculate the widest gap he can cross.
10. A shell is fired at an angle of elevation of 30° with a velocity of 500 m s^{-1}. Calculate the vertical and horizontal components of the velocity, the maximum height that the shell reaches, and its range.
11. An aeroplane drops a food packet from a height of 2000 m above the ground while in

horizontal flight at a constant speed of 200 kmh^{-1}. How long does the packet take to fall to the ground? How far ahead (horizontally) of the point of release does the packet land?

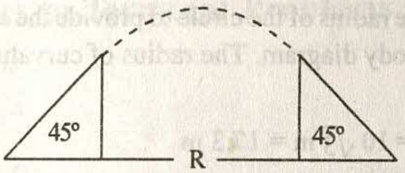


Fig. 4.9

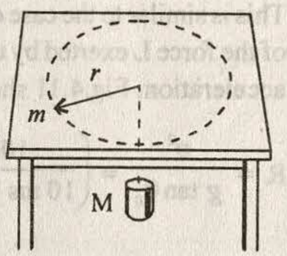


Fig. 4.10

12. A mass m moving in a circle at speed υ on a frictionless table is attached to a hanging mass M by a string through a hole in the table (Fig.4.10). Determine the speed of the mass m for which the mass M would remain at rest.
13. A car is rounding a curve of radius 200 m at a speed of 60 kmh^{-1}. What is the centripetal force on a passenger of mass m = 90 kg?

Answers to Intext Questions

4.1

(1) (a), (b), (d)

(2) (a) Yes (b) Yes (c) The ball with the maximum range.

(3) Maximum Range

$$\frac{\upsilon_0^2}{g} = \frac{(9.5\ \text{ms}^{-1})^2}{9.78\ \text{ms}^{-2}} = 9.23\ \text{m}$$

Thus, the difference is 9.23 m – 8.90 m = 0.33 m.

4.2

(1) (a) Yes (b) No (c) Yes (d) No

The velocity and acceleration are not constant because their directions are changing continuously.

(2) Since

$$a = \frac{\upsilon^2}{r}, r = \frac{\upsilon^2}{a} = \frac{(9.0\ \text{ms}^{-1})^2}{3\ \text{ms}^{-2}} = 27\ \text{m}$$

(3) $$a = \frac{c^2}{r} = \frac{(3\times 10^8\ \text{ms}^{-1})^2}{10\times 10^3\ \text{m}}$$

$$= 9 \times 10^{13}\ \text{ms}^{-2}$$

Notes

4.3

(1) This is similar to the case of banking of roads. If the aircraft banks, there is a component of the force L exerted by the air along the radius of the circle to provide the centripetal acceleration. Fig.4.11 shows the free body diagram. The radius of curvature is

$$R = \frac{\upsilon^2}{g \tan\theta_0} = \left(\frac{100 \text{ ms}^{-1}}{10 \text{ ms}^{-2} \times \tan 30^\circ}\right)^2 = 10\sqrt{3} \text{ m} = 17.3 \text{ m}$$

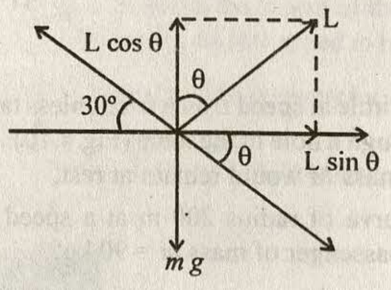


Fig.4.11

(2) The force of friction provides the necessary centripetal acceleration :

$$F_s = \mu_s N = \frac{m\upsilon^2}{r}$$

Since the road is horizontal N – mg

$$\text{Thus } \mu_s mg = \frac{m\upsilon^2}{r}$$

or $\upsilon^2 = \mu_s g r$

or $\upsilon = (0.9 \times 10 \text{ m s}^{-2} \times 100 \text{ m})^{1/2}$

$\upsilon = 30 \text{ ms}^{-1}$.

(3) Refer to Fig. 4.12 showing the free body diagram for the bucket at the top of the circle. In order that water in the bucket does not spill but keeps moving in the circle, the force *mg* should provide the centripetal acceleration. At the top of the circle.

$$mg = \frac{m\upsilon^2}{r}$$

or $\upsilon^2 = Rg$

$\therefore \ \upsilon = \sqrt{Rg}$

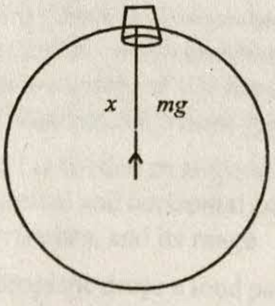


Fig. 4.12

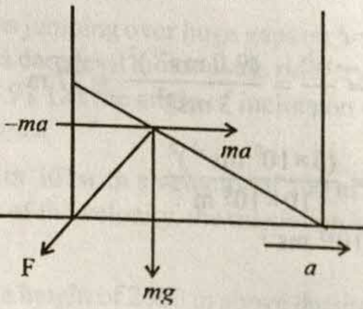


Fig. 4.13

This is the minimum value of the bucket's speed at the top of the vertical circle. For $R = 1.0$ m and taking $g = 10$ ms^{-2} we get

$\upsilon = 10$ m s^{-1} = 3.2 ms^{-1}

Answers to Terminal Problems

5. 10 ms^{-1}

6. 2 ms^{-2}

7. $900\sqrt{3}$ m

8. 1.05×10^{-3} ms^{-1}

9. 77.1 m

10. $\upsilon_x = 250\sqrt{3}$ ms^{-1}

    $\upsilon y = 250$ ms^{-1}

    Vertical height = 500 m

    Horizontal range = 3125 m

11. $t = 20$ s, 999.9 m

12. $\upsilon = \sqrt{\dfrac{m\,g\,r}{m}}$

13. 125 N

5

GRAVITATION

Have you ever thought why a ball thrown upward always comes back to the ground? Or a coin tossed in air falls back on the ground. Since times immemorial, human beings have wondered about this phenomenon. The answer was provided in the 17th century by Sir Isaac Newton. He proposed that the gravitational force is responsible for bodies being attracted to the earth. He also said that it is the same force which keeps the moon in its orbit around the earth and planets bound to the Sun. It is a universal force, that is, it is present everywhere in the universe. In fact, it is this force that keeps the whole universe together.

In this lesson you will learn Newton's law of gravitation. We shall also study the acceleration caused in objects due to the pull of the earth. This acceleration, called acceleration due to gravity, is not constant on the earth. You will learn the factors due to which it varies. You will also study Kepler's laws of planetary motion and orbits of artificial satellites of various kinds in this lesson. Finally, we shall recount some of the important programmes and achievements of India in the field of space research.

Objectives

After studying this lesson, you should be able to:

- *state the law of gravitation;*
- *calculate the value of acceleration due to gravity of a heavenly body;*
- *analyse the variation in the value of the acceleration due to gravity with height, depth and latitude;*
- *identify the force responsible for planetary motion and state Kepler's laws of planetary motion;*
- *calculate the orbital velocity and the escape velocity;*
- *explain how an artificial satellite is launched;*
- *distinguish between polar and equatorial satellites;*
- *state conditions for a satellite to be a geostationary satellite;*
- *calculate the height of a geostationary satellite and list their applications; and*
- *state the achievements of India in the field of satellite technology.*

5.1 Law of Gravitation

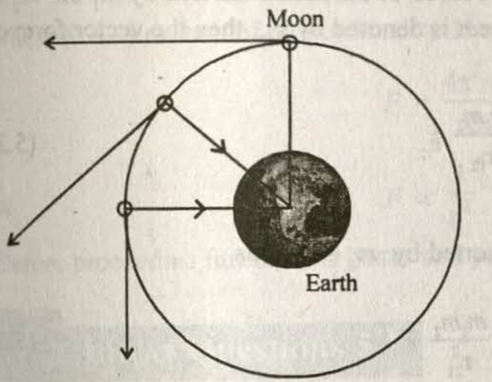


Fig. 5.1 : At each point on its orbit, the moon would have flown off along a tangent but the attraction of the earth keeps it in its orbit.

It is said that Newton was sitting under a tree when an apple fell on the ground. This set him thinking: since all apples and other objects fall to the ground, there must be some force from the earth acting on them. He asked himself: Could it be the same force which keeps the moon in orbit around the earth? Newton argued that at every point in its orbit, the moon would have flown along a tangent, but is held back to the orbit by some force (Fig. 5.1). Could this continuous 'fall' be due to the same force which forces apples to fall to the ground? He had deduced from Kepler's laws that the force between the Sun and planets varies as $1/r^2$. Using this result he was able to show that it is the same force that keeps the moon in its orbit around the earth. Then he generalised the idea to formulate the universal law of gravitation as.

Every particle attracts every other particle in the universe with a force which varies as the product of their masses and inversely as the square of the distance between them. Thus, if m_1 and m_2 are the masses of the two particles, and r is the distance between them, the magnitude of the force F is given by.

$$F \propto \frac{m_1m_2}{r^2}$$

or

$$F = G\frac{m_1m_2}{r_2} \qquad (5.1)$$

The constant of proportionality, G, is called **the universal constant of gravitation. Its value remains the same between any two objects everywhere in the universe.** This means that if the force between two particles is F on the earth, the force between these particles kept at the same distance anywhere in the universe would be the same.

One of the extremely important characteristics of the gravitational force is that it is always attractive. It is also one of the fundamental forces of nature.

Remember that the attraction is mutual, that is, particle of mass m_1 attracts the particle of mass m_2 and m_2 attracts m_1. Also, the force is along the line joining the two particles.

Knowing that the force is a vector quantity, does Eqn. (5.1) need modification? The answer to this question is that the equation should reflect both magnitude and the direction of the force. As stated, the gravitational force acts along the line

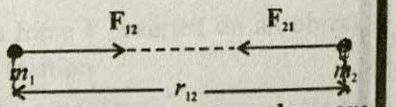


Fig. 5.2 : The masses m_1 and m_2 are placed at a distance r_{12} from eact other. The mass m_1 attracts m_2 with a force F_{12}.

joining the two particles. That is, m_2 attracts m_1 with a force which **is along the line joining the two particles** (Fig. 5.2). If the force of attraction exerted by m_1 on m_2 is denoted by $\mathbf{F}_{12}$ and the distance between them is denoted by $\mathbf{r}_{12}$, then the vector form of the law of gravitation is

$$\mathbf{F}_{12} = G\frac{m_1 m_2}{\mathbf{r}_{12}}\,\hat{\mathbf{r}}_{12} \tag{5.2}$$

Here $\hat{\mathbf{r}}_{12}$ is a unit vector from m_1 to m_2

In a similar way, we may write the force exerted by m_2 on m_1 as

$$\mathbf{F}_{21} = -G\frac{m_1 m_2}{\mathbf{r}_{21}^2}\,\hat{\mathbf{r}}_{21} \tag{5.3}$$

As $\hat{\mathbf{r}}_{12} = -\hat{\mathbf{r}}_{21}$, from Eqns. (5.2) and (5.3) we find that

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \tag{5.4}$$

The forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are equal and opposite and form a pair of forces of action and reaction in accordance with Newton's third law of motion. Remember that $\hat{\mathbf{r}}_{12}$ and $\hat{\mathbf{r}}_{21}$ have unit magnitude. However, the directions of these vectors are opposite to each other.

Unless specified, in this lesson we would use only the magnitude of the gravitational force.

The value of the constant G is so small that it could not be determined by Newton or his contemporary experimentalists. It was determined by Cavendish for the first time about 100 years later. Its accepted value today is 6.67×10^{-11} Nm^{2}kg^{-2}. It is because of the smallness of G that the gravitational force due to ordinary objects is not felt by us.

Example 5.1 : Kepler's third law states (we shall discuss this in greater details later) that if r is the mean distance of a planet from the Sun, and T is its orbital period, then $r^3/T^2 =$ const. Show that the force acting on a planet is inversely proportional to the square of the distance.

Solution : Assume for simplicity that the orbit of a planet is circular. (In reality, the orbits are nearly circular.) Then the centripetal force acting on the planet is

$$F = \frac{mv^2}{r}$$

where v is the orbital velocity. Since $v = r\omega = \frac{2\pi r}{T}$, where T is the period, we can rewrite above expression as

$$F = m\left(\frac{2\pi r}{T}\right)^2 \Big/ r$$

or

$$F = \frac{4\pi^2 mr}{T^2}$$

But $T^2 \propto r^3$ or $T^2 = Kr^3$ (Kelpler's 3rd law)

where K is a constant of proportionality. Hence

$$\therefore \quad F = \frac{4\pi^2 mr}{Kr^3} = \frac{4\pi^2}{K} \times \frac{m}{r^2} = \frac{4\pi^2 m}{K} \cdot \frac{1}{r^2}$$

or

$$F \propto \frac{1}{r^2} \qquad (\because \frac{4\pi^2 m}{K} \text{ is constant for a planet})$$

Before proceedins further, it is better that you check your progress.

Intext Questions 5.1

1. The period of revolution of the moon around the earth is 27.3 days. Remember that this is the period with respect to the fixed stars (the period of revolution with respect to the moving earth is about 29.5 days; it is this period that is used to fix the duration of a month in some calendars). The radius of moon's orbit is 3.84×10^8 m (60 times the earth's radius). Calculate the centripetal acceleration of the moon and show that it is very close to the value given by 9.8 ms^{-2} divided by 3600, to take account of the variation of the gravity as $1/r^2$.

....

2. From Eqn. (5.1), deduce dimensions of G.

....

3. Using Eqn. (5.1), show that G may be defined as the magnitude of force between two masses of 1 kg each separated by a distance of 1 m.

....

4. The magnitude of force between two masses placed at a certain distance is F. What happens to F if (i) the distance is doubled without any change in masses, (ii) the distance remains the same but each mass is doubled, (iii) the distance is doubled and each mass is also doubled?

....

5. Two bodies having masses 50 kg and 60 kg are seperated by a distance of 1m. Calculate the gravitational force between them.

....

5.2 Acceleration Due to Gravity

From Newton's second law of motion you know that a force **F** exerted on an object produces an acceleration **a** in the object according to the relation

$$\mathbf{F} = m\mathbf{a} \tag{5.5}$$

The force of gravity, i.e., the force exerted by the earth on a body lying on or near its surface, also produces an acceleration in the body. The acceleration produced by the force of gravity is called the **acceleration due to gravity**. It is denoted by the symbol g. According to Eq. (5.1), the magnitude of the force of gravity on a particle of mass m on the earth's surface is given by

$$F = G\frac{mM}{R^2} \tag{5.6}$$

where M is the mass of the earth and R is its radius. From Eqns. (5.5) and (5.6), we get

$$mg = G\frac{mM}{R^2}$$

or

$$g = G\frac{M}{R^2} \tag{5.7}$$

Remember that the force due to gravity on an object is directed towards the center of the earth. It is this direction that we call vertical. Fig. 5.3 shows vertical directions at different places on the earth. The direction perpendicular to the vertical is called the **horizontal** direction.

Once we know the mass and the radius of the earth, or of any other celestial body such as a planet, the value of g at its surface can be calculated using Eqn. (5.7). On the surface of the earth, the value of g is taken as 9.8 ms^{-2}.

Given the mass and the radius of a satellite or a planet, we can use Eqn. (5.7) to find the acceleration due to the gravitational attraction of that satellite or planet.

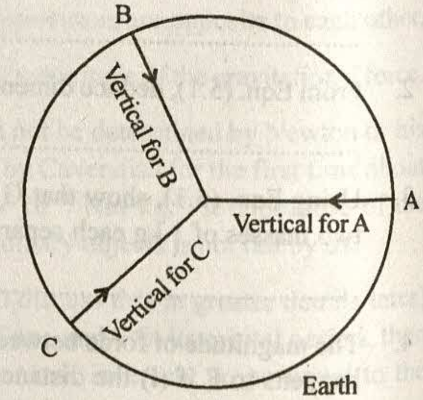


Fig. 5.3 : The vertical direction at any place is the direction towards the centre of earth at that point

Before proceeding further, let us look at Eqn. (5.7) again. The acceleration due to gravity produced in a body is independent of its mass. This means that a heavy ball and a light ball will fall with the same velocity. **If we drop these balls from a certain height at the same time, both would reach the ground simultaneously.**

Activity 5.1

Take a piece of paper and a small pebble. Drop them simultaneously from a certain height. Observe the path followed by the two bodies and note the times at which they touch the ground. Then take two pebbles, one heavier than the other. Release them simultaneously from a height and observe the time at which they touch the ground.

Fall Under Gravity

The fact that a heavy pebble falls at the same rate as a light pebble, might appear a bit strange. Till sixteenth century it was a common belief that a heavy body falls faster than a light body. However, the great scientist of the time, Galileo, showed that the two bodies do indeed fall at the same rate. It is said that he went up to the top of the Tower of Pisa and released simultaneously two iron balls of considerably different masses. The balls touched the ground at the same time. But when feather and a stone were made to fall simultaneously, they reached the ground at different times. Galileo argued that the feather fell slower because it experienced greater force of buoyancy due to air. He said that if there were no air, the two bodies would fall together. In recent times, astronauts have performed the feather and stone experiment on the moon and verified that the two fall together. Remember that the moon has no atmosphere and so no air.

Under the influence of gravity, a body falls vertically downwards towards the earth. For small heights above the surface of the earth, the acceleration due to gravity does not change much. Therefore, the equations of motion for initial and final velocities and the distance covered in time t are given by

$$v = u + gt$$

$$s = ut + \left(\frac{1}{2}\right)gt^2$$

and

$$v^2 = u^2 + 2gs. \tag{5.8}$$

It is important to remember that g **is always directed vertically downwards, no matter what the direction of motion of the body is.** A body falling with an acceleration equal to g is said to be in **free-fall.**

From Eqn. (5.8) it is clear that if a body begins to fall from rest, it would fall a distance $h = (1/2)gt^2$ in time t. So, a simple experiment like dropping a heavy coin from a height and measuring its time of fall with the help of an accurate stop watch could give us the value of g. If you measure the time taken by a five-rupee coin to fall through a distance of 1 m, you will find that the average time of fall for several trials is 0.45 s. From this data, the value of g can be calculated. However, in the laboratory you would determine g by an indirect method, using a simple pendulum.

You must be wondering as to why we take radius of the earth as the distance between the earth and a particle on its surface while calculating the force of gravity on that particle. When we consider two discreet particles or mass points, the separation between them is just the distance between them. But when we calculate gravitational force between extended bodies, what distance do we take into account? To resolve this problem, the concept of **centre of gravity of a body is introduced**. This is a point such that, as far as the gravitational effect is concerned, we may replace the whole body by just this point and the effect would be the same. For geometrically regular bodies of uniform density, such as spheres, cylinders, rectangles, the geometrical center is also the centre of gravity. That is why we choose the center of the earth to measure distances to other bodies. For irregular bodies, there is no easy way to locate their centres of gravity.

Where is the center of gravity of metallic ring located? It should lie at the centei the ring. But this point is outside the mass of the body. It means that the centre of gravity of a body may lie outside it. Where is your own centre of gravity located? Assuming that we have a regular shape, it would be at the centre of our body, somewhere beneath the navel.

Later on in this course, you would also learn about the **centre of mass** of a body. This is a point at which the whole mass of the body can be assumed to be concentrated. In a **uniform gravitational field**, the kind we have near the earth, the centre of gravity coincides with the centre of mass.

The use of centre of gravity, or the center of mass, makes our calculations extremely simple. Just imagine the amount of calculations we would have to do if we have to calculate the forces between individual particles a body is made of and then finding the resultant of all these forces.

You should remember that G and *g* represent different physical quantities. G **is the universal constant of gravitation** which remains the same everywhere, while *g* is **acceleration due to gravity,** which may change from place to place, as we shall see in the next section.

You may like to answer a few questions to check your progress.

Intext Questions 5.2

1. The mass of the earth is 5.97×10^{24} kg and its mean radius is 6.371×10^{6} m. Calculate the value of *g* at the surface of the earth.

   ....

2. Careful measurements show that the radius of the earth at the equator is 6378 km while at the poles it is 6357 km. Compare values of *g* at the poles and at the equator.

   ....

3. A particle is thrown up. What is the direction of *g* when (i) the particle is going up, (ii) when it is at the top of its journey, (iii) when it is coming down, and (iv) when it has come back to the ground?

   ....

4. The mass of the moon is 7.3×10^{22} kg and its radius is 1.74×10^{6} m. Calculate the gravitational acceleration at its surface.

   ....

5.3 Variation in the Value of *g*

5.3.1 Variation with Height

The quantity R^2 in the denominator on the right hand side of Eqn. (5.7) suggests that **the magnitude of *g* decreases as square of the distance from the centre of the earth increases**. So, at a distance *R* from the surface, that is, at a distance 2*R* from the centre of the earth, the value of *g* becomes (1/4) th of the value of *g* at the surface. However, if the distance h above the surface of the earth, called **altitude,** is small compared with the radius of the earth, the value of g, denoted by g_h, is given by

$$g_h = \frac{GM}{(R+h)^2}$$

$$= \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$= \frac{g}{\left(1+\frac{h}{R}\right)^2} \qquad (5.9)$$

where $g = GM/R^2$ is the value of acceleration due to gravity at the surface of the earth. Therefore,

$$\frac{g}{g_h} = \left(1+\frac{h}{R}\right)^2 = 1 + \frac{2h}{R} + \left(\frac{h}{R}\right)^2$$

Since (h/R) is a small quantity, $(h/R)^2$ will be a still smaller quantity. So it can be neglected in comparison to (h/R). Thus

$$g_h = \frac{g}{\left(1+\frac{2h}{R}\right)} \qquad (5.10)$$

Let us take an example to understand how we apply this concept.

Example 5.2 : Modern aircrafts fly at heights upward of 10 km. Let us calculate the value of g at an altitude of 10 km. Take the radius of the earth as 6400 km and the value of g on the surface of the earth as 9.8 ms^{-2}.

Solution : From Eqn. (5.8), we have

$$g_h = \frac{g}{\left(1+\frac{2.(10)\ \text{km}}{6400\ \text{km}}\right)} = \frac{9.8\ \text{ms}^{-2}}{1.003} = 9.77\ \text{ms}^{-2}.$$

5.3.2. Variation of *g* with Depth

Consider a point P at a depth d inside the earth (Fig. 5.4). Let us assume that the earth is a sphere of uniform density ρ. The distance of the point P from the center of the earth is $r = (R - d)$. Draw a sphere of radius $(r - d)$. A mass placed at P will experience gravitational force from particles in (i) the shell of thickness d, and (ii) the sphere of radius

r. It can be shown that the forces due to all the particles in the shell cancel each other. That is, the net force on the particle at P due to the matter in the shell is zero. Therefore, in calculating the acceleration due to gravity at P, we have to consider only the mass of the sphere of radius $(r - d)$. The mass M' of the sphere of radius $(r - d)$ is

$$M' = \frac{4\pi}{3}\rho\,(R - d)^3 \qquad (5.10)$$

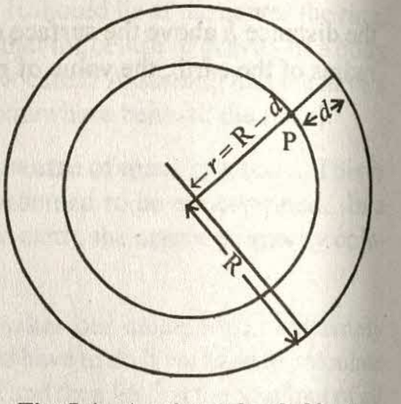


Fig. 5.4 : A point at depth d is at a distance $r = R - d$ from the centre of the earth

The acceleration due to gravity experienced by a particle placed at P is, therefore,

$$g_d = G\frac{M'}{(R-d)^2} = \frac{4\pi G}{3}\rho\,(R-d) \qquad (5.11)$$

Note that as d increases, $(R - d)$ decreases. This means that **the value of g decreases as we go below the earth**. At d = R, that is, at the centre of the earth, the acceleration due to gravity will vanish. Also note that $(R - d) = r$ is the distance from the centre of the earth. Therefore, acceleration due to gravity is linearly proportional to r. The variation of g from the centre of the earth to distances far from the earth's surface is shown in Fig. 5.5.

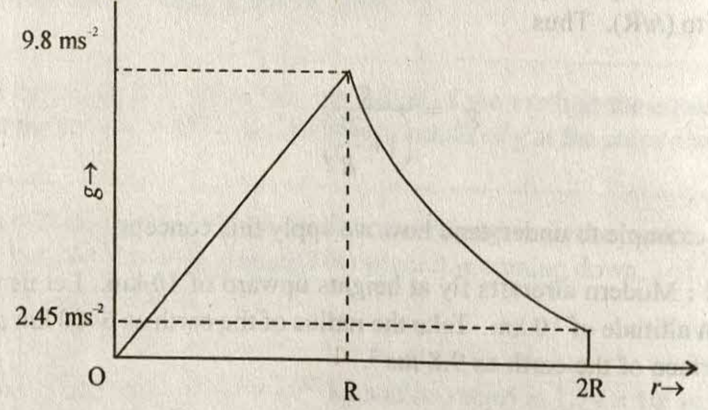


Fig. 5.5 : Variation of g with distance from the centre of the earth

We can express g_d in terms of the value at the surface by realizing that at $d = 0$, we get the surface value: $g = \frac{4\pi G}{3}\rho R$. It is now easy to see that

$$g_d = g\frac{(R-d)}{R} = g\left(1 - \frac{d}{R}\right),\ 0 \le d \le R \qquad (5.12)$$

On the basis of Eqns. (5.9) and (5.12), we can conclude that **g decreases with both height as well as depth.**

Internal Structure of the Earth

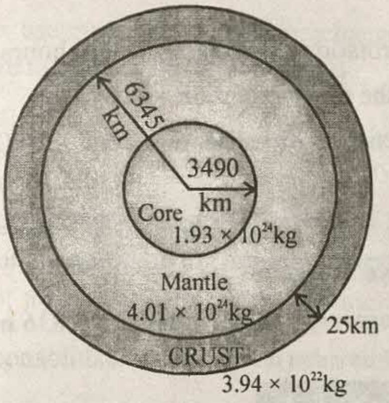


Fig.5.6 : Structure of the earth (not to scale). Three prominent layers of the earth are shown along with their estimated masses.

Refer to Fig. 5.6 You will note that most of the mass of the earth is concentrated in its core. The top surface layer is very light. For very small depths, there is hardly any decrease in the mass to be taken into account for calculating g, while there is a decrease in the radius. So, the value of g increases up to a certain depth and then starts decreasing. It means that assumption about earth being a uniform sphere is not correct.

5.3.3 Variation of *g* with Latitude

You know that the earth rotates about its axis. Due to this, every particle on the earth's surface excecutes circular motion. In the absence of gravity, all these particles would be flying off the earth along the tangents to their circular orbits. Gravity plays an important role in keeping us tied to the earth's surface. You also know that to keep a particle in circular motion, it must be supplied centripetal force. A small part of the gravity force is used in supplying this centripetal force. As a result, the force of attraction of the earth on objects on its surface is slightly reduced. The maximum effect of the rotation of the earth is felt at the equator. At poles, the effect vanishes completely. We now quote the formula for variation in g with latitude without derivation. If g_λ denotes the value of g at latitude λ and g is the value at the poles, then

$$g_\lambda = g - R\omega^2 \cos\lambda, \qquad (5.13)$$

where ω is the angular velocity of the earth and R is its radius. You can easily see that at the poles, $\lambda = 90$ degrees, and hence $g_\lambda = g$.

Example 5.3 : Let us calculate the value of g at the poles.

Solution : The radius of the earth at the poles = 6357 km = 6.357×10^6 m

The mass of the earth = 5.97×10^{24} kg

Using Eqn. (5.7), we get

$$g \text{ at the poles} = [6.67 \times 10^{-11} \times 5.97 \times 10^{24} / (6.357 \times 10^6)^2] \text{ ms}^{-2}$$

$$= 9.853 \text{ ms}^{-2}$$

Example 5.4 : Now let us calculate the value of g at $\lambda = 60°$, where radius of earth is 6371 km.

Solution : The period of rotation of the earth, $T = 24$ hours $= (24 \times 60 \times 60)$ s

$\therefore$ frequency of the earth's rotation $= 1/T$

angular frequency of the earth $\omega = 2\pi/T = 2\pi/(24 \times 60 \times 60)$

$= 7.27 \times 10^{-5}$

$\therefore R\omega^2 \cos\lambda = 6.371 \times 10^6 \times (7.27 \times 10^{-5})^2 \times 0.5 = 0.017 \text{ ms}^{-2}$

Since $g_0 = g - R\omega^2 \cos\lambda$, we can write

g_λ (at latitude 60 degrees) $= 9.853 - 0.017 = 9.836 \text{ ms}^{-2}$

Intext Questions 5.3

1. At what height must we go so that the value of g becomes half of what it is at the surface of the earth?

....

2. At what depth would the value of g be 80% of what it is on the surface of the earth?

....

3. The latitude of Delhi is approximately 30 degrees north. Calculate the difference between the values of g at Delhi and at the poles.

....

4. A satellite orbits the earth at an altitude of 1000 km. Calcultate the acceleration due to gravity acting on the satellite (i) using Eqn. (5.9) and (ii) using the relation g is proportional to $1/r^2$, where r is the distance from the centre of the earth. Which method do you consider better for this case and why?

....

5.4 Weight and Mass

The force with which a body is pulled towards the earth is called its weight. If m is the mass of the body, then its weight W is given by

$$W = mg \tag{5.14}$$

Since weight is a force, its unit is newton. If your mass is 50 kg, your weight would be 50 kg × 9.8 ms^{-2} = 490 N.

Since g varies from place to place, weight of a body also changes from place to place. The weight is maximum at the poles and minimum at the equator. This is because the radius of the earth is minimum at the poles and maximum at the equator. The weight decreases when we go to higher altitudes or inside the earth.

The mass of a body, however, does not change. Mass is an intrinsic property of a body.

Therefore, it stays constant wherever the body may be situated.

Note: In everyday life we often use mass and weight interchangeably. Spring balances, though they measure weight, are marked in kg (and not in N).

Activity 5.2

Calculate the weight of an object of mass 50 kg at distances of 2R, 3R, 4R, 5R and 6R from the centre of the earth. Plot a graph showing the weight against distance. Show on the same graph how the mass of the object varies with distance.

Try the following questions to consolidate your ideas on mass and weight.

Intext Questions 5.4

1. Suppose you land on the moon. In what way would your weight and mass be affected?

   ....

2. Compare your weight at Mars with that on the earth? What happens to your mass? Take the mass of Mars = 6×10^{23} kg and its radius as 4.3×10^{6} m.

   ....

3. You must have seen two types of balances for weighing objects. In one case there are two pans. In one pan, we place the object to be weighed and in the other we place weights. The other type is a spring balance. Here the object to be weighed is suspended from the hook at the end of a spring and reading is taken on a scale. Suppose you weigh a bag of potatoes with both the balances and they give the same value. Now you take them to the moon. Would there be any change in the measurements made by the two balances?

   ....

5.5 Kepler's Laws of Planetary Motion

In ancient times it was believed that all heavenly bodies move around the earth. Greek astronomers lent great support to this notion. So strong was the faith in the earth-centred universe that all evidences showing that planets revolved around the Sun were ignored. However, Polish Astronomer Copernicus in the 15th century proposed that all the planets revolved around the Sun. In the 16th century, Galileo, based on his astronomical observations, supported Copernicus. Another European astronomer, Tycho Brahe, collected a lot of observations on the motion of planets. Based on these observations, his assistant Kepler formulated **laws of planetary motion**.

Johannes Kepler

German by birth, Johannes Kepler, started his career in astronomy as an assistant to Tycho Brahe. Tycho religiously collected the data of the positions of various planets on the daily basis for more than 20 years. On his death, the data was passed on to Kepler who spent 16 years to analyse the data. On the basis of his analysis, Kepler arrived at the three laws of planetary motion.

He is considered as the founder of geometrical optics as he was the first person to describe the working of a telescope through its ray diagram.

For his assertion that the earth revolved around the Sun, Galileo came into conflict with the church because the Christian authorities believed that the earth was at the centre of the universe. Although he was silenced, Galileo kept recording his observations quietly, which were made public after his death. Interestingly, Galileo was freed from that blame recently by the present Pope.

Kepler formulated three laws which govern the motion of planets. These are:

1. The orbit of a planet is an ellipse with the Sun at one of the foci (Fig. 5.7). (An ellipse has two foci.)

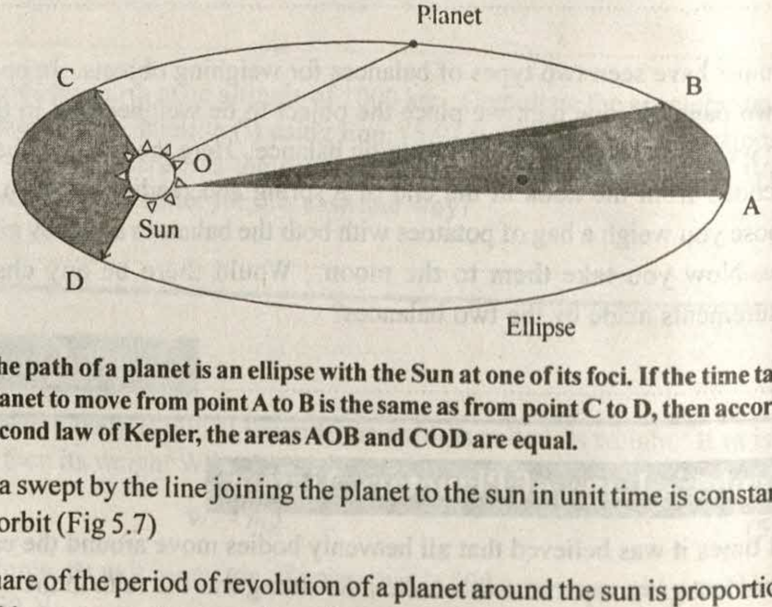


Fig. 5.7 : The path of a planet is an ellipse with the Sun at one of its foci. If the time taken by the planet to move from point A to B is the same as from point C to D, then according to the second law of Kepler, the areas AOB and COD are equal.

2. The area swept by the line joining the planet to the sun in unit time is constant throughout the orbit (Fig 5.7)
3. The square of the period of revolution of a planet around the sun is proportional to the cube of its average distance from the Sun. If we denote the period by T and the average distance from the Sun as r, $T^2 \alpha r^3$.

Let us look at the third law a little more carefully. You may recall that Newton used this law to deduce that the force acting between the Sun and the planets varied as $1/r^2$ (Example 5.1). Moreover, if T_1 and T_2 are the orbital periods of two planets and r_1 and r_2 are their mean distances from the Sun, then the third law implies that

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \qquad (5.15)$$

The constant of proportionality cancels out when we divide the relation for one planet by the relation for the second planet. This is a very important relation. For example, it can be used to get T_2, if we know T_1, r_1 and r_2.

Example 5.5 : Calculate the orbital period of planet mercury, if its distance from the Sun is 57.9×10^9 m. You are given that the distance of the earth from the Sun is 1.5×10^{11} m.

Solution : We know that the orbital period of the earth is 365.25 days. So, $T_1 = 365.25$ days and $r_1 = 1.5 \times 10^{11}$ m. We are told that $r_2 = 57.9 \times 10^9$ m for mercury. Therefore, the orbital period of mercury is given by T_2

$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

On substituting the values of various quantities, we get

$$T_2 = \sqrt{\frac{T_1^2 r_2^3}{r_1^3}} = \sqrt{\frac{(365.25)^2 \times (57.9 \times 10^9)^3 \text{ m}^3}{(1.5 \times 10^{11})^3 \text{ m}^3}} \text{ days}$$

$$= 87.6 \text{ days.}$$

In the same manner you can find the orbital periods of other planets. The data is given below. You can also check your results with numbers in Table 5.1.

Table 5.1: Some data about the planets of solar system

Name of the planet	Mean distance from the Sun (in terms of the distance of earth)	Radius (x10³ km)	Mass (Earth Masses)
Mercury	0.387	2.44	0.53
Venus	0.72	6.05	0.815
Earth	1.0	6.38	1.00
Mars	1.52	3.39	0.107
Jupiter	5.2	71.40	317.8
Saturn	9.54	60.00	95.16
Uranus	19.2	25.4	14.50
Neptune	30.1	24.3	17.20
Pluto	39.4	1.50	0.002

Kepler's laws apply to any system where the force binding the system is gravitational in nature. For example, they apply to Jupiter and its satellites. They also apply to the earth and its satellites like the moon and artificial satellites.

Example 5.6 : A satellite has an orbital period equal to one day. (Such satellites are called geosynchronous satellites.) Calculate its height from the earth's surface, given that the distance of the moon from the earth is 60 R_E (R_E is the radius of the earth), and its orbital period is 27.3 days. [This orbital period of the moon is with respect to the fixed stars. With respect to the earth, which itself is in orbit round the Sun, the orbital period of the moon is about 29.5 day.]

Solution : A geostationary satellite has a period T_2 equal to 1 day. For moon T_1 = 27.3 days and $r_1 = 60\ R_E$, T_2 = 1 day. Using Eqn. (5.15), we have

$$r_2 = \left[\frac{r_1^3 T_2^2}{T_1^2}\right]^{1/3} = \left[\frac{(60^3\ R_E^3)(1^2\ \text{day}^2)}{27.3^2\ \text{day}^2}\right]^{1/3} = 6.6\ R_E.$$

Remember that the distance of the satellite is taken from the centre of the earth. To find its height from the surface of the earth, we must subtract R_E from 6.6 R_E. The required distance from the earth's surface is 5.6 R_E. If you want to get this distance in km, multiply 5.6 by the radius of the earth in km.

5.5.1 Orbital Velocity of Planets

We have so for talked of orbital periods of planets. If the orbital period of a planet is T and its distance from the Sun is r, then it covers a distance $2\pi r$ in time T. Its orbital velocity is, therefore,

$$\upsilon_{orb} = \frac{2\pi r}{T} \tag{5.16}$$

There is another way also to calculate the orbital velocity. The centripetal force experienced by the planet is $m\upsilon_{orb}^2/r$, where m is its mass. This force must be supplied by the force of gravitation between the Sun and the planet. If M is the mass of the Sun, then the gravitational force on the planet is $\frac{GmM_s}{r^2}$. Equating the two forces, we get

$$\frac{m\upsilon_{orb}^2}{r} = \frac{G\ M_s}{r^2},$$

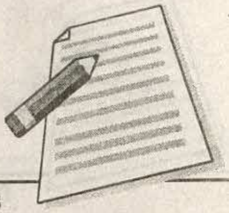
so that,

$$\upsilon_{orb} = \sqrt{\frac{G\ M_s}{r^2}} \tag{5.17}$$

Notice that the mass of the planet does not enter the above equation. The orbital velocity depends only on the distance from the Sun. Note also that if you substitute υ from Eqn. (5.16) in Eqn. (5.17), you get the third law of Kepler.

Intext Questions 5.5

1. Many planetary systems have been discovered in our Galaxy. Would Kepler's laws be applicable to them?



Notes

2. Two artificial satellites are orbiting the earth at distances of 1000 km and 2000 km from the surface of the earth. Which one of them has the longer period? If the time period of the former is 90 min, find the time period of the latter.

....

3. A new small planet, named Sedna, has been discovered recently in the solar system. It is orbiting the Sun at a distance of 86 AU. (An AU is the distance between the Sun and the earth. It is equal to 1.5×10^{11} m.) Calculate its orbital period in years.

....

4. Obtain an expression for the orbital velocity of a satellite orbiting the earth.

....

5. Using Eqns. (5.16) and (5.17), obtain Kepler's third law.

....

5.6 Escape Velocity

You now know that a ball thrown upwards always comes back due to the force of gravity. If you throw it with greater force, it goes a little higher but again comes back. If you have a friend with great physical power, ask him to throw the ball upwards. The ball may go higher than what you had managed, but it still comes back. You may then ask: Is it possible for an object to escape the pull of the earth? The answer is 'yes'. The object must acquire what is called the **escape velocity. It is defined as the minimum velocity required by an object to escape the gravitational pull of the earth.**

It is obvious that the escape velocity will depend on the mass of the body it is trying to escape from, because the gravitational pull is proportional to mass. It will also depend on the radius of the body, because smaller the radius, stronger is the gravitational force.

The escape velocity from the earth is given by

$$v_{esc} = \sqrt{\frac{2GM}{R}} \qquad (5.18)$$

where M is the mass of the earth and R is its radius. For calculating escape velocity from any other planet or heavenly body, mass and radius of that heavenly body will have to be substituted in the above expression.

It is not that the force of gravity ceases to act when an object is launched with escape velocity. The force does act. Both the velocity of the object as well as the force of gravity acting on it decrease as the object goes up. It so happens that the force becomes zero before the velocity becomes zero. Hence the object escapes the pull of gravity.

Try the following questions to grasp the concept.

Intext Questions 5.6

1. The mass of the earth is 5.97×10^{24} kg and its radius is 6371 km. Calculate the escape velocity from the earth.

   ....

2. Suppose the earth shrunk suddenly to one-fourth its radius without any change in its mass. What would be the escape velocity then?

   ....

3. An imaginary planet X has mass eight times that of the earth and radius twice that of the earth. What would be the escape velocity from this planet in terms of the escape velocity from the earth?

   ....

5.7 Artificial Satellites

A cricket match is played in Sydney in Australia but we can watch it live in India. A game of Tennis played in America is enjoyed in India. Have you ever wondered what makes it possible? All this is made possible by artificial satellites orbiting the earth. You may now ask : How is an artificial satellite put in an orbit?

You have already studied the motion of a projectile. If you project a body at an angle to the horizontal, it follows a parabolic path. Now imagine launching bodies with increasing force. What happens is shown in Fig. 5.8. Projectiles travel larger and larger distances before falling back to the earth. Eventually, the projectile goes into an orbit around the earth. It becomes an **artificial satellite**. Remember that such satellites are man-made and launched with a particular purpose in mind. Satellites like the moon are natural satellites.

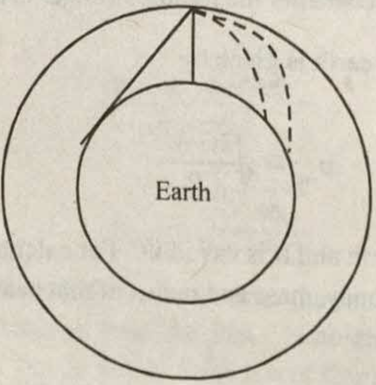


Fig. 5.8 : A projectile to orbit the earth

In order to put a satellite in orbit, it is first lifted to a height of about 200 km to minimize loss of energy due to friction in the atmosphere of the earth. Then it is given a horizontal push with a velocity of about 8 kms^{-1}.

The orbit of an artificial satellite also obeys Kepler's laws because the controlling force is gravitational force between the satellite and the earth. The orbit is elliptic in nature and its plane always passes through the center of the earth.

Remember that the orbital velocity of an artificial satellite has to be less than the escape velocity; otherwise it will break free of the gravitational field of the earth and will not orbit around the earth. From the expressions for the orbital velocity of a satellite close to the earth and the escape velocity from the earth, we can write

$$\upsilon_{orb} = \frac{\upsilon_{sec}}{\sqrt{2}} \qquad (5.19)$$

Artificial satellites have generally two types of orbits (Fig. 5.9) depending on the purpose for which the satellite is launched. Satellites used for tasks such as remote sensing have **polar orbits**. The altitude of these orbits is about 800 km. If the orbit is at a height of less than about 300 km, the satellite loses energy because of friction caused by the particles of the atmosphere. As a result, it moves to a lower height where the density is high. There it gets burnt. The time period of polar satellites is around 100 minutes. It is possible to make a polar satellite **sun-synchronous,** so that it arrives at the same latitude at the same time every day. During repeated crossing, the satellite can scan the whole earth as it spins about its axis (Fig. 5.10). Such satellites are used for collecting data for weather prediction, monitoring floods, crops, bushfires, etc.

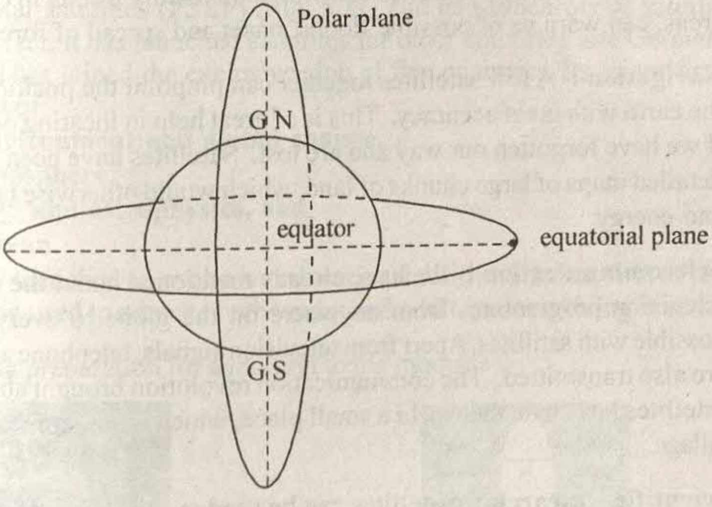


Fig.5.9: Equitorial and polar orbits

Satellites used for communications are put in equatorial orbits at high altitudes. Most of these satellites are **geo-synchronous, the ones which have the same orbital period as the period of rotation of the earth,** equal to 24 hours. Their height, as you saw in Example 5.6 is fixed at around 36000 km. Since their orbital period matches that of the earth, they appear to be hovering above the same spot on the earth. A combination of such satellites covers the entire globe, and signals can be sent from any place on the globe to any other place. Since a geo-synchronous satellite observes the same spot on the earth all the time, it can also be used for monitoring any peculiar happening that takes a long time to develop, such as severe storms and hurricanes.

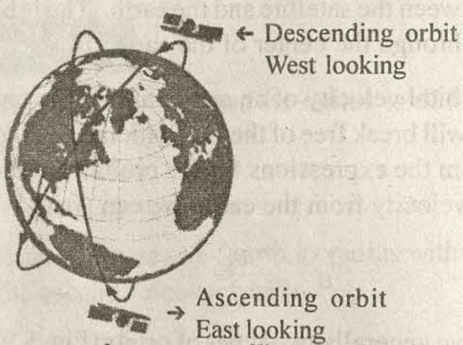


Fig. 5.10: A sun synchromous satellite scanning the earth

Applications of Satellites

Artificial satellites have been very useful to mankind. Following are some of their applications:

1. **Weather Forecasting :** The satellites collect all kinds of data which is useful in forecasting long term and short term weather. The weather chart that you see every day on the television or in newspapers is made from the data sent by these satellites. For a country like India, where so much depends on timely rains, the satellite data is used to watch the onset and progress of monsoon. Apart from weather, satellites can watch unhealthy trends in crops over large areas, can warn us of possible floods, onset and spread of forest fire, etc.

2. **Navigation :** A few satellites together can pinpoint the position of a place on the earth with great accuracy. This is of great help in locating our own position if we have forgotten our way and are lost. Satellites have been used to prepare detailed maps of large chunks of land, which would otherwise take a lot of time and energy.

3. **Telecommunication :** We have already mentioned about the transmission of television programmes from anywhere on the globe to everywhere became possible with satillites. Apart from television signals, telephone and radio signals are also transmitted. The communication revolution brought about by artificial satellites has made the world a small place, which is sometimes called a global village.

4. **Scientific Research :** Satellites can be used to send scientific instruments in space to observe the earth, the moon, comets, planets, the Sun, stars and galaxies. You must have heard of Hubble Space Telescope and Chandra X-Ray Telescope. The advantage of having a telescope in space is that light from distant objects does not have to go through the atmosphere. So there is hardly any reduction in its intensity. For this reason, the pictures taken by Hubble Space Telescope are of much superior quality than those taken by terrestrial telescopes.

   Recently, a group of Europeon scientists have observed an earth like planet outside our solar system at a distance of 20 light years.

5. **Monitoring Military Activities :** Artificial satellites are used to keep an eye on the enemy troop movement. Almost all countries that can afford cost of these satellites have them.

Notes

Vikram Ambalal Sarabhai

Born in a family of industrialists at Ahmedabad, Gujarat, India. Vikram Sarabhai grew to inspire a whole generation of scientists in India. His initial work on time variation of cosmic rays brought him laurels in scientific fraternity. A founder of Physical Research Laboratory, Ahmedabad and a pioneer of space research in India, he was the first to realise the dividends that space research can bring in the fields of communication, education, metrology, remote sensing and geodesy, etc.

5.7.1 Indian Space Research Organization

India is a very large and populous country. Much of the population lives in rural areas and depends heavily on rains, particularly the monsoons. So, weather forecast is an important task that the government has to perform. It has also to meet the communication needs of a vast population. Then much of our area remains unexplored for minerals, oil and gas. Satellite technology offers a cost-effective solution for all these problems. With this in view, the Government of India set up in 1969 the Indian Space Research Organization (ISRO) under the dynamic leadership of Dr. Vikram Sarabhai. Dr. Sarabhai had a vision for using satellitis for educating the nation. ISRO has pursued a very vigorous programme to develop space systems for communication, television broadcasting, meteorological services, remote sensing and scientific research. It has also developed successfully launch vehicles for polar satellites ***(PSLV)*** (Fig. 5.11) and geo-synchronous satellites ***(GSLV)*** (Fig. 5.12). In fact, it has launched satellites for other countries like Germany, Belgium and Korea. and has joined the exclusive club of five countries. Its scientific programme includes studies of

(i) climate, environment and global change,

(ii) upper atmosphere,

(iii) astronomy and astrophysics, and

(iv) Indian Ocean.

Recently, ISRO launched an exclusive educational satillite EduSat, first of its kind in the world. It is being used to educate both young and adult students living in remote places.

It is now making preparation for a mission to the moon.

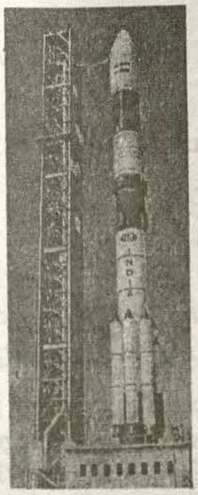


Fig. 5.11: PSLV

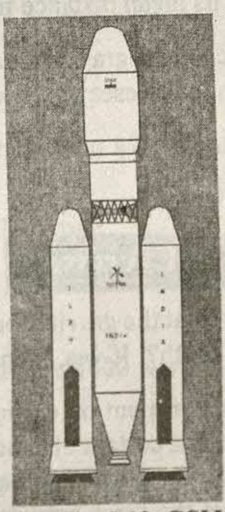


Fig. 5.12: GSLV

Intext Questions 5.7

1. Some science writers believe that some day human beings will establish colonies on the Mars. Suppose people living this desire to put in orbit a Mars synchronous satellite. The rotation period of Mars is 24.6 hours. The mass and radius of Mars are 6.4×10^{23} kg and 3400 km, respectively. What would be the height of the satellite from the surface of Mars?

   ....

2. List the advantages of having a telescope in space.

   ....

What You Have Learnt

- The force of gravitation exists between any two particles in the universe. It varies as the product of their masses and inversely as the square of distance between them.
- The constant of gravitation, G, is a universal constant.
- The force of gravitation of the earth attracts all bodies towards it.
- The acceleration due to gravity near the surface of the earth is 9.8 ms^{-2}. It varies on the surface of the earth because the shape of the earth is not perfectly spherical.
- The acceleration due to gravity varies with height, depth and latitude.
- The weight of a body is the force of gravity acting on it.
- Kepler's first law states that the orbit of a planet is elliptic with sun at one of its foci.
- Kepler's second law states that the line joining the planet with the Sun sweeps equal areas in equal intervals of time.
- Kepler's third law states that the square of the orbital period of a planet is proportional to the cube of its mean distance from the Sun.
- A body can escape the gravitational field of the earth if it can acquire a velocity equal to or greater than the escape velocity.
- The orbital velocity of a satellite depends on its distance from the earth.

Terminal Exercise

1. You have learnt that the gravitational attraction is mutual. If that is so, does an apple also attract the earth? If yes, then why does the earth not move in response?
2. We set up an experiment on earth to measure the force of gravitation between two particles placed at a certain distance apart. Suppose the force is of magnitude F. We take the same set up to the moon and perform the experiment again. What would be the magnitude of the force between the two particles there?

3. Suppose the earth expands to twice its size without any change in its mass. What would be your weight if your present weight were 500 N?

4. Suppose the earth loses its gravity suddenly. What would happen to life on-this plant?

5. Refer to Fig. 5.6 which shows the structure of the earth. Calculate the values of g at the bottom of the crust (depth 25 km) and at the bottom of the mantle (depth 2855 km).

6. Derive an expression for the mass of the earth, given the orbital period of the moon and the radius of its orbit.

7. Suppose your weight is 500 N on the earth. Calculate your weight on the moon. What would be your mass on the moon?

8. A polar satellite is placed at a height of 800 km from earth's surface. Calculate its orbital period and orbital velocity.

Answers to Intext Questions

5.1

1. Moon's time period $T = 27.3\text{d}$

$$= 27.3 \times 24 \times 3600 \text{ s}$$

Radius of moon's orbit $R = 3.84 \times 10^8$ m

Moon's orbital speed $\upsilon = \dfrac{2\pi R}{T}$

Centripetal accleration = υ^2/R

$$= \frac{4\pi^2 R^2}{T^2} \cdot \frac{1}{R} = \frac{4\pi^2 R}{T^2}$$

$$= \frac{4\pi^2 \times 3.84 \times 10^8 \text{ m}}{(27.3 \times 24 \times 3600)^2 \text{ s}^2}$$

$$= \frac{4\pi^2 \times 3.84}{(27.3 \times 2.4 \times 3.6)^2} \times 10^{-2} \text{ ms}^{-2}$$

$$= .00272 \text{ ms}^{-2}$$

If we calculate centripetal acceleration on dividing g by 3600, we get the same value :

$$= \frac{9.8}{3600} \text{ ms}^{-2}$$

$$= 0.00272 \text{ ms}^{-2}$$

2. $F = \dfrac{G\, m_1\, m_2}{r^2}$

F is force $\therefore\ G = \dfrac{\text{Force} \times r^2}{(\text{mass})^2} = \dfrac{\text{Nm}^2}{\text{kg}^2}$

3. $F = G\,\dfrac{m_1\, m_2}{r^2}$

If $m_1 = 1\text{kg}$, $m_2 = 1\text{kg}$, $r = 1\text{m}$, then F = G

or G is equal to the force between two masses of 1kg each placed at a distance of 1m from each other

4. (i) $F \propto 1/r^2$, if r is doubled, force becomes one-fourth.

(ii) $F \propto m_1 m_2$, if m_1 and m_2 are both doubled then F becomes 4 times.

(iii) $F \propto \dfrac{m_1\, m_2}{r^2}$,

if each mass is doubled, and distance is also doubled, then

F remains unchanged.

5. $F = G\,\dfrac{50 \text{ kg} \times 60 \text{ kg}}{1 \text{ m}^2}$; $G = 6.68 \times 10^{-11}\,\dfrac{\text{Nm}^2}{\text{kg}^2}$

$$= 6.67 \times 10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{3000 \text{ kg}^2}{1 \text{ m}^2}$$

$$= 6.67 \times 10^{-11} \times 3 \times 10^3 \text{ N}$$

$$= 2 \times 10^{-7} \text{ N}$$

5.2

1. $g = \dfrac{GM}{R^2}$

$$= 6.67 \times 10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{5.97 \times 10^{24} \text{kg}}{(6.371 \times 10^6)^2 \text{m}^2}$$

$$= \frac{6.97 \times 59.7}{6.371 \times 6.371}\ \frac{\text{N}}{\text{kg}} = 9.81 \text{ m s}^{-2}$$

$$g_{\text{pole}} = \frac{GM}{R_{\text{pole}}^2}$$

$$= 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{5.97 \times 10^{24}\,\text{kg}}{(6.371 \times 10^6)^2\,\text{m}^2}$$

$$= \frac{6.97 \times 59.7}{6.371 \times 6.371} \frac{\text{N}}{\text{kg}} = 9.81\ \text{ms}^{-2}$$

Similarly,

$$g_{\text{eguator}} = \frac{6.97 \times 59.7}{6.378 \times 6.378} \frac{\text{N}}{\text{kg}} = 9.79\ \text{ms}^{-2}$$

3. The value of g is always vertically downwards.

4. $g_{\text{moon}} = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \times \frac{7.3 \times 10^{22}\,\text{kg}}{(1.74 \times 10^6)^2\,\text{m}^2}$

$$= \frac{6.67 \times 7.3}{1.74 \times 1.74} \times 10^{-1} \frac{\text{N}}{\text{kg}} = 1.61\ \text{m s}^{-2}$$

5.3

1. Let g at distance r from the centre of the earth be called g_1.

Outside the earth,

then $\frac{g}{g_1} = \frac{r^2}{R^2}$

If $g_1 = g/2 \Rightarrow r^2 = 2R^2 \Rightarrow r = \sqrt{2}R = 1.412\,R$

$\therefore$ Height from earth's surface$= 1.4142\,R - R$

$$= 0.4142\,R$$

2. Inside the earth g varies as distance from the centre of the earth. Suppose at depth d, g is called g_d.

Then $\frac{g_d}{g} = \frac{R-d}{R}$

If $g_d = 80\%$, then

$$\frac{0.8}{1} = \frac{R-d}{R}$$

$$\frac{0.8}{1} = \frac{R - d}{R}$$

$$\therefore \quad d = 0.2\,R$$

3. In example 5.3, we calculated $\omega = 7.27 \times 10^{-5}$ rad s^{-1}

$$\therefore R\omega^2 \cos 30^\circ = 6.37 \times 10^6 \times (7.27 \times 10^{-5})^2\ s^{-2} \,.\, \sqrt{3}/2 = 0.029\ ms^{-2}$$

g at poles $= 9.853$ m s^2

(Calculated in example 5.2)

$\therefore$ g at Delhi $= 9.853\ ms^{-2} - 0.029\ ms^{-2}$

$= 9.824\ ms^{-2}$

4. Using formula (5.9),

$$g_h = \frac{g}{1 + \frac{2h}{R}} = \frac{9.81\ m\,s^{-2}}{1 + \frac{2000\ km}{6371\ km}}$$

$$= \frac{9.81\ m\,s^{-2}}{\frac{28371\ km}{6371\ km}} = 7.47\ m\,s^{-2}$$

Using variation with r

$$g = \frac{GM}{(R+h)^2}$$

$$= 6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2} \,.\, \frac{5.97 \times 10^{24}\ kg}{(7.371 \times 10^6)^2\ m^2}$$

$= 7.33\ ms^{-2}$

This gives more accurate results because formula (5.9) is for the case $h << R$. In this case h is not $<< R$.

5.4

1. On the moon the value of g is only 1/6th that on the earth. So, your weight on moon will become 1/6th of your weight on the earth. The mass, however, remains constant.

2. Mass of Mars $= 6 \times 10^{23}$ kg

Radius of Mars $= 4.3 \times 10^6$ m

$$\frac{\text{Weight on Mars}}{\text{Weight on Earth}} = \frac{m.2.16}{m.9.81} = 0.22$$

So, your weight will become roughly 1/4th that on the earth. Mass remains constant.

3. Balances with two pans actually compare masses because g acts on both the pans and gets cancelled. The other type of balance, spring balance, measures weight. The balance with two pans gives the same reading on the moon as on the earth. Spring balance with give weight as 1/6th that on the earth for a bag of potatoes.

5.5

1. Yes. Wherever the force between bodies is gravitational, Kepler's laws will hold.

2. According to Kepler's third law

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \quad \text{or} \quad T^2 \alpha r^3 \Rightarrow T \alpha r^{3/2}$$

So, the satellite which is farther off has higher period.

Let $T_1 = 90$ min,, $r_1 = 1000 \text{ km} + 6371 \text{ km}$

$$r_2 = 2000 \text{ km} + 6371 \text{ km}$$

[From the centre of the earth]

$$\therefore \quad T_2^2 = \frac{T_1^2 . r_2^3}{r_1^3} = (90 \text{ min})^2 \left(\frac{8371 \text{ km}}{7371 \text{ km}}\right)^3$$

$$T_2 = 108.9 \text{ min}$$

3. According to Kepler's third law

$$\frac{T_{earth}^2}{T_{sedna}^2} = \frac{r_{earth}^3}{r_{sedna}^3}$$

[Distance from the Sun]

$T_{earth} = 1$ yr., $r_{earth} = 1$ AU

$$T_{sedna}^2 = \frac{(1\text{yr})^2 (86 \text{ AU})^3}{(1\text{AU})^3} (86)^3 \text{yr}^2$$

$$\therefore T_{sedna} = 797.5 \text{ yr}$$

4. If υ is the orbital velocity of the satellite of mass m at a distance r from the centre of the earth, then equating centripltal force with the gravitational force, we have

$$\frac{m\upsilon^2}{r} = \frac{GmM}{r^2} \Rightarrow \upsilon = \sqrt{\frac{GM}{r}}$$

where M is the mass of the earth.

$$\frac{m\upsilon^2}{r} = \frac{GmM}{r^2} \Rightarrow \upsilon = \sqrt{\frac{GM}{r}}$$

where *M* is the mass of the earth.

5. From Eqs. (5.16) and (5.17),

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM.}$$

or $T^2 \alpha r^3$.

5.6

1. $\upsilon_{esc} = \sqrt{\frac{2GM}{R}}$

$$= \sqrt{2 \times 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} . \frac{5.97 \times 10^{24} \text{kg}}{6.371 \times 10^6 \text{ m}}}$$

$$= \sqrt{\frac{2 \times 6.67 \times 5.97 \times 10}{6.371}} \; 10^3 \text{ ms}^{-1}$$

$$= 11.2 \times 10^3 \text{ ms}^{-1} = 11.3 \text{ kms}^{-1}$$

2. $\upsilon_{esc} \alpha \sqrt{\frac{1}{R}}$

If *R* becomes 1/4th, υ_{esc} becomes double.

3. $\upsilon_{esc} \alpha \sqrt{\frac{M}{R}}$

If *M* becomes eight times, and *R* twice,

then

$\upsilon_{esc} \alpha \sqrt{4}$ or υ_{esc} becomes double.

5.7

1. $(R + h) \frac{4\pi^2}{T^2} = \frac{GM}{(R + h)^2}$

$$\Rightarrow (R + h)^3 = \frac{GM}{4\pi^2} T^2$$

$$R + h = 20300 \text{ km}$$

$$h = 26900 \text{ km}$$

2. (a) Images are clearer

(b) x-ray telescopy etc. also work.

Answers to Terminal Problems

3. 125 N

5. $\simeq g$, 5.5 ms^{-2}

7. Weight = $\frac{500}{6}$ N, mass 50 kg on moon as well as on earth

8. $T \simeq 1\frac{1}{2}$ h, $\upsilon = 7.47$ km s^{-1}

6

WORK ENERGY AND POWER

You know that motion of objects arises due to application of force and is described by Newton's laws of motion. You also know how the velocity (speed and direction) of an object changes when a force acts on it. In this lesson, you will learn the concepts of work and energy. Modern society needs large amounts of energy to do many kinds of work. Primitive man used muscular energy to do work. Later, animal energy was harnessed to help people do various kinds of tasks. With the invention of various kinds of machines, the ability to do work increased greatly. Progress of our civilization now critrcally depends the on the availability of usable energy. Energy and work are, therefore, closely linked.

From the above discussion you will appreciate that the rate of doing work improved with newer modes, i.e. as we shifted from humans → animals → machines to provide necessary force. The rate of doing work is known as **power**.

Objectives

After studying this lesson, you should be able to:

- *define work done by a force and give unit of work;*
- *calculate the work done by an applied force;*
- *state work-energy theorem;*
- *define power of a system;*
- *calculate the work done by gravity when a mass moves from one point to another;*
- *explain the meaning of energy;*
- *obtain expressions for gravitational potential energy and elastic potential energy;*
- *apply the principle of conservation of energy for physical system; and*
- *apply the laws of conservation of momentum and energy in elastic collisions.*

6.1 Work

The word 'work' has different meaning for different people. When you study, you do mental work. When a worker carries bricks and cement to higher floors of a building, he is doing physical work against the force of gravity. But in science, work has a definite meaning. The technical meaning of work is not always the same as the common meaning. The work is defined in the following way :

Let us suppose that a **constant force F** acting on an object results in displacement **d** i.e. moves it by a distance d along a straight line on a horizontal surface, as shown in Fig. 6.1. *The work done by a force is the product of the magnitude of force component in the direction of displacement and the displacement of this object.*

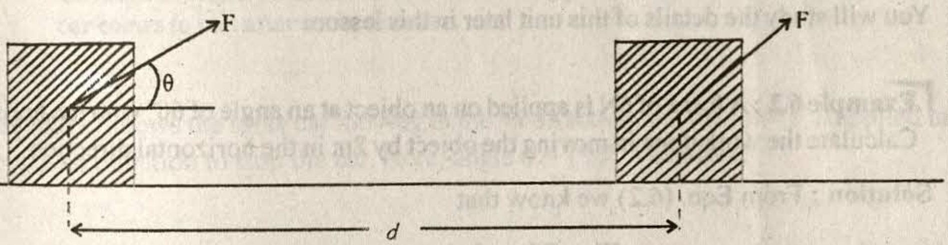


Fig 6.1 : A force F on a block moves it by a horizontal distance d. The direction of force makes an angle θ with the horizontal direction.

If force **F** is acting at angle θ with respect to the displacement **d** of the object, its component along **d** will be $F \cos \theta$. Then work done by force **F** is given by

$$W = F \cos\theta . d \qquad (6.1)$$

In vector form, the work done is given by:

$$W = \mathbf{F} . \mathbf{d} \qquad (6.2)$$

Note that if $d = 0$, $W = 0$. That is, no work is done by a force, whatever its magnitude, if there is no displacement of the object. Also note that though both force and displacement are vectors, work is a scalar.

Activity 6.1

You and your friends may try to push the wall of a room. Irrespective of the applied force, the wall will not move. Thus we say that no work is done.

The unit of work is defined using Eqn.(6.2). If the applied force is in newton and displacement is in metre, then the unit of work is joule.

(Unit of Force)×(Unit of displacement) = newton . metre = Nm (6.3)

This unit is given a special name, **joule,** and is denoted by J.

One joule is defined, as the work done by a force of one newton when it produces a displacement of one metre. Joule is the SI unit of work.

Example 6.1 : Find the dimensional formula of work.

Solution : W = Force × Distance

= Mass × Acceleration × distance

$$\text{Dimension of work} = [M] \times [LT^{-2}] \times [L]$$

$$= [ML^2T^{-2}]$$

In electrical measurements, kilowatt-hour (kWh) is used as unit of work. It is related to joule as

$$1\text{kWh} = 3.6 \times 10^6 \text{ J}$$

You will study the details of this unit later in this lesson.

Example 6.2 : A force of 6N is applied on an object at an angle of 60° with the horizontal. Calculate the work done in moving the object by 2m in the horizontal direction.

Solution : From Eqn. (6.2) we know that

$$W = Fd\cos\theta$$

$$= 6\times2\times\cos 60°$$

$$= 6\times2\times(½)$$

$$= 6 \text{ J}$$

Example 6.3 : A person lifts 5 kg potatoes from the ground floor to a height of 4m to bring it to first floor. Calculate the work done.

Solution : Since the potatoes are lifted, work is being done against gravity. Therefore, we can write

$$\text{Force} = mg$$

$$= 5 \text{ kg} \times 9.8 \text{ ms}^{-2}$$

$$= 49 \text{ N}$$

$$\text{Work done} = 49 \times 4 \text{ (Nm)}$$

$$= 196 \text{ J}$$

6.1.1 Positive and Negative Work

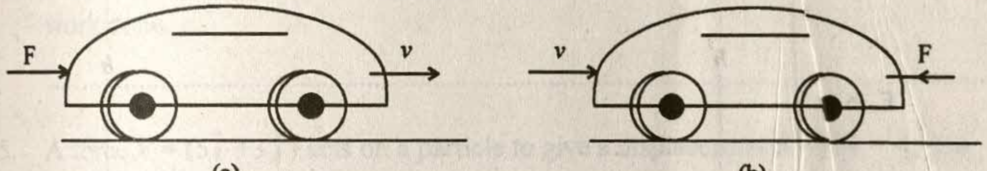
As you have seen, work done is defined by Eqn.(6.2), where the angle θ between the force and the displacement is also important. In fact, it leads us to the situation in which work becomes a positive or a negative quantity. Consider the examples given below:

Fig. 6.2 (a) shows a car moving in $+x$ direction and a force F is applied in the same direction. The speed of the car keeps increasing. The force and the displacement both are in the same direction, i.e. $\theta = 0°$. Therefore, the work done is given by

$$W = Fd\cos 0^\circ$$

$$= Fd \qquad (6.4)$$

The work is this case is positive.



(a) (b)

Fig. 6.2 : A car is moving on a horizontal road. a) A force F is applied in the direction of the moving car. It gets accelerated. b) A force F is applied in opposite direction so that the car comes to rest after some distance.

Figure 6.2 (b) shows the same car moving in the +x direction, but the force **F** is applied in the opposite direction to stop the car. Here, angle $\theta = 180^\circ$. Therefore,

$$W = Fd\cos 180^\circ.$$

$$= -Fd \qquad (6.5)$$

Hence, the work done by the force is negative. In fact, the work done by a force shall be negative for θ lying between 90° and 270°.

From the above examples, we can conclude that

a) When we press the accelerator of the car, the force is in the direction of motion of the car. As a result, we increase the speed of the car. The work done is positive.

b) When we apply brakes of a car, the force is applied in a direction opposite to its motion. The car loses speed and may finally come to rest. Negative work is said to have been done.

c) In case the applied force and displacement are as right angles, i.e. $\theta = 90^\circ$, no work is said to be done.

6.1.2 Work Done by the Force of Gravity

Fig.6.3(a) shows a mass *m* being lifted to a height *h* and Fig. 6.3(b) shows the same mass being lowered by a distance *h*. The weight of the object is *mg* in both cases. You may recall from the previous lesson that weight is a force.

In Fig. 6.3 (a), the work is done against the force *mg* (downwards) and the displacement is upward ($\theta = 180^\circ$). Therefore,

$$W = Fd\cos 180^\circ$$

$$= -\,mgh$$

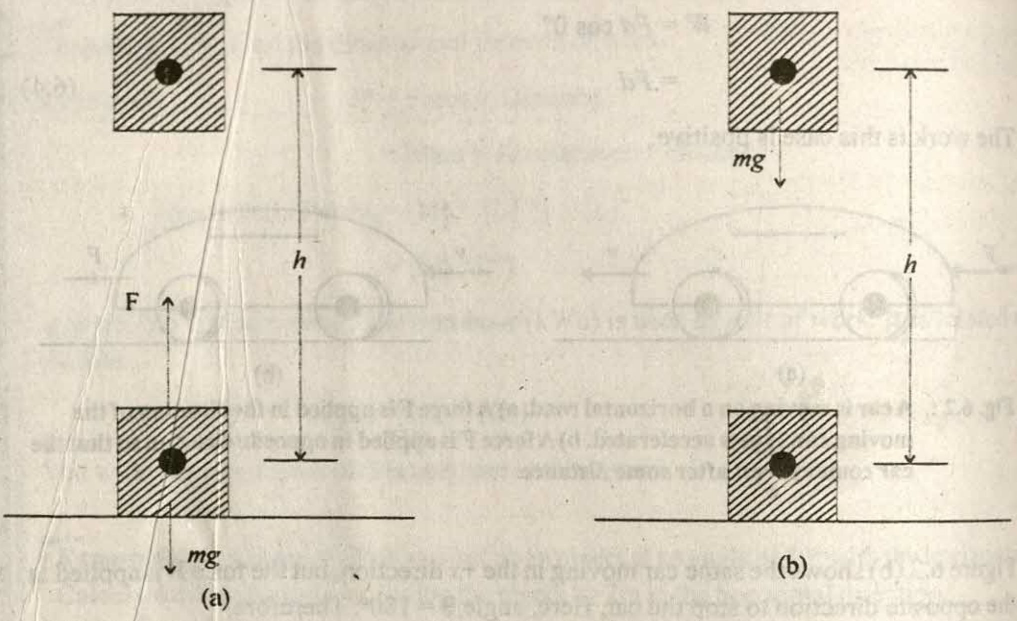


Fig 6.3 : **a) The object is lifted up against the force of gravity,**
b) The object is lowered towards the earth.

In the Fig. 6.3(b), the mass is being lowered. The force *mg* and the displacement *d* are in the same direction ($\theta = 0^\circ$). Therefore, the work done

$$W = Fd \cos 0^\circ$$

$$= + mgh \qquad (6.6)$$

You must be very careful in interpreting the results obtained above. When the object is lifted up, the work done by the gravitational force is **negative** but the work done by the person lifting the object is **positive.** When the object is being lowered, the work done by the gravitational force is **positive** but the workdone by the person lowering the object is **negative.** In both of these cases, it is assumed that the object is being moved without acceleration.

Intext Questions 6.1

1. When a particle rotates in a circle, a force acts on the particle. Calculate the work done by this force on the particle.

....

2. Give one example of each of the following. Work done by a force is
   a) zero
   b) negative
   c) positive

....

3. A bag of grains of mass 2 kg. is lifted through a height of 5m.

a) How much work is done by the lift force?
b) How much work is done by the force of gravity?

....

4. A force $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N produces a displacements $d = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ m. Calculate the work done.

....

5. A force $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ acts on a particle to give a displacement $\mathbf{d} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ m

a) Calculate the magnitude of displacement

b) Calculate the magnitude of force.

c) How much work is done by the force?

....

6.2 Work Done by A Variable Force

You have so far studied the cases where the force acting on the object is constant. This may not always be true. In some cases, the force responsible for doing work may keep varying with time. Let us now consider a case in which the magnitude of force $F(x)$ changes with the position x of the object. Let us now calculate the work done by a variable force. Let us assume that the displacement is from x_i to x_f, where x_i and x_f are the initial and final positions. In such a situation, work is calculated over a large number of small intervals of displacements Δx. In fact, Δx is taken so small that the force $F(x)$ can be assumed to be constant over each such interval. The work done during a small displacements Δx is given by

$$\Delta W = F(x)\,\Delta x \tag{6.7}$$

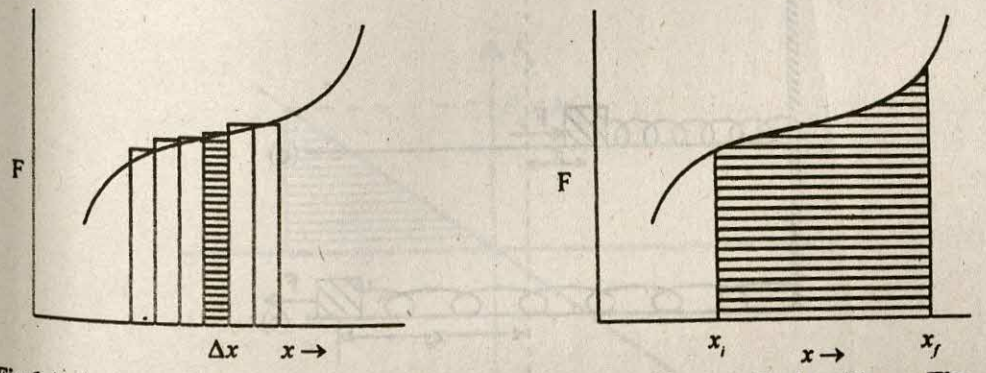


Fig 6.4 : A varying force F moves the object from the initial position x_i to final position x_f. The variation of force with distance is shown by the solid curve (arbitrary) and work done is numerically equal to the shaded area.

$F(x)\,\Delta x$ is numerically equal to the small area shown shaded in the Fig. 6.4(a). The total work done by the force between x_i and x_f is the sum of all such areas (area of all strips added together):

$$W = \Sigma \Delta W$$
$$= \Sigma F(x)\,\Delta x \qquad (6.8)$$

The width of the strips can be made as small as possible so that the areas of all strips added together are equal to the total area enclosed between x_i and x_f. It will give the total work done by the force between x_i and x_f :

$$W = \lim_{\Delta x \to 0} \sum F(x)\,\Delta x \qquad (6.9)$$

6.2.1 Work done by a Spring

A very simple example of a variable force is the force exerted by a spring. Let us derive the expression for work done in this case.

Fig. 6.5(a) shows the equilibrium position of a light spring whose one end is attached to a rigid wall and the other end is attached to a block of mass *m*. The system is placed on a smooth horizontal table. We take *x*-axis along the horizontal direction. Let mass *m* be at position $x = 0$. The spring is now compressed (or elongated) by an external force **F**. An internal force $\mathbf{F}_s$ is called into play in the spring due to its elastic property. This force $\mathbf{F}_s$ keeps increasing with increasing *x* and becomes equal to **F** when the compression (or elongation) is maximum at $x = x_m$.

According to Hooke's law (true for small *x* only), $|\mathbf{F}_s| = kx$, where *k* is known as spring constant. Since the direction of $\mathbf{F}_s$ is always opposite to compression (or extension), it is written as :

$$\mathbf{F} = \mathbf{F}_s = -kx \qquad (6.10)$$

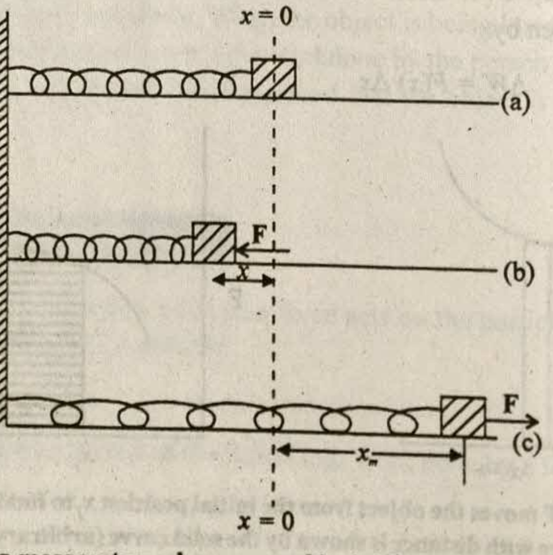


Fig. 6.5 : **A spring-mass system whose one end is rigidly fixed and mass m, rests on a smooth horizontal surface. a) The relaxed position of the spring's, free end at $x = 0$; b) The spring is compressed by applying external force *F* and c) Pulled or elongated by an external force *F*. The maximum compression/ elongation is x_m.**

Let us now calculate the work done and also examine, if it is positive or negative. In the event of compression of the spring, the external force **F** is directed towards left and the displacement **x** is also towards left. Hence, the work done by the **external force** is positive. However, for the same direction of displacement, the restoring force generated in the spring is towards right, i.e. **F** and **x** are oppositely directed. The work done by the **spring force** is negative. You can yourself examine the case of extension of the spring and arrive at the same result: ***"the work done by the external force is positive but the work done by the spring force is negative and its magnitude is** (½) kx_m^2**"***

A simple calculation can be done to derive an expression for the work done. At $x = 0$, the force $\mathbf{F}_s = 0$. As x increases, the force $\mathbf{F}_s$ increases and becomes equal to **F** when $x = x_m$. Since the variation of the force is linear with displacement, the average force during compression (or extension) can be approximated to $\left(\frac{0+\mathbf{F}_s}{2}\right) = \frac{\mathbf{F}_s}{2}$. The work done by the force is given by

$$W = \text{force . displacement}$$

$$= \frac{\mathbf{F}_s}{2} \cdot \mathbf{x},$$

But $|\mathrm{F}_s| = k\,|\,x_m|$. Hence

$$W = \frac{1}{2} k\, x_m \times x_m$$

$$= \frac{1}{2} k\, x^2_m \qquad (6.11)$$

The work done can also be obtained graphically. It is shown in Fig. 6.6.

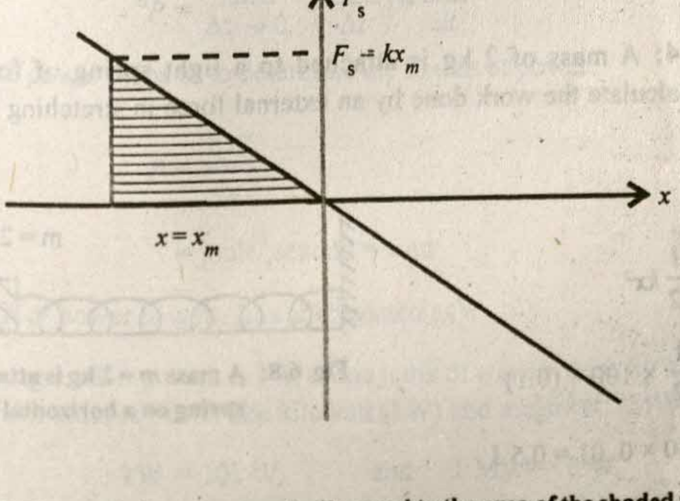


Fig. 6.6: The work done is numerically equal to the area of the shaded triangle.

The area of the shaded triangle is:

$$= \frac{1}{2} \text{ base} \times \text{height}$$

$$W = \frac{1}{2} x_m \times kx_m$$

$$= \frac{1}{2} kx_m^{\ 2} \qquad (6.12)$$

This is the same as that obtained analytically in Eqn. (6.11)

Activity 6.2

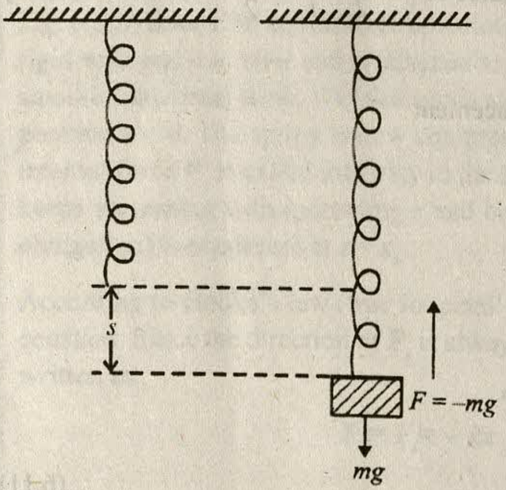


Fig. 6.7 : Extension in a spring under a load.

Measuring spring constant

Suspend the spring vertically, as shown in Fig. 6.7 (a). Now attach a block of mass *m* to the lower end of the spring. On doing so, the spring extends by some distance. Measure the extension. Suppose it is *s*, as shown in Fig 6.7 (b). Now think why does the spring not extend further. This is because the spring force (restoring force) acting upwards balances the weight *mg* of the block in equilibrium state. You can calculate the spring constant by putting the values in

$$F_s = k.s$$

or $$mg = k.s$$

Thus, $$k = \frac{mg}{s} \qquad (6.13)$$

Example 6.4: A mass of 2 kg is attached to a light spring of force constant k=100 Nm^{-1}. Calculate the work done by an external force in stretching the spring by 10 cm.

Solution:

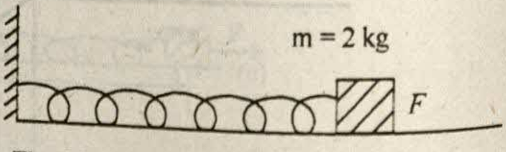


Fig. 6.8: A mass m = 2 kg is attached to a spring on a horizontal surface.

$$W = \frac{1}{2} kx^2$$

$$= \frac{1}{2} \times 100 \times (0.1)^2$$

$$= 50 \times 0.01 = 0.5 \text{ J}$$

As explained earlier, the work doen by the restioning force in the spring = – 0.5 J.

Intext Questions 6.2

1. Define spring constant. Give its SI unit.

....

2. A force of 10 N extends a spring by 1cm.How much force is needed to extend this spring by 5 cm? How much work will be done by this force?

....

6.3 Power

You have already learnt to calculate the work done by a force. In such calculations, we did not consider whether the work is done in one second or in one hour. In our daily life, however, the time taken to perform a particular work is important. For example, a man may take several hours to load a truck with cement bags, whereas a machine may do this work in much less time. Therefore, it is important to know the rate at which work is done. **The rate at which work is done is called power.**

If ΔW work is done in time Δt, the average power is defined as

$$\text{Average Power} = \frac{\text{Work done}}{\text{time taken}}$$

Mathematically, we can write

$$P = \frac{\Delta W}{\Delta t} \qquad (6.14)$$

If the rate of doing work is not constant, this rate may vary. In such cases, we may define instantaneous power P

$$P = \underset{\Delta t \to 0}{\text{limit}} \left(\frac{\Delta W}{\Delta t}\right) = \frac{dW}{dt} \qquad (6.15)$$

The definition of power helps us to determine the SI unit of power:

$$P = \frac{\Delta W}{\Delta t}$$

$$= \text{joule/ second} = \text{watt}$$

Thus, the SI unit of power is watt. It is abbreviated asW.

The power of an agent doing work is 1W, if one joule of work is done by it in one second. The more common units of power are kilowatt (kW) and megawatt (MW).

$1 \text{ kW} = 10^3 \text{ W},$ and $1 \text{ MW} = 10^6 \text{W}$

James Watt
(1736–1819)

Scottish inventor and mechanical engineer, James Watt is renowned for improving the efficiency of a steam engine. This paved the way for industrial revolution.

He, introduced horse power as the unit of power. SI unit of power watt is named in his honour. Some of the important inventions by James Watt are : a steam locomotive and an attachment that adapted telescope to measure distances.

Example 6.5 : Determine the dimensions of power.

Solution : Since $P = \frac{\text{work}}{\text{time}}$

$$= \text{Force} \times \frac{\text{Distance}}{\text{Time}}$$

$$\therefore \text{ Dimension of } P = [\text{Mass}] \times [\text{Acceleration}] \times \frac{[\text{Distance}]}{[\text{Time}]}$$

$$= [\text{M}] \times \left[\frac{\text{L}}{\text{T}^2}\right] \times \left[\frac{\text{L}}{\text{T}}\right]$$

$$= [\text{ML}^2\text{T}^{-3}]$$

You may have heard electricians discussing the power of a machine in terms of the horse power, abbreviated as hp. This unit of power was under British system. It is a larger unit:

$$1\text{hp} = 746 \text{ W} \qquad (6.16)$$

The unit of power is used to define a new unit of work (energy). One such unit of work is **kilowatt hour.** This unit is commonly used in electrical measurement.

$$\text{kilowatt. hour (kWh)} = \text{kW. hour}$$

$$= 10^3 \text{ W. hour}$$

$$= \frac{10^3 \text{ J}}{1\text{s}} \times 3600 \text{ s}$$

$$= 36,00,000 \text{ J} = 3.6 \times 10^6 \text{ J}$$

Or $$1 \text{ kWh} = 3.6 \text{ MJ (mega joules)} \qquad (6.17)$$

The electrical energy that is consumed in homes is measured in kilowatt-hour. In common man's language : 1kWh = 1 Unit of electrical consumption.

Intext Questions 6.3

1. A body of mass 100 kg is lifted through a distance of 8 m in 10s. Calculate the power of the lifter.

....

2. Convert 10 horse power into kilo watt.

....

6.5 Work and Kinetic Energy

As you know, the capacity to do work is called energy. If a system (object) has energy, it has ability to do work. An automobile moving on a road uses chemical energy of fuel (CNG, petrol, diesel). It can push an object which comes on its way to some distance. Thus it can do work. All moving objects possess energy because they can do work before they come to rest. We call this kind of energy as **kinetic energy**. Kinetic energy is the energy of an object because of its motion.

Let us consider an object of mass m moving along a straight line when a constant force of magnitude F acts on it along the direction of motion. This force produces a uniform acceleration a such that $F = ma$. Let υ_1 be the speed of the object at time t_1. This speed becomes υ_2 at another instant of time t_2. During this interval of time $t = (t_2 - t_1)$, the object covers a distance, s. Using Equations of Motion, we can write

$$\upsilon_2^2 = \upsilon_1^2 + 2as$$

or

$$a = \frac{\upsilon_2^2 - \upsilon_1^2}{2s} \tag{6.18}$$

Combining this result with Newton's second law of motion, we can write

$$F = m \times \frac{\upsilon_2^2 - \upsilon_1^2}{2s}$$

We know that work done by the force is given by

$$W = Fs$$

Hence,

$$W = m \times \frac{\upsilon_2^2 - \upsilon_1^2}{2s} s$$

$$= \frac{1}{2}m\upsilon_2^2 - \frac{1}{2}m\upsilon_1^2$$

$$= K_2 - K_1 \tag{6.19}$$

where $K_2 = \frac{1}{2}m\upsilon_2^2$ and $K_1 = \frac{1}{2}m\upsilon_1^2$ respectively denote the final and initial kinetic energies.

$(K_2 - K_1)$ denotes the change in kinetic energy, which is equal to the work done by the force.

Kinetic Energy is a scalar quantity. It depends on the product of mass and the square of the speed. It does not matter which one of the two (m and υ) is small and which one is large. It is the total value $\frac{1}{2} m \upsilon^2$ that determines the kinetic energy.

Example 6.6 : A body of mass 10 kg is initially moving with a speed of 4.0 ms^{-1}. A force of 30 N is now applied on the body for 2 seconds.

i) What is the final speed of the body after 2 seconds?

ii) How much work has been done during this period?

iii) What is the initial kinetic energy?

iv) What is the final kinetic energy?

v) What is the distance covered during this period?

vi) Show that the work done is equal to the change in kinetic energy?

Solution :

i) Force $(F) = ma$

or $a = F/m$

$= 30/10$

$= 3 \text{ ms}^{-2}$

The final speed $\upsilon_2 = \upsilon_1 + at$

$= 4 + (3 \times 2) = 10 \text{ ms}^{-1}$

ii) The distance covered in 2 seconds:

$$s = ut + \frac{1}{2} at^2$$

$$= (4\times2) + \frac{1}{2} (3\times4)$$

$$= 8 + 6 = 14 \text{ m}$$

Work done $W = F \times S$

$= 30 \times 14 = 420$ J

iii) The initial Kinetic Energy

$$K_1 = \frac{1}{2} m\upsilon_1^2$$

$$= \frac{1}{2} (10 \times 16) = 80 \text{ J}$$

Notes

iv) The final kinetic energy

$$K_2 = \frac{1}{2}mv_2^2$$

$$= \frac{1}{2}(10 \times 100) = 500 \text{ J}$$

v) The distance covered as calculated above = 14m

vi) The change in kinetic energy is:

$$K_2 - K_1 = (500 - 80) = 420 \text{ J}$$

As may be seen, this is same as wok done.

Work-Energy Theorem

The work-energy theorem states that the work done by the resultant of all forces acting on a body is equal to the change in kinetic energy of the body.

Intext Question 6.4

1. Is it possible for a particle to have a negative value of kinetic energy? Why?

....

2- What happens to the kinetic energy of a particle if

a) The speed υ of the particle is made 2υ.

b) The mass m of the particle is made $m/2$?

....

3- A particle moving with a kinetic energy 3.6 J collides with a spring of force constant 180 Nm^{-1}. Calculate the maximum compression of the spring.

....

4- A car of mass 1000 kg is moving at a speed of 90 kmh^{-1}. Brakes are applied and the car stops at a distance of 15 m from the braking point. What is the average force applied by brakes? If the car stops in 25s after braking, calculate the average power of the brakes?

....

5. If an external force does 375 J of work in compressing a spring, how much work is done by the spring itself?

....

6.6 Potential Energy

In the previous section we have discussed that a moving object has kinetic energy associated with it. Objects possess another kind of energy due to their position in space. This energy is known as **Potential Energy.** Familiar example is the **Gravitational Potential Energy possessed by a body in Gravitational Field.** Let us understand it now.

6.6.1 Potential Energy in Gravitational Field

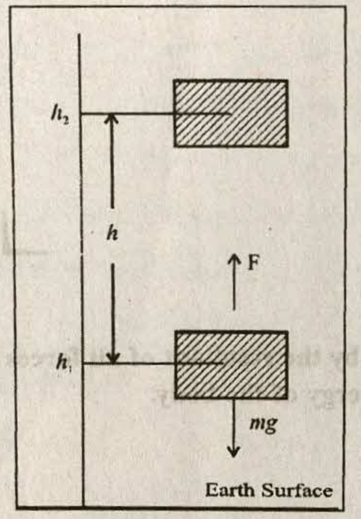


Fig. 6.9: Object of mass m originally at height h_1 above the earth's surface is moved to a height h_2.

Suppose that a person lifts a mass m from a given height h_1 to a height h_2 above the earth's surface. Let us also assume that the value of acceleration due to gravity remains constant. The mass has been displaced by a distance $h = (h_2 - h_1)$ against the force of gravity. The magnifude of this force is mg and it acts downwards. Therefore, the work done by the person is

$$W = \text{force} \times \text{distance}$$

$$= mgh \qquad (6.20)$$

The work is positive and is stored in mass m as energy. This energy by virtue of the position in space is called **gravitational potential energy**. It has capacity to do work. If this mass is left free, it will fall down and during the fall it can be made to do work. For example, it can lift another mass if properly connected by a string, which is passing over a pulley.

The selection of the initial height h_1 is arbitrary. The important concept is the change in height, i.e. $(h_2 - h_1)$. We, therefore, say that the point of **zero potential energy** is arbitrary. Any point in space can be chosen as a point of zero potential energy. Normally, a point on the surface of the earth is assumed to be the reference point with zero potential energy.

Example 6.7 : A truck is loaded with sugar bags. The total mass of the load and the truck together is 100,000 kg. The truck moves on a winding path up a mountain to a height of 700 m in 1 hour. What average power must the engine produce to lift the material?

Solution : $W = mgh$

$$= (100{,}000\ \text{kg}) \times (9.8\ \text{m s}^{-2} \times 700\ \text{m})$$

$$= 9.8 \times 7 \times 10^7\ \text{J}$$

$$= 68.6 \times 10^7 \text{J}$$

$$\text{Time taken} = 1\ \text{hour} = 60 \times 60\text{s}$$

$$= 3600\ \text{s}$$

$$\text{Average Power, } P = W/t$$

$$= \frac{68.6 \times 10^7\ \text{J}}{3600\ \text{s}}$$

$$= 1.91 \times 10^5 \text{ W}$$

We know that 746 W = 1 hp

$$\therefore \quad P = \frac{1.91\times10^5}{746} = 2.56 \times 10^2 = 256 \text{ hp.}$$

Notes

Example 6.8 : Hydroelectric power generation uses falling water as a source of energy to turn turbine blades and generate electrical power. In a power station, 1000×10^3 kg water falls through a height of 51 m in one second.

i) Calculate the work done by the falling water?

ii) How much power can be generated under ideal conditions?

Solution :

i) The potential energy of the water at the top = mgh

$$\text{P.E.} = (1000 \times 10^3 \text{ kg}) \times (9.8 \text{ ms}^{-2}) \times (51 \text{ m}).$$

$$= 9.8 \times 51 \times 10^6 \text{ J}$$

$$= 500 \times 10^6 \text{ J}$$

Water loses all its potential energy. The same is converted into work in moving the turbine blades. Therefore

$$W = \text{Force} \times \text{distance}$$

$$= mg \times h$$

$$= 1000 \times 10^3 \times (9.8) \times 51 \text{ J}$$

$$= 500 \times 10^6 \text{ J}$$

$$= 500 \text{ M J}$$

ii) The work done per second is given by

$$P = W/t$$

$$= \frac{500 \text{ M J}}{1 \text{ s}}$$

$$= 500 \text{ MW}$$

Ideal conditions mean that there is no loss of energy due to frictional forces. In practice, there is the always some loss in machines. Such losses can be minimized but can never be eliminated.

6.6.2 Potential energy of springs

You now know that an external force is required to compress or stretch a given spring. These situations are shown in Fig. 6.5. Let there be a spring of force constant k. This spring is compressed by a distance x. From Eqn.(6.11) we recall that work done by the external force to compress the spring is given by

$$W = \frac{1}{2} kx^2$$

This work is stored in the spring as elastic potential energy. When the spring is left free, it bounces back and the elastic potential energy of the spring is converted into kinetic energy of the mass *m*.

6.6.3 Conservation of Energy

We see around us various forms of energy but we are familiar with some forms more than others. Examples are Electrical Energy, Thermal Energy, Gravitational Energy, Chemical Energy and Nuclear Energy etc. These forms of energy are very closely related in the sense that one can be changed to another. There is a very fundamental law about energy. It is known as **Law of Conservation of Energy**. It states, " **The total energy of an isolated system always remains constant.**" The energy may change its form. It can be converted from one form to other. But the total energy of the system remains unchanged. In an isolated system, if there is any loss of energy of one form, there is a gain of an equal amount of another form of energy. Thus energy is neither created nor destroyed. The universe is also an isolated system as there is nothing beyond this. It is therefore said that the total energy of the universe always remains constant in spite of the fact that variety of changes are taking place in the universe every moment. It is a law of great importance. It has led to many new discoveries in science and it has not been found to fail.

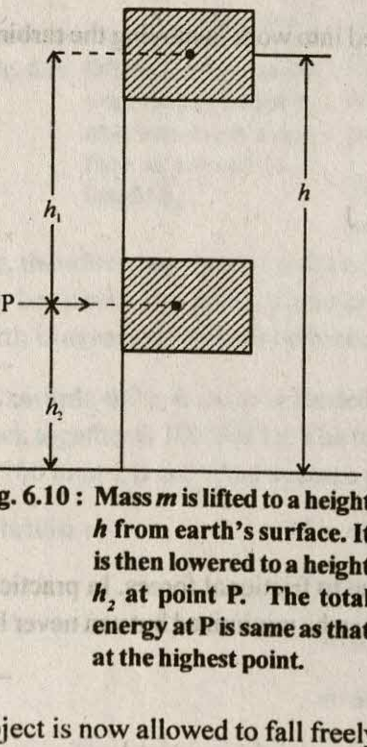


Fig. 6.10 : Mass *m* is lifted to a height *h* from earth's surface. It is then lowered to a height h_2 at point P. The total energy at P is same as that at the highest point.

In a Thermal Power Station, the chemical energy of coal is changed into electrical energy. The electrical energy runs machines. In these machines, the electrical energy changes into mechanical energy, light energy or thermal energy.

The law of conservation of energy is more general than we can think of. It applies to systems ranging from big planets and stars to the smallest nuclear particles.

(a) Conservation of mechanical energy during the free fall of a body

We now wish to test the validity of the law of conservation of energy in case of mechanical energy, which is of immediate interest.

Let us suppose that an object of mass *m* lying on the ground is lifted to a height *h*. The work done is *mgh*, which is stored in the object as potential energy. This object is now allowed to fall freely. Let us calculate the energy of this object when it has fallen through a distance h_1. The height of the object now above the earth surface is $h_2 = h - h_1$ (Fig 6. 10). At this point P, the potential energy = mgh_2.

When the object falls freely, it gets accelerated and gains in speed. We can calculate the speed of the object when it has fallen through a height h_1 from the top positions using the equation

$$v^2 = u^2 + 2gs \qquad (6.21)$$

Notes

where u is the initial speed at the height h_1, i.e. $u = 0$ and $s = h_1$. Then, we have

$$v^2 = 2gh_1$$

The kinetic energy at point P is given by

$$\begin{aligned} \text{K.E} &= \frac{1}{2}mv^2 \\ &= \frac{m}{2} \times 2gh_1 \\ &= mgh_1 \qquad (6.22) \end{aligned}$$

The total energy at the point P is

$$\begin{aligned} \text{Kinetic Energy + Potential Energy} &= mgh_1 + mgh_2 \\ &= mgh \qquad (6.23) \end{aligned}$$

This is same as the potential energy at the highest point. **Thus, the total Energy is conserved.**

(b) Conservation of Mechanical Energy for a Mass Oscillating on a Spring

Fig. 6.11 shows a spring whose one end is fixed to a rigid wall and the other end is connected to a wooden block lying on a smooth horizontal table. This free end is at x_0 in the relaxed position of the spring. A block of mass m moving with speed v along the line of the spring collides with the spring at the free end, and compresses it by x_m. This is the maximum compression. At x_0, the total energy of the spring-mass system is $\frac{1}{2}mv^2$. It is the kinetic energy of the mass. The potential energy of the spring is zero. At the point of extreme compression, the potential energy of the spring is $\frac{1}{2}kx_m^2$ and the kinetic energy of the mass is zero. The total energy now is $\frac{1}{2}kx_m^2$. Obviously, this means that

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 \qquad (6.24)$$

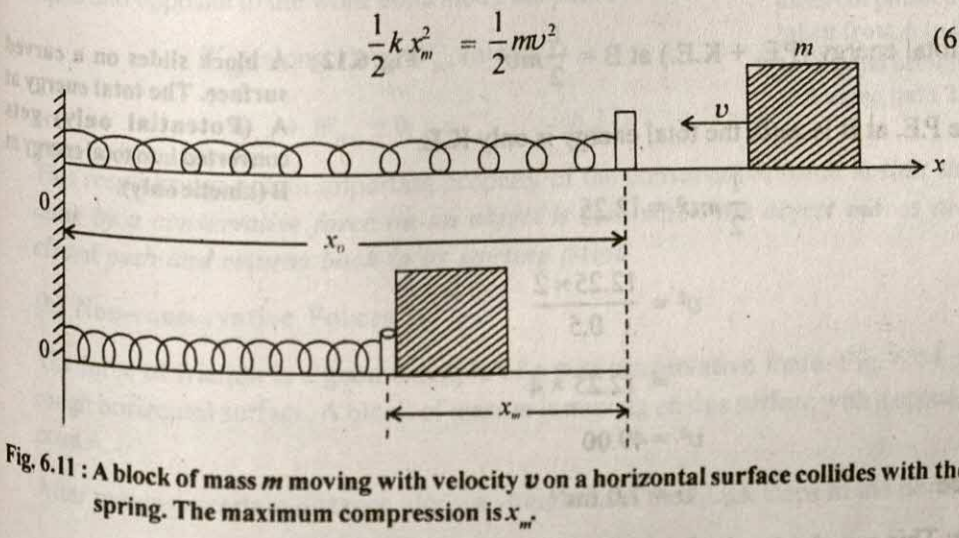


Fig. 6.11 : A block of mass *m* moving with velocity *v* on a horizontal surface collides with the spring. The maximum compression is x_m.

K.E + P.E (Before collision) = K.E. + P.E. (After collision)

$$\frac{1}{2}mv^2 + 0 = 0 + \frac{1}{2}kx_m^2 \qquad (6.25)$$

i.e., *the total energy is conserved.*

Conservation of mass-energy in nuclear reactions

Nuclear energy is different from other forms of energy in the sense that it is not obtained by the transformation of some other form of energy. On the contrary, it is obtained by transformation of mass into energy.

Hence, in nuclear reacions, the law of conservation of mass and the law of conservation of energy merge into a single law of conservation of mass-energy.

Example 6.9 : A block of mass 0.5 kg slides down a smooth curved surface and falls through a vertical height of 2.5m to reach a horizontal surface at B (Fig 6.12). On the basis of energy conservtion, calculate, i) the energy of the block at point A, and ii) the speed of the block at point B.

Solution :

i) Potential energy at A = mgh $= (0.5) \times (9.8) \times 2.5$ J

$= 4.9 \times 2.5$ J

$= 12.25$ J

The kinetic energy at A = 0 and

Total Energy = 12.25 J

ii) The total energy of the block at A must be the same as the total energy at B.

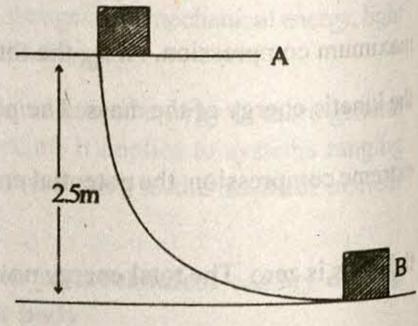


Fig. 6.12 : A block slides on a curved surface. The total energy at A (Potential only) gets converted into total energy at B (kinetic only).

The total energy (P.E. + K.E.) at A = 12.25 J

The total energy (P.E. + K.E.) at B = $\frac{1}{2}mv^2$

Since P.E. at B is zero, the total energy is only K.E.

$$\therefore \quad \frac{1}{2}mv^2 = 12.25$$

$$v^2 = \frac{12.25 \times 2}{0.5}$$

$$= 12.25 \times 4$$

$$v^2 = 49.00$$

Hence $v = 7.0 \text{ ms}^{-1}$

Note: This can also be obtained from the equations of motion:

$$v^2 = v_0^2 + 2gx$$

$$= 0 + 2 \times 9.8 \times 2.5$$

$$v^2 = 49$$

$$v = 7\ \text{ms}^{-1}$$

6.5.4 Conservative and dissipative (Non conservative) Forces

(a) Conservative forces

We have seen that the work done by the gravitational force acting on an object depends on the product of the weight of the object and its vertical displacement. If an object is moved from a point A to a point B under gravity, (Fig 6.13), the work done by gravity depends on the vertical separation between the two points. It does not depend on the path followed to reach B starting from A. When a force obeys this rule, it is called **a conservative force.** Some of the examples of conservative forces are gravitational force, elastic force and electrostatic force.

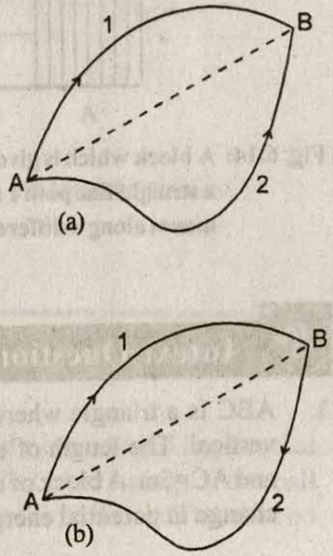


Fig. 6.13 : a) The object is moved from A to B along two different paths. b) It is taken from A to B along path 1 and brought back to A along path 2.

A conservative force has a property that *the work done by a conservative force is independent of path. In* Fig 6.13 (*a*)

$$W_{AB}\ (\text{along } 1) = W_{AB}\ (\text{along } 2)$$

Fig. 6.13 (b) shows the same two positions of the object. The object moves from A to B along the path 1 and returns back to A along the path 2. By definition, the work done by a conservative force along path 1 is equal and opposite to the work done along the path 2.

$$W_{AB}\ (\text{along } 1) = -W_{BA}\ (\text{along } 2)$$

or
$$W_{AB} + W_{BA} = 0 \qquad (6.27)$$

This result brings out an important property of the conservative force in ***that the work done by a conservative force on an object is zero when the object moves around a closed path and returns back to its starting point.***

(b) Non-conservative Forces

The force of friction is a good example of a non-conservative force. Fig. 6.14 shows a rough horizontal surface. A block of mass m is moving on this surface with a speed v at the point A.

After moving a certain distance along a straight line, the block stops at the point B. The block had a kinetic energy $E = \frac{1}{2} m v^2$ at the point A. It has neither kinetic energy nor potential energy at the point B. It has lost all its energy. Do you know where did the energy go? It has changed its form. Work has been done against the frictional force or we can say that force of friction has done negative work on the block. The kinetic energy has changed to thermal energy of the system. The block with the same kinetic energy E is now taken from A to B through a longer path 2. It may not even reach the point B. It may stop much before reaching B. This obviously means that more work has to be done along this path. Thus, it canbe said that the work done depends on the path.

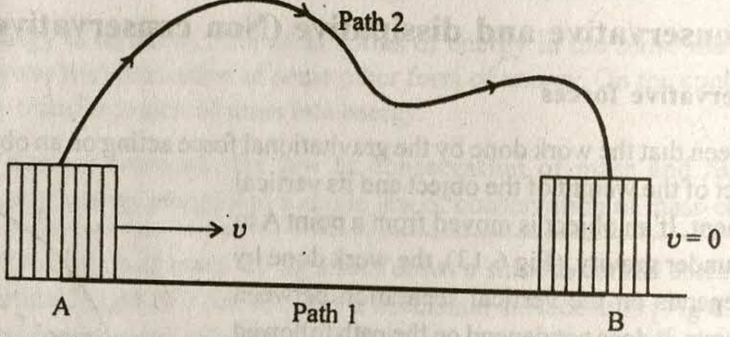


Fig. 6.14: A block which is given an initial speed υ on a rough horizontal surface, moves along a straight line path 1 and comes to rest at B. It starts with the same speed υ at A but now moves along a different path 2.

Intext Questions 6.5

1. ABC is a triangle where AB is horizontal and BC is vertical. The length of the sides AB = 3m, BC = 4m and AC=5m. A block of mass 2 kg is at A. What is the change in potential energy of the block when

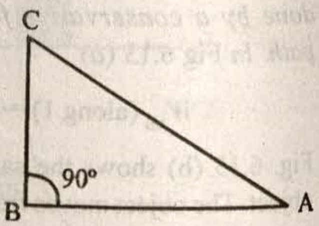


Fig. 6.15

a) it is taken from A to B

b) from B to C

c) from C to A

d) How much work is done by gravitational force in moving the mass form B to C (positive or Negative work)?

....

2. A ball of mass 0.5 kg is at A at a height of 10m above the ground. Solve the following questions by applying work-energy principle. In free fall

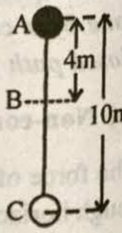


Fig. 6.16

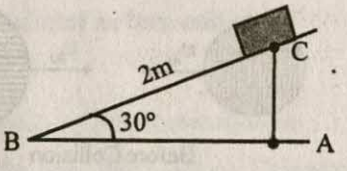
a) What is the speed of the ball at B?

b) What is the speed of the ball at the point C?

c) How much work is done by gravitational force in bringing the ball from A to C (give proper sign)?

....

3. A block at the top of an inclined plane slides down. The length of the plane BC = 2m and it makes an angle of 30° with horizontal. The mass of the block is 2 kg. The kinetic energy of the block at the point B is 15.6 J. How much of the potential energy is lost due to non-conservative forces (friction). How much is the magnitude of the frictional force?



4. The Figure shows two curves A and B between energy E and displacement x of the bob of a simple pendulum. Which one represents the P.E. of the bob and why?

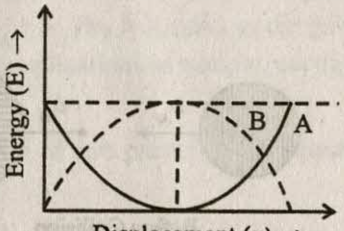


Fig. 6.18

....

5. When non- conservative forces work on a system, does the total mechanical energy remain constant?

....

6.6 Elastic and inelastic collisions

Let us consider a system of two bodies. The system is a closed system which implies that no external force acts on it. The system may consist of two balls or two springs or one ball and one spring and so on. When two bodies interact, it is termed as ***collision***. There are no external forces acting on the system.

Let us start with a collision of two balls and to make the analysis simpler, let there be a **"head-on"** or **"central collision"**. In such collisions, colliding bodies move along the line joining their centres. The collisions are of two kinds :

(i) **Perfectly Elastic Collision:** If the forces of interaction between the two bodies are **conservative,** the total kinetic energy is conserved i.e. the total kinetic energy before collision is same as that after the collision. Such collisions are termed as **completely elastic collisions.**

(ii) **Perfectly Inelastic collision:** When two colliding bodies stick together after the collision and move as one single unit, it is termed as **perfectly inelastic collision.** It is like motion of a bullet embedded in a target.

You should remember that the momentum is conserved in all types of collisions. Why? But kinetic energy is conserved in elastic collisions only.

6.6.1 Elastic Collision (Head-on)

Let two balls A and B having masses m_A and m_B respectively collide "head-on", as shown in Fig. (6.19). Let v_{Ai} and v_{Bi} be the velocities of the two balls before collision and v_{Af} and v_{Bf} be their velocities after the collision .

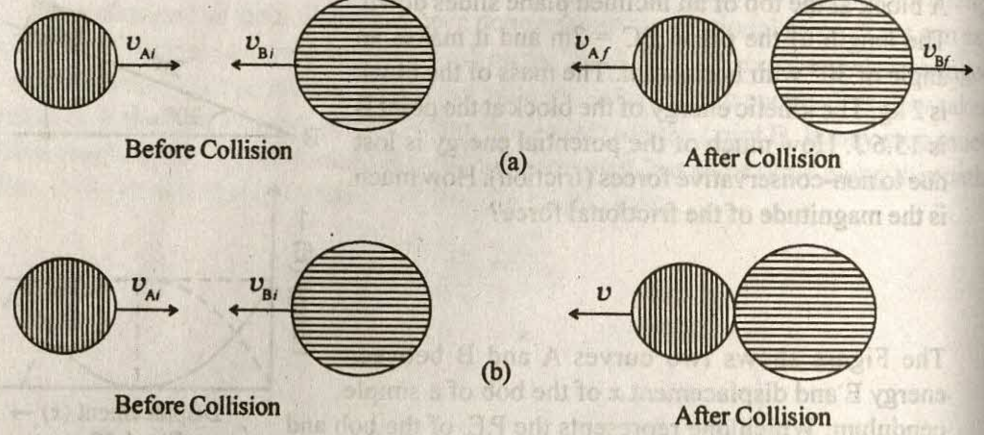


Fig.6.19 : Schematic representation of Head-on collision (a) Elastic collision; (b) In elastic collision

Now applying the laws of conservation of momentum and kinetic energy, we get

For conservation of momentum

$$m_A v_{Ai} + m_A v_{Bi} = m_A v_{Af} + m_B v_{Bf} \tag{6.28}$$

For conservation of kinetic energy

$$\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_{Bi} = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 \tag{6.29}$$

There are only two unknown quantities (velocities of the balls after collision) and there are two independent equations [Eqns. (6.28) and (6.29)]. The solution is not difficult, but a lengthy one. Therefore, we quote the results only

$$(v_{Bf} - v_{Af}) = -(v_{Bi} - v_{Ai}) \tag{6.30}$$

$$v_{Af} = \frac{2m_B v_{Bi}}{m_A + m_B} + \frac{v_{Ai}(m_A - m_B)}{m_A + m_B} \tag{6.31}$$

$$v_{Bf} = -\frac{2m_A v_{Af}}{m_A + m_B} + \frac{(m_B - m_A) v_{Bi}}{(m_A + m_B)} \tag{6.32}$$

We now discuss some special cases.

CASE I : Suppose that the two balls colliding with each other are identical i.e. $m_A = m_B = m$. Then the second term in Eqns. (6.31 and (6.32) will drop out resulting in

$$v_{Af} = v_{Bi} \tag{6.33}$$

and

$$v_{Bf} = v_{Ai} \tag{6.34}$$

That is, if two identical balls collide "head-on", their velocities after collision get interchanged.

After collision: i) the velocity of A is same as that of B before collision.

ii) the velocity of B is same as that of A before collision.

Now, think what would happen if one of the balls is at rest before collision?

Let B be at rest so that $\upsilon_{Bi} = 0$. Then $\upsilon_{Af} = 0$ and $\upsilon_{Bf} = \upsilon_{Ai}$

After collision, A comes to rest and B moves with the velocity of A before collision.

Similar conclusion can be drawn about the kinetic energy of the balls after collision. Complete loss of kinetic energy or partial loss of kinetic energy $(m_A \# m_B)$ by A is same as the gain in the kinetic energy of B. These facts have very important applications in nuclear reactors in slowing down neutrons.

CASE II : The second interesting case is that of collision of two particles of unequal masses.

i) Let us assume that m_B is very large compared to m_A and particle B is initially at rest :

$$m_B >> m_A \text{ and } \upsilon_{Bi} = 0$$

Then, the mass m_A can be neglected in comparison to m_B. From Eqns. (6.31) and (6.32), we get

$$\upsilon_{Af} \approx -\upsilon_{Ai}$$

and

$$\upsilon_{Bf} \approx 0$$

After collision, the heavy particle continues to be at rest. The light particle returns back on its path with a velocity equal to its the initial velocity.

This is what happens when a child hits a wall with a ball.

These results find applications in Physics of atoms, as for example in the case where an α – particle hits a heavy nucleus such as uranium.

Intext Questions 6.6

1 Two hard balls collide when one of them is at rest.

a) Is it possible that both of them remain at rest after collision?

b) Is it possible that one of them remains at rest after collision?

....

2. There is a system of three identical balls A B C on a straight line as shown here. B and C are in contact and at rest. A moving with a velocity υ collides "head-on" with B. After collision, what will be the velocities of A, B and C separately? Explain.

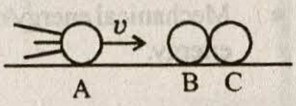


Fig. 6.20

....

3. Ball A of mass 2 kg collides head-on with ball B of mass 4 kg. A is moving in + x direction with speed 50ms^{-1} and B is moving in –x direction with speed 40ms^{-1}. What are the velocities of A and B after collision? The collision is elastic.

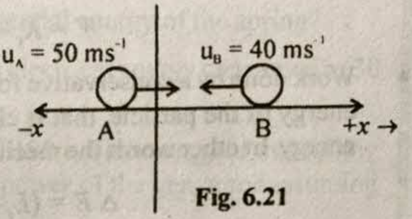


Fig. 6.21

....

4. A bullet of mass 1 kg is fired and gets embedded into a block of wood of mass 1 kg initially at rest. The velocity if the bullet before collision is 90m/s.

   a) What is the velocity of the system after collision.

   b) Calculate the kinetic energies before and after the collision?

   c) Is it an elastic collision or inelastic collision?

   d) How much energy is lost in collision?

   ....

5. In an elastic collision between two balls, does the kinetic energy of each ball change after collision?

   ....

What You Have Learnt

- Work done by a constant force F is

  $$W = \mathbf{F}.\mathbf{d} = Fd\cos\theta$$

  Where θ is the angle between F and d. The unit of work is joule. Work is a scalar quantity.

- Work is numerically equal to the area under the F versus x graph.

- Work done by elastic force obeying Hooke's law is $W = \frac{1}{2}kx^2$ where k is force constant of the elastic material (spring or wire). The sign of W is **positive** for the external force acting on the spring and **negative** for the restoring force offered by spring. x is compression or elongation of the spring.

- The unit of k is newton per metre ($N\ m^{-1}$.)

- Power is the time rate of doing work. $P = W/t$ its unit is J/s i.e., watt (W)

- Mechanical energy of a system exists in two forms (i) kinetic energy and (ii) Potential energy.

- Kinetic energy of mass m moving with speed v is $E = \frac{1}{2}mv^2$. It is a scalar quantity.

- The **Work-Energy Theorem** states that the work done by all forces is equal to the change in the kinetic energy of the object.

  $$W = K_f - K_i = \Delta K$$

- Work done by a conservative force on a particle is equal to the change in mechanical energy of the particle, that is change in the kinetic energy + the change in potential energy. In other words the mechanical energy is conserved under conservative forces.

  $$\Delta E = (E_f - E_i) + (E_f - E_i)$$

  $$= (\Delta E)_p + (\Delta E)_k$$

- Work done by a conservative force on an object is zero for a round trip of the object (object returning back to its starting point).
- Work done by a conservative force does not depend on the path of the moving object. It depends only on its initial and final positions.
- Work done is path dependent for a non-conservative force. The total mechanical energy is not conserved.
- The potential energy of a particle is the energy because of its position in space in a conservative field.
- Energy stored in a compressed or stretches spring is known as elastic potential energy. It has a value $\frac{1}{2}kx^2$, where k is spring constant and x is diplacement.
- The energy stored in a mass m near the earth's surface is mgh. It is called the gravitational potential energy. Here h is change in vertical co-ordinate of the mass. The reference level of zero potential energy is arbitrary.
- Energy may be transformed from one kind to another in an isolated system, but it can neither be created nor destroyed. The total energy always remains constant.
- Laws of conservation of momentum always hold good in any type of collision.
- The kinetic energy is also conserved in elastic collision while it is not conserved in inelastic collision.

Terminal Exercise

1. If two particles have the same kinetic energy, are their momenta also same? Explain.
2. A particle in motion collides with another one at rest. Is it possible that both of them are at rest after collision?
3. Does the total mechanical energy of a system remain constant when dissipative forces work on the system?
4. A child throws a ball vertically upwards with a velocity 20 m s^{-1}.

   (a) At what point is the kinetic energy maximum?

   (b) At what point is the potenital energy maximum?
5. A block of mass 3kg moving with a velocity 20ms^{-1} collides with a spring of force constant 1200 Nm^{-1}. Calculate the maximum compression of the spring.
6. What will be the compression of the spring in question 5 at the moment when kinetic energy of the block is equal to twice the elastic potential energy of the spring?
7. The power of an electric bulb is 60W. Calculate the electrical energy consumed in 30 days if the bulb is lighted for 12 hours per day.
8. 1000kg of water falls every second from a height of 120m. The energy of this falling water is used to generate electricity. Calculate the power of the generator assuming no losses.

9. The speed of a 1200 kg car is 90 km h^{-1} on a highway. The driver applies brakes to stop the car. The car comes to rest in 3 seconds. Calculate the average power of the brakes.

10. A 400g ball moving with speed 5 m s^{-1} has elastic head-on collision with another ball of mass 600g initially at rest. Calculate the speed of the balls after collision.

11. A bullet of mass 10g is fired with an initial velocity 500 m s^{-1}. It hits a 20kg wooden block at rest and gets embedded into the block.

    (a) Calculate the velocity of the block after the impact

    (b) How much energy is lost in the collision?

12. An object of mass 6kg. is resting on a horizontal surface. A horizontal force of 15N is constantly applied on the object. The object moves a distance of 100m in 10 seconds.
    (a) How much work does the applied force do?
    (b) What is the kinetic energy of the block after 10 seconds?
    (c) What is the magnitude and direction of the frictional force (if there is any)?
    (d) How much energy is lost during motion?

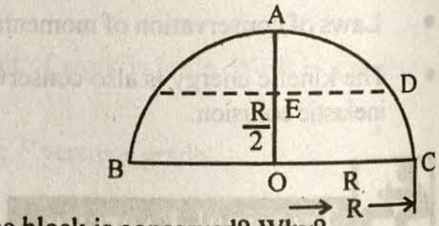
13. A, B, C and D are four point on a hemispherical cup placed inverted on the ground. Diameter BC = 50cm. A 250g particle at rest at A, slide down along the smooth surface of the cup. Calculate it's

    (a) Potential energy at A relative to B.

    (b) Speed at the point B (Lowest point).

    (c) Kinetic and potential energy at D.

    Do you find that the mechanical energy of the block is conserved? Why?



14. The force constant of a spring is 400N/m. How much work must be done on the spring to stretch it (a) by 6.0cm (b) from $x = 4.0$cm to $x = 6.0$ cm, where $x = 0$ is the relaxed position of the spring.

15. The mass of a car is 1000kg. It starts from rest and attains a speed of 15 m s^{-1} in 3.0seconds. Calculate

    (a) The average power of the engine.

    (b) The work done on the car by the engine.

Answers to Intext Questions

6.1

1. The force always works at right angle to the motion of the particle. Hence no work is done by the force.

2. (a) Work done is zero (i) when there is no displacement of the object. (ii) When the angle between force and the displacement is 90°.
   When a mass moves on a horizontal plane the work done by gravitation force is zero.

2. (a) Work done is zero (i) when there is no displacement of the object. (ii) When the angle between force and the displacement is 90°.
   When a mass moves on a horizontal plane the work done by gravitation force is zero.

   (b) When a particle is thrown verically upwards, the work done by gravitational force is **negative.**

   (c) When a particle moves in the direction of force, the work done by force is **positive.**

3. (a) $W = mgh = 2 \times 9.8 \times 5 = +98$ J

   (b) The work done by gravity is –98 J

4. $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ $\quad$ $\mathbf{d} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$

   $W = \mathbf{F.d} \qquad = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) . (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$

   $-2 + 6 = 4$

5. $\mathbf{F} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ $\quad$ $\mathbf{d} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

   (a) $|\mathbf{d}| = \sqrt{9+16} = \sqrt{25} = \mathbf{5\ m}$

   (b) $|\mathbf{F}| = \sqrt{25+9} = \sqrt{34} = 5.83$

   (c) $W = \mathbf{F.d} \qquad = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) . (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$

   $= 15 + 12 = 27$ J

6.2

1. Spring constant is defined as the restoring force per unit displacement. Thus, it is measured in Nm^{-1}.

2. $k = \frac{10\text{N}}{1\text{cm}} = \frac{10\text{N}}{1/100\text{m}} = 100$ N m

As $F = kx$ for $x = 50$ cm. $F = \left(100\frac{\text{N}}{\text{m}}\right)(0.5\text{ m})$

$= 50$ N.

$W = \frac{1}{2}kx^2 = \frac{1}{2} \times \frac{100\text{N}}{\text{m}} \times \left(\frac{5}{100} \times \frac{5}{100}\right)\text{m}^2$

$= 1.25$ N m $= 1.25$ J.

6.3

1. $P = \frac{mgh}{t} = \frac{(100 \times 9.8 \times 8)}{10s}\text{J} = 784\text{W}.$

2. $10\text{ H.P} = (10 \times 746)\text{ W} = \frac{10 \times 746}{1000}\text{ W}$

$= 7.46 \text{ kW}$

6.4

1. $\text{k.E.} = \frac{1}{2}m\upsilon^2$. It is never negative because

   (i) m is never negative

   (ii) υ^2 is always positive.

2. (a) $\text{K.E} = \frac{1}{2}m\upsilon^2 = E$

   When υ is made 2υ, K.E becomes 4 times and E becomes $4E$

   (b) When m becomes $\frac{m}{2}$, E becomes $\frac{E}{2}$

3. P.E. of spring $= \frac{1}{2}kx^2 = 3.6 \text{ J}$

$$\therefore \quad x^2 = \frac{2\times 3.6}{k} = \frac{2\times 3.6}{180} = 0.04\text{m}$$

and $\quad x = 0.2\text{m} = 20\text{cm}.$

4. $\upsilon^2 = u^2 - 2as$ Final velocity is zero and initial velocity is $\frac{90\text{km}}{\text{h}} = 25\text{ms}^{-1}$

$$\therefore \frac{u^2}{2s} = a = \frac{25\times 25}{2\times 15} = 20.83 \text{ ms}^{-2}$$

$F = ma = 1000 \times 20.83 = 20830\text{N}.$

$$\text{Power} = \frac{W}{t} = \frac{20830\times 15}{25} = 12498\text{W}$$

5. Work done by external force = 375 J

   Work done by spring = – 375 J

6.5

1. (a) O, no change in P.E.

   (b) Change in P.E. $= mgh = 2\times 9.8\times 4 = 78.4$ J

   (c) Change in P.E. = 78.4 J.

   (d) – 78.4 J.

2. (a) Change in P.E. from $= mgh = 0.5\times 9.8\times 4 = 19.6$ J

b) $\upsilon = 14 \text{ m s}^{-1}$

(c) $mgh = 0.5 \times 9.8 \times 10 = 49.0$ J (+ positive)

W = +49 J

3. BC = 2m

$$\frac{AC}{BC} = \sin 30^\circ$$

$$AC = BC \sin 30^\circ$$

$$= 2 \times \frac{1}{2} = 1$$

Change in P.E. from C to B = mgh = 2 × 9.8 × 1 = 19.6 J

But the K.E. at B is = 15.6 J

Energy lost = 19.6 – 15.6 = 4J

This loss is due to frictional force

$4\text{J} = F \times d = F \times 2$

$F = 2\text{N}$

4. When the bob of a simple pendulum oscillates, its K.E. is max at $x = 0$ and min at $x = x_m$. The P.E. is min at $x = 0$ and max at $x = x_m$. Hence A represents the P.E. curve.

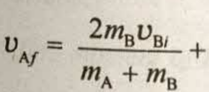
5. No.

6.6

1. (a) No, because, it will go against the low of conservation of linear momentum.

(b) yes.

2.



$\upsilon_A = 0,\ \upsilon_B = 0,\ \upsilon_C = \upsilon$

∵ This condition only satisfies the laws of conservation of (i) linear momentum and (ii) total kinetic energy.

3. $$\upsilon_{Af} = \frac{2m_B \upsilon_{Bi}}{m_A + m_B} + \frac{\upsilon_{Ai}(m_A - m_B)}{m_A + m_B}$$

$$= \frac{2 \times 4 \times (-40)}{6} - \frac{50(-2)}{6}$$

$$= -\frac{320}{6} + \frac{100}{6}$$

$$= -\frac{220}{6}$$

$$= \frac{2\times4\times(-40)}{6} - \frac{50(-2)}{6}$$

$$= -\frac{320}{6} + \frac{100}{6}$$

$$= -\frac{220}{6}$$

$$= -35\text{ms}^{-1}.$$

$$\upsilon_{Bf} = -\frac{2m_A\upsilon_{A_i}}{m_A + m_B} + \frac{(m_B - m_A)\,\upsilon_{B_i}}{(m_A + m_B)}$$

$$= \frac{2\times2\times50}{6} + \frac{(-40)(4-2)}{6}$$

$$= \frac{200}{6} - \frac{80}{6}$$

$$= \frac{120}{6} = 20\text{ ms}^{-1}.$$

Thus ball A returns back with a velocity of 35 ms^{-1} and ball B moves on with a velocity of 20 ms^{-1}.

4. (a) 1.76 ms^{-1}.

   (b) 81 J and 1.58 J

   (c) Inelastic collision

   (d) 79.42 J

5. yes, but the total energy of both the balls together after collision is the same as it was before collision.

Answers to Terminal Problem

5. 1 m.
6. 0.707 m
7. 21.6 kW
8. 1.2 mega watt
9. 125 kW
10. $\frac{1}{4}\text{ m s}^{-1}$, $\frac{19}{6}\text{ ms}^{-1}$
11. (a) 0.25 m s^{-1}

    (b) 1249.4 J
12. (a) 1500 J (b) 1200 J (c) 3 N opposite to the direction of motion

    (d) 300 J

7

MOTION OF RIGID BODY

So far you have learnt about the motion of a single object, usually taken as a point mass. This simplification is quite useful for learning the laws of mechanics. But in real life, objects consist of very large number of particles. A tiny pebble contains millions of particles. Do we then write millions of equations. one for each particle? Or is there a simpler way? While discovering answer to this question you will learn about centre of mass and moment of inertia, which plays the same role in rotational motion as does mass in translational motion.

You will also study an important concept of physics, the angular momentum. If no external force acts on a rotating system, its angular momentum in conserved. This has very important implications in physics. It enables us to understand how a swimmer is able to somersault while diving from a diving board into the water below.

Objectives

After studying this lesson, you should be able to :

- *define the centre of mass of a rigid body;*
- *explain why motion of a rigid body is a combination of translational and rotational motions;*
- *define moment of inertia and state theorems of parallel and perpendicular axes;*
- *define torque and find the direction of rotation produced by it;*
- *write the equation of motion of a rigid body;*
- *state the principle of conservation of angular momentum; and*
- *calculate the velocity acquired by a rigid body at the end of its motion on an inclined plane.*

7.1 Rigid Body

As mentioned earlier, point masses are ideal constructs, brought in for simplicity in discussion. In practice, when extended bodies interact with each other and the distances between them are very large compared to their sizes, their sizes can be ignored and they may be treated as point masses. *Can you give two examples of such cases where the sizes of the bodies are not important?* Sizes of stars are small as compared to the size of the galaxy. So, stars can be considered as point masses. Similarly, in the earth-moon system, moon's size can be ignored. But when we have to consider the rotation of a body about an axis, the size of the body becomes important. When we consider the rotation of a system, we generally assume that during rotation, the distances between its constituent particles remain fixed. Such a system of particles is called a ***rigid body***.

A rigid body is one in which the separation between the constituent particles does not change with its motion.

This definition implies that the shape of a rigid body is preserved during its motion. However, like a point mass, a rigid body is also an idealisation because, if we apply large forces, the distances between particles do change, may be infinitesimally. Therefore, in nature there is nothing like a perfectly rigid body. For most purposes, a solid body is good enough approximation to a rigid body. A cricket ball, a wooden block, a steel disc, even the earth and the moon could be considered as rigid bodies in this lesson.

Can water in a bucket be considered a rigid body? Obviously, water in a bucket cannot be a rigid body because it changes shape as bucket is pushed around.

You may now like to check what you have understood about a rigid body.

Intext Questions 7.1

1. A frame is made of six wooden rods. The rods are firmly attached to each other. Can this system be considered a rigid body?

   ....

2. Can a heap of sand be considered a rigid body? Explain your answer.

   ....

7.2 Centre of Mass (C.M.) of a Rigid Body

Before we deal with rigid bodies consisting of several particles, let us consider a simpler case. Suppose we have a system of two particles having same mass joined by weightless and inextensible rod. *Can we consider this system as a rigid body?*

In this system, the distance between the two particles is fixed. So it is a rigid body.

Suppose that the two particles are at heights z_1 and z_2 from a horizontal surface (Fig. 7.1). Suppose further that the gravitational force is uniform in the small region in which the two particles move about. The force on each particle will be mg. The total force acting on the system is therefore $2mg$. We have now to find a point C somewhere in the system so that if a force $2mg$ acts at that point located at a height z from the horizontal surface, the motion of the system would be the same as with two forces. The potiential energies of particles 1 and 2 are mgz_1 and mgz_2, respectively. The potential energy of the particle at C is $2mgz$. Since this must be equal to the combined potential energy of the two particles, can write

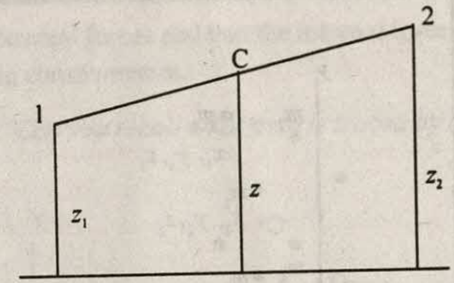


Fig. 7.1 : Two particle system

$$2\,mgz = mgz_1 + mgz_2 \tag{7.1}$$

or

$$z = \frac{z_1 + z_2}{2} \tag{7.2}$$

Note that the point C lies midway between the two particles. If the two masses were unequal, this point would not have been in the middle. If the mass of particle 1 is m_1 and that of particle 2 is m_2, Eqn. (7.1) modifies to

$$(m_1 + m_2)\,gz = m_1 gz_1 + m_2 gz_2 \tag{7.3}$$

so that

$$z = \frac{m_1 z_1 + m_2 z_2}{(m_1 + m_2)} \tag{7.4}$$

The point C is called the **centre of mass** (CM) of the system. As such, it is a mathematical tool and there is no physical point as CM.

To grasp this concept, study the following example carefully.

Example 7.1 : If in the above case, the mass of one particle is twice that of the other, let us locate the CM.

Solution : $m_1 = m$ and $m_2 = 2\,m$, Then Eqn. (7.4) gives

$$z = \frac{m\,z_1 + 2\,m\,z_2}{(m + 2\,m)} = \frac{z_1 + 2\,z_2}{3}$$

When a body consists of several particles, we generalise Eqn (7.4) to define its CM : *If the particle with mass m_1 has coordinates (x_1, y_1, z_1) with respect to some coordinate system, mass m_2 has coordinates (x_2, y_2, z_2) and so on (Fig.7.2), the coordinates of CM are given by*

$$x = \frac{m_1\,x_1 + m_2\,x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum_i^N m_i\,x_i}{\sum_{i=1}^N m_i} \tag{7.5}$$

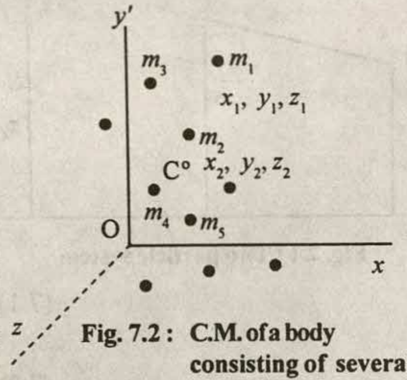


Fig. 7.2 : C.M. of a body consisting of several particles

$$= \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

Similarly $$y = \frac{\sum_{i=1}^{N} m_i y_i}{M} \tag{7.6}$$

and $$z = \frac{\sum_{i=1}^{N} m_i z_i}{M} \tag{7.7}$$

where $\sum_{i=1}^{N} m_i$ denotes the sum over all the particles and, therefore, $\sum_{i=1}^{N} m_i$ is the total mass of the body, M.

Why should we define CM so precisely?

Recall that the rate of change of displacement is velocity, and the rate of change of velocity is acceleration. If a_{1x} denotes the component of acceleration of particle 1 along the x-axis and so on, from Eqn. (7.5), we can write

$$M a_x = m_1 a_{1x} + m_2 a_{2x} + \ldots \tag{7.8}$$

where a_x is the acceleration of the centre of mass along x-axis. Similar equations can be written for accelerations along y- and z-axes. These equations can, however, be combined into a single equation using vector notation :

$$M\mathbf{a} = m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2 + \ldots \tag{7.9}$$

But the product of mass and acceleration is force. $m_1 \mathrm{a}_1$ is therefore the sum of all forces acting on particle 1. Similarly, $m_2 \mathbf{a}_2$ gives the net force acting on particle 2. The right hand side is, thus, the total force acting on the body.

The forces acting on a body can be of two kinds. Some forces can be due to sources outside the body. These forces are called the ***external*** forces. A familiar example is the force of gravity. Some other forces arise due to the interaction among the particles of the body. These are called ***internal*** forces. A familiar example is cohesive force.

In the case of a rigid body, the sum of the internal forces is zero because they cancel each other in pairs. Therefore, the acceleration of individual particles of the body are due to the sum or resultant of the external forces. In the light of this, we may write Eqn. (7.9) as

$$M\mathbf{a} = \mathbf{F}_{ext} \tag{7.10}$$

This shows that ***the CM of a body moves as though the entire mass of the body were located at that point and it was acted upon by the sum of all the external forces acting on the body.*** Note the simplification introduced in the derivation by defining the centre of mass. We donot have to deal with millions of individual particles now, only the centre of mass needs to be located to determine the motion of the given body. The fact that the motion of the CM is determined by the external forces and that the internal forces have no role in this at all leads to very interesting consequences.

You are familiar with the motion of a projectile. *Can you recall what path is traced by a projectile?*

The motion is along a parabolic path. Suppose the projectile is a bomb which explodes in mid air and breaks up into several fragments. The explosion is caused by the internal forces. There is no change in the external force, which is the force of gravity. The centre of mass of the projectile, therefore, continues to be the same parabola on which the bomb would have moved if it had not exploded (Fig. 7.3). The fragments may fly in all directions on different parabolic paths but the centre of mass of the various fragments will lie on the original parabola.

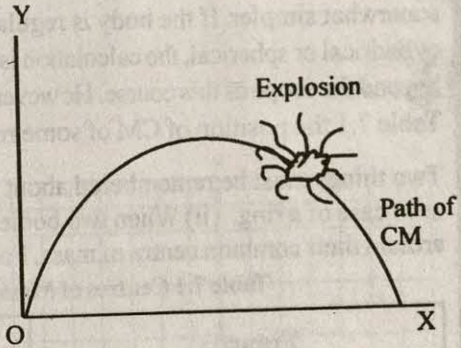


Fig.7.3 : Centre of mass of a projectile

You might have now understood the importance of the concept of centre of mass of a rigid body. You will encounter more examples of importance in subsequent sections. Let us therefore see how the centre of mass of a system is obtained by taking a simple example.

Example 7.2 : Suppose four masses, 1.0 kg, 2.0 kg, 3.0 kg and 4.0 kg are located at the corners of a square of side 1.0 m. Locate its centre of mass?

Solution : We can always make the square lie in a plane. Let this plane be the (x,y) plane. Further, let us assume that one of the corners coincides with the origin of the coordinate system and the sides are along the x and y axes. The coordinates of the four masses are : m_1 (0,0), m_2 (1.0,0), m_3 (1.0,1.0) and m_4 (0, 1.0), where all distances are expressed in metres (Fig.7.4).

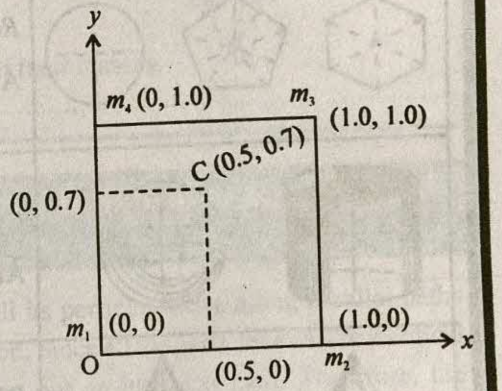


Fig. 7.4 : Locating CM of four masses placed at the corners of a square

From Eqns. (7.5) and (7.6), we get

$$x = \frac{1.0 \times 0 + 2.0 \times 1.0 + 3.0 \times 1.0 + 4.0 \times 0}{1.0 + 2.0 + 3.0 + 4.0} \text{ m}$$

$$= 0.5 \text{ m}$$

and

$$y = \frac{1.0 \times 0 + 2.0 \times 0 + 3.0 \times 1.0 + 4.0 \times 1.0}{1.0 + 2.0 + 3.0 + 4.0} \text{ m}$$

$$= 0.7 \text{ m}$$

The CM has coordinates (0.5 m, 0.7 m) and is marked C in Fig.7.4. Note that the CM is not at the centre of the square although the square is a symmetrical figure.

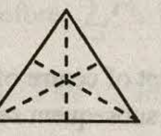
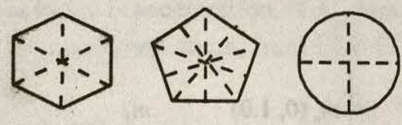
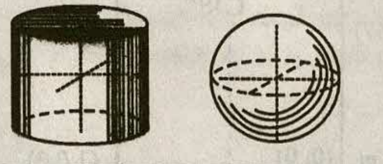
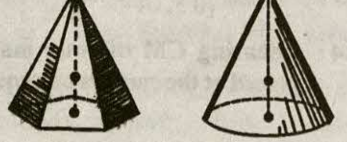
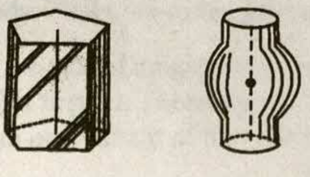
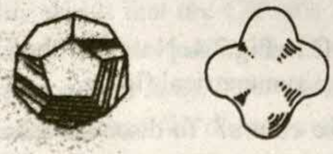
What could be the reason for the CM not being at the centre? To discover answer to this question, calculate the coordinates of CM if all masses are equal.

7.2.1 CM of Some Bodies

The position of centre of mass of extended bodies can not be easily calculated because a very large number of particles constituting the body have to be considered. The fact that all the particles of a rigid body have same mass and are uniformly distributed makes things somewhat simpler. If the body is regular in shape and possesses some symmetry, say it is cylindrical or spherical, the calculation is a little bit simplified. But even such calculations are beyond the scope of this course. However, keeping in mind the importance of CM, we give in Table 7.1 the position of CM of some regular, symmetrical bodies.

Two things must be remembered about the centre of mass : (i) It may be outside the body as in case of a ring. (ii) When two bodies revolve around each other, they actually revolve around their common centre of mass. For example, stars in a binary system revolve around

Table 7.1 Centres of Mass of some regular symmetrical bodies

Figure	Position of Centre of Mass
	*Triangular plate* Point of intersection of the three medians
	*Regular polygon and circular plate* At the geometrical centre of the figure
	*Cylinder and sphere* At the geometrical centre of the figure
	*Pyramid and cone* On line joining vertex with centre of base and at h/4 of the height measured from the base.
	*Figure with axial symmetry* Some point on the axis of symmetry
	*Figure with centre of symmetry* At the centre of symmetry

their common centre of mass. The Earth-Sun system also revolves around its common centre of mass. But since mass of the Sun is very large as compared to the mass of earth, the centre of mass of the system is very close to the centre of the Sun.

Now it is time to check your progress.

Intext Questions 7.2

1. The grid shown here has particles A, B, C, D and E respectively have masses 1.0 kg, 2.0kg, 3.0 kg, 4.0 kg and 5.0 kg. Calculate the coordinates of the position of the centre of mass of the system (Fig. 7.5).

   ....

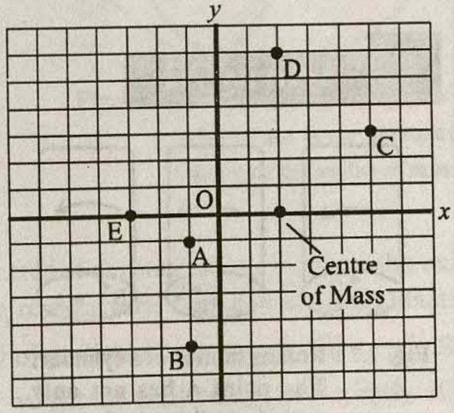


Fig. 7.5

2. If three particles of masses $m_1 = 1$ kg, $m_2 = 2$ kg, and $m_3 = 3$ kg are situated at the corners of an equilateral triangle of side 1.0 m, obtain the position coordinates of the centre of mass of the system.

   ....

3. Show that the ratio of the distances of the two particles from their common centre of mass is inversely proportional to the ratio of their masses.

   ....

7.3 Translational and Rotational Motion of a Rigid Body : A Comparison

When a rigid body moves in such a way that all its particles move along parallel paths (Fig.7.6), its motion is called **translational** motion. Since all the particles execute identical motion, its centre of mass must also be tracing out an identical path. Therefore, the translational motion of a body may be characterised by the motion of its centre of mass. We have seen that this motion is given by Eqn.(7.10) :

$$M\mathbf{a} = \mathbf{F}_{ext}$$

Do you now see the advantage of defining the centre of mass of a body? With its help, the translational motion of body can be described by an equation for a single particle having mass equal to the mass of the whole body. It is located at the centre of mass and is acted upon by the sum of all the external forces which are acting on the rigid body. To understand the concept clearly, perform the following activities.

Activity 7.1

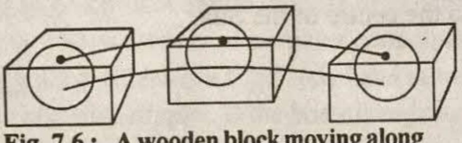


Fig. 7.6 : A wooden block moving along the floor performs translational motion.

Take a wooden block. Make two or three marks on any of its surfaces. Now keep the marked surface in front of you and push the block along a horizontal floor. Note the paths traced by the marks. All these marks have paths parallel to the floor and, therefore, parallel to one another (Fig. 7.6). You can easily see that the lengths of the paths are also equal.

Activity 7.2

Let us now perform another simple experiment. Take a cylindrical piece of wood. On its plane face make a mark or two. Now roll the cylinder slowly on the floor, keeping the plane face towards you. You would notice that the mark such as A in Fig. 7.7, has not only moved parallel to the floor, but has also performed circular motion. So, the body has performed both translational and rotational motion.

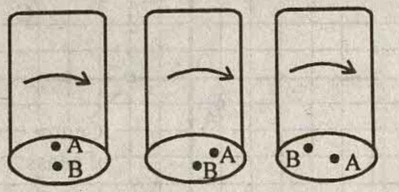


Fig. 7.7 : Rolling motion of a cylinder: The point A has not only moved parallel to the floor but also performed circular motion

While the general motion of a rigid body consists of both translation and rotation, it cannot have translational motion if one point in the body is fixed; it can then only rotate. The most convenient point to fix for this purpose is the CM of the body.

You might have seen a grinding stone (the chakki). The handle of the stone moves in a circular path. All the points on the stone also move in circular paths around an axis passing through the centre of the stone (Fig.7.8).

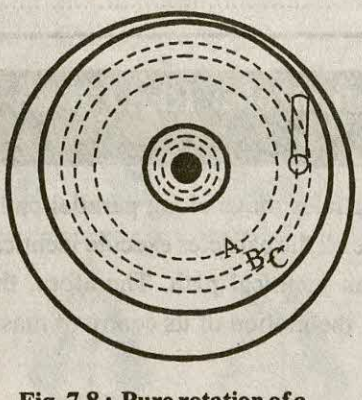


Fig. 7.8 : Pure rotation of a grinding stone

The motion of a rigid body in which all its constituent particles describe concentric circular paths is known as rotational motion.

We have noted above that translational motion of a rigid body can be described by an equation similar to that of a single particle. You are familiar with such equations. Therefore, in this lesson we concentrate only on the rotational motion of a rigid body. The rotational motion can be obtained by keeping a point of the body fixed so that it cannot have any translational motion. For the sake of

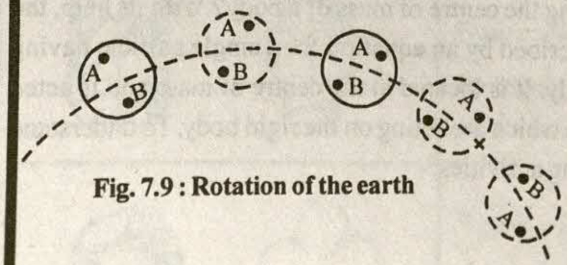


Fig. 7.9 : Rotation of the earth

mathematical convenience, this point is taken to be the CM. The rotation is then about an axis passing through the CM. A good example of rotational motion is the earth's rotation about its own axis (Fig. 7.9). You have studied in earlier lessons that the mass of the body plays a very important role. It determines the acceleration acquired by the body for a given force. Can we define a similar quantity for rotational motion also? Let us find out.

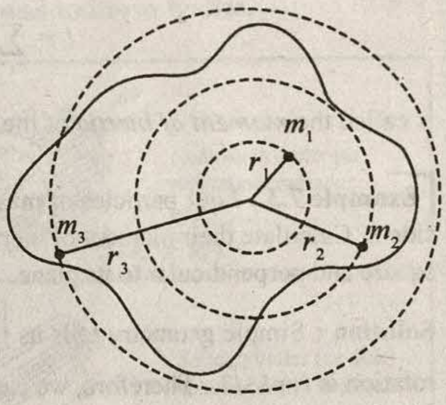


Fig. 7.10 : Rotation of a plane lamina about an axis passing through its centre of mass

7.3.1 Moment of Inertia

Let C be the centre of mass of a rigid body. Suppose it rotates about an axis through this point (Fig.7.10).

Suppose particles of masses m_1, m_2, m_3...are located at distances r_1, r_2, r_3...from the axis of rotation and are moving with speeds v_1, v_2, v_3 respectively. Then particle 1 has kinetic energy (½) $m_1 v_1^2$. Similarly, the kinetic energy of particle of mass m_2 is (½) $m_2 v_2^2$. By adding the kinetic energies of all the particles, we get the total energy of the body. If T denotes the total kinetic energy of the body, we can write

$$T = (½)\, m_1 v_1^2 + (½)\, m_2 v_2^2 + ...$$

$$= \sum_{i=1}^{\overset{n}{i=1}} \left(\frac{1}{2}\right) m_i v_i^2 \qquad (7.11)$$

where $\sum\limits_{i=1}^{i=n}$ indicates the sum over all the particles of the body.

You have studied in lesson 4 that angular speed (ω) is related to linear speed (v) through the equation $v = r\,\omega$. Using this result in Eqn. (7.11), we get

$$T = \sum_{i=1}^{i=n} \left(\frac{1}{2}\right) m_i (r_i \omega)^2 \qquad (7.12)$$

Note that we have not put the subscript i with w because all the particles of a rigid body have the same angular speed. Eqn. (7.12) can now be rewritten as

$$T = \frac{1}{2}\left(\sum_{i-1}^{i=n} m_i\, r_i^2\right)\omega^2$$

$$= \frac{1}{2}\, I\, \omega^2 \qquad (7.13)$$

The quantity

$$I = \sum_i m_i r_i^2 \qquad (7.14)$$

is called the ***moment of inertia*** of the body.

Example 7.3 : *Four particles of mass m each are located at the corners of a square of* side *L*. Calculate their moment of inertia about an-axis passing through the centre of the square and perpendicular to its plane.

Solution : Simple geometry tells us that the distance of each particle from the axis of rotation is $r = L\sqrt{2}$. Therefore, we can write

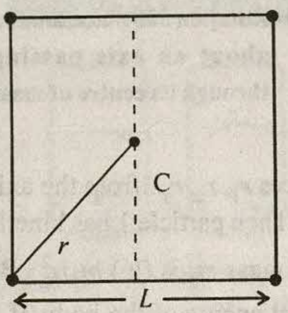


Fig. 7.11

$$I = mr^2 + mr^2 + mr^2 + mr^2$$

$$= 4mr^2$$

$$= 4m\left(\frac{L}{\sqrt{2}}\right)^2 \qquad \text{(Since } r = \frac{L}{\sqrt{2}}\text{).}$$

$$= 2mL^2$$

It is important to remember that ***moment of inertia*** is defined with reference to an axis of rotation. Therefore, whenever you mention moment of inertia, the axis of rotation must also be specified. In the present case, *I* is the moment of inertia about an axis passing through the centre of the square and normal to the plane containing four perfect masses. (Fig. 7.10) The moment of inertia is expressed in kg m^2.

The moment of inertia of a rigid body is often written as

$$I = MK^2 \qquad (7.15)$$

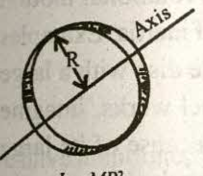
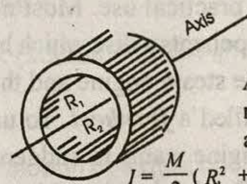
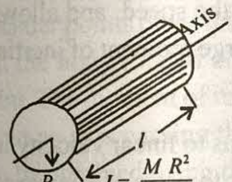
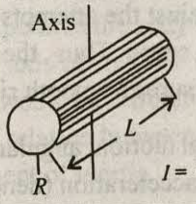
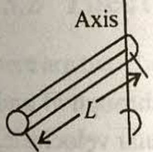
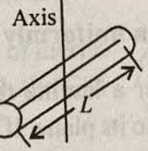
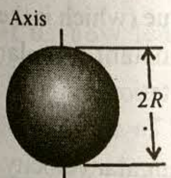
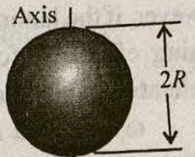
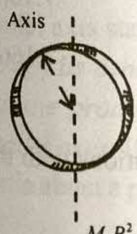
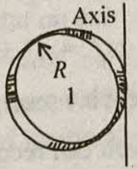
where *M* is the total mass of the body and *K* is called the ***radius of gyration*** of the body. ***The radius of gyration is that distance from the axis of rotation where the whole mass of the body can be assumed to be placed to get the same moment of inertia which the body actually has.*** It is important to remember that the moment of inertia of a body about an axis depends on the distribution of mass around that axis. If the distribution of mass changes, the moment of inertia will also change. This can be easily seen from Example 7.3. Suppose we place additional masses at one pair of opposite corners of amount *m* each. Then the moment of inertia of the system about the axis through C and perpendicular to the plane of square would be

$$I = mr^2 + 2mr^2 + mr^2 + 2mr^2$$

$$= 6mr^2$$

Note that moment of inertia has changed from $2mL^2$ to $3\ mL^2$.

Table 7.2 Moments of inertia of a few regular and uniform bodies.

 Hoop about central axis $I = MR^2$	 Annular cylinder (or ring) about cylinder axis $I = \frac{M}{2}(R_1^2 + R_2^2)$
 Solid cylinder about cylindrical axis $I = \frac{MR^2}{2}$	 Solid cylinder (or disk) about a central diameter $I = \frac{MR^2}{4} + \frac{M^2\ell^2}{12}$
 Thin rod about an axis passing through its centre and normal to its length $I = \frac{ML^2}{12}$	 Thin rod about an axis passing through one end and perpendicular to length $I = \frac{ML^2}{3}$
 Solid sphere about any diameter $I = \frac{2MR^2}{5}$	 Thin spherical shell about any diameter $I = \frac{2MR^2}{3}$
 Hoop about any diameter $I = \frac{MR^2}{2}$	 Hoop about any tangent line $I = \frac{3MR^2}{2}$

Refer to Eqn.(7.13) again and compare it with the equation for kinetic energy of a body in linear motion. Can you draw any analogy? You will note that in rotational motion, the role of mass has been taken over by the moment of inertia and the angular speed has replaced the linear speed.

A. Physical significance of moment of inertia

The physical significance of moment of inertia is that it performs the same role in rotational motion that the mass does in linear motion.

Just as the mass of a body resists change in its state of linear motion, the moment of inertia resists a change in its rotational motion. This property of the moment of inertia has been put to a great practical use. Most machines, which produce rotational motion have as one of their components a disc which has a very large moment of inertia. Examples of such machines are the steam engine and the automobile engine. The disc with a large moment of inertia is called a ***flywheel***. To understand how a flywheel works, imagine that the driver of the engine wants to suddenly increase the speed. Because of its large moment of inertia, the flywheel resists this attempt. It allows only a gradual increase in speed. Similarly, it works against the attempts to suddenly reduce the speed, and allows only a gradual decrease in the speed. Thus , the flywheel, with its large moment of inertia, prevents jerky motion and ensures a smooth ride for the passengers.

We have seen that in rotational motion, angular velocity is analogous to linear velocity in linear motion. Since angular acceleration (denoted usually by α) is the rate of change of angular velocity, it must correspond to acceleration in linear motion.

B. Equations of motion for a uniformly rotating rigid body

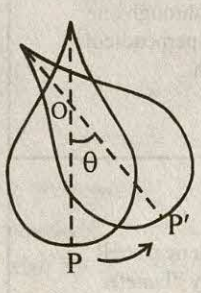


Fig. 7.12 : Rotation of a lamina about a fixed nail

Consider a lamina rotating about an axis passing through O and normal to its plane. If it is rotating with a constant angular velocity ω, as shown, then it will turn through an angle θ in t seconds such that

$$\theta = \omega t \qquad 7.16(a)$$

However, if the lamina is subjected to constant torque (which is the turning effect of force), it will undergo a constant angular acceleration. The following equations describe its rotational motion:

$$\omega_f = \omega_i + \alpha t \qquad 7.16(b)$$

where ω_i is initial angular velocity and ω_f is final angular velocity. Similarly, we can write

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2 \qquad 7.16(c)$$

$$\omega_f^2 = \omega_i^2 + 2 \alpha \theta \qquad 7.16(d)$$

where θ is angular dispalcement in t seconds.

On a little careful scrutiny, you can recognise the similarity of these equations with the corresponding equations of kinematics for translatory motion.

Example 7.4 : A wheel of a bicycle is free to rotate about a horizontal axis (Fig. 7.11). It is initially at rest. Imagine a line OP drawn on it. By what angle would the line OP move in 2 s if it had a uniform angular acceleration of 2.5 rad s^{-2}.

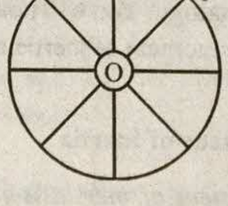


Fig.7.13 : Rotation of bicycle wheel

Solution : Angular displacement of line OP is given by

$$\theta = \omega_0 t + (½)\,\alpha\, t^2$$

$$= 0 + (½) \times (2.5 \text{ rad s}^{-2}).\times 4 \text{ s}^2$$

$$= 5 \text{ rad}$$

We have mentioned above that for rotational motion of a rigid body, its CM is kept fixed. However, it is just a matter of convenience that we keep CM fixed. But many a time, we consider points other than the CM. That is, a point in the body which can also be kept fixed and the body rotated about it. But then the axis of rotation will pass through this fixed point. The moment of inertia about this axis would be different from the moment of inertia about an axis passing through the CM. The relation between the two moments of inertia can be obtained using the theorems of moment of inertia.

7.3.2 Theorems of moment of inertia

There are two theorems which connect moments of inertia about two axes; one of which is passing through the CM of the body. These are :

(i) the theorem of parallel axes, and

(ii) the theorem of perpendicular axes.

Let us now learn about these theorems and their applications.

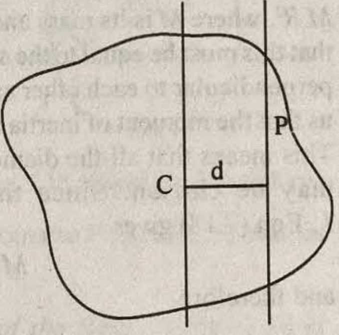


Fig.7.14 : Parallel axes through CM and another point P

(*i*) Theorem of parallel axes

Suppose the given rigid body rotates about an axis passing through any point P other than the centre of mass. The moment of inertia about this axis can be found from a knowledge of the moment of inertia about a parallel axis through the centre of mass. Theorem of parallel axis states that *the moment of inertia about an axis parallel to the axis passing through its centre of mass is equal to the moment of inertia about its centre of mass plus the product of mass and square of the perpendicular distance between the parallel axes*. If I denotes the required moment of inertia and I_C denotes the moment of inertia about a parallel axis passing through the CM, then

$$I = I_C + M d^2 \qquad (7.17)$$

where M is the mass of the body and d is the distance between the two axes (Fig. 7.12). This is known as the **theorem of parallel axes.**

(*ii*) Theorem of perpendicular axes

Let us choose three mutually perpendicular axes, two of which, say x and y are in the plane of the body, and the third, the z–axis, is perpendicular to the plane (Fig.7.13). The perpendicular axes theorem states that ***the sum of the moments of inertia about x and y axes is equal to the moment of inertia about the z–axis.***

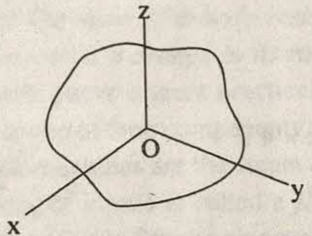


Fig.7.15 : Theorem of perpendicular axes

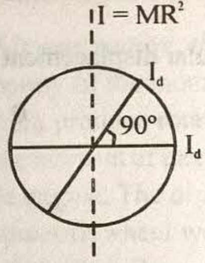


Fig.7.16 : Moment of inertia of a hoop

That is,

$$I_z = I_x + I_y \tag{7.18}$$

We now illustrate the use of these theorems by the following example.

Let us take a hoop shown in Fig. 7.16. From Table 7.2 you would recall that moment of inertia of a hoop about an axis passing through its centre and perpendicular to the base is $M R^2$, where M is its mass and R is its radius. The theorem of perpendicular axes tells us that this must be equal to the sum of the moments of inertia about two diameters which are perpendicular to each other as well as to the central axis. The symmetry of the hoop tells us that the moment of inertia about any diameter is the same as about any other diameter. This means that all the diameters are **equivalent** and any two perpendicular diameters may be chosen.. Since the moment of inertia about each is the same, say I_d, Eqn.(7.18) gives

$$M R^2 = 2 I_d$$

and therefore

$$I_d = (½)\ MR^2$$

So, the moment of inertia of a hoop about any of its diameter is (½) $M R^2$.

Let us now take a point P on the rim. Consider a tangent to the hoop at this point which is parallel to the axis of the hoop. The distance between the two axes is obviously equal to R. The moment of inertia about the tangent can be calculated using the theorem of parallel axes. It is given by

$$I_{\tan} = M R^2 + M R^2 = 2\ M R^2.$$

It must be mentioned that many of the entries in Table 7.2 have been computed using the theorems of parallel and perpendicular axes.

7.3.3 Torque and Couple

Activity 7.3

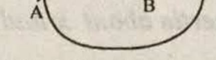


Fig.7.17 : Rotation of a body

Have you ever noticed that it is easy to open the door by applying force at a point far away from the hinges? What happens if you try to open a door by applying force near the hinges? Carry out this activity a few times. You would realise that much more effort is needed to open the door if you apply force near the hinges than at a point away from the hinges. Why is it so? Similarly, for turning a screw we use a spanner with a long handle. What is the purpose of keeping a long handle? Let us seek answers to these questions now.

Suppose O is a fixed point in the body and it can rotate about an axis passing through this point (Fig.7.17). Let a force of magnitude F be applied at the point A along the line AB. If AB passes through the point O, the force F will not be able to rotate the body. The farther is the line AB from O, the greater is the ability of the force to turn the body about the axis through O. The ***turning effect of a force is called torque.*** Its magnitude is given by

$$\tau = F s = F r \sin\theta \qquad (7.19)$$

where s is the distance between the axis of rotation and the line along which the force is applied.

The units of torque are newton-metre, or Nm. The torque is actually a vector quantity. The vector from of Eqn.(7.19) is

$$\tau = \mathbf{r} \times \mathbf{F} \qquad (7.20)$$

which gives both magnitude and direction of the torque. What is the direction in which the body would turn? To discover this, we recall the rules of vector product (refer to lesson 1) : τ is perpendicular to the plane containing vectors **r** and **F**, which is the plane of paper here (Fig.7.18). If we extend the thumb of the right hand at right angles to the fingers and curl the fingers so as to point from **r** to **F** through the smaller angle, the direction in which thumb points is the direction of τ.

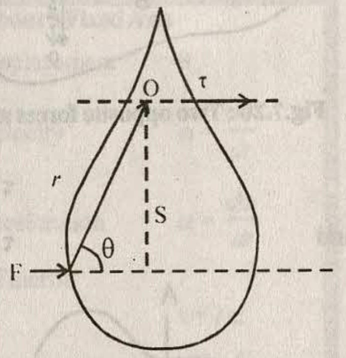


Fig. 7.18 : Right hand thumb rule

Apply the above rule and show that the turning effect of the force in Fig. 7.18 is normal to the plane of paper downwards. This corresponds to clockwise rotation of the body.

Example 7.5 : Fig.7.19 shows a bicycle pedal. Suppose your foot is at the top and you are pressing the pedal downwards. (*i*) What torque do you produce? (ii) Where should your foot be for generating maximum torque?

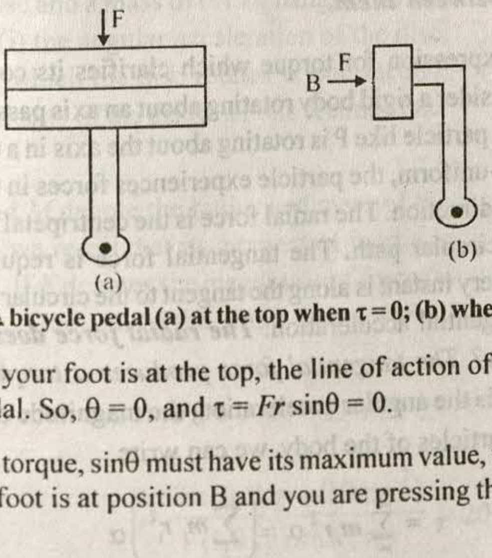


Fig.7.19 : A bicycle pedal (a) at the top when τ = 0; (b) when τ is maximum

Solution : (i) When your foot is at the top, the line of action of the force passes through the centre of the pedal. So, $\theta = 0$, and $\tau = Fr\sin\theta = 0$.

(ii) To get maximum torque, $\sin\theta$ must have its maximum value, that is θ must be 90°. This happens when your foot is at position B and you are pressing the pedal downwards.

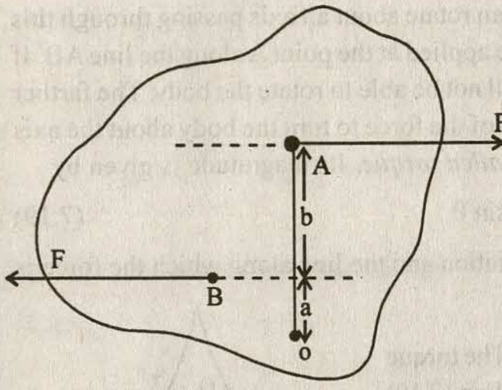


Fig.7.20 : Two opposite forces acting on body

If there are several torques acting on a body, the net torque is the vector sum of all the torques. Do you see any correspondence between the role of torque in the rotational motion and the role of force in the linear motion? Consider two forces of equal magnitude acting on a body in opposite directions (Fig.7.20). Assume that the body is free to rotate about an axis passing through O. The two torques on the body have magnitudes

$$\tau_1 = (a + b)\,F$$

and $$\tau_2 = a\,F.$$

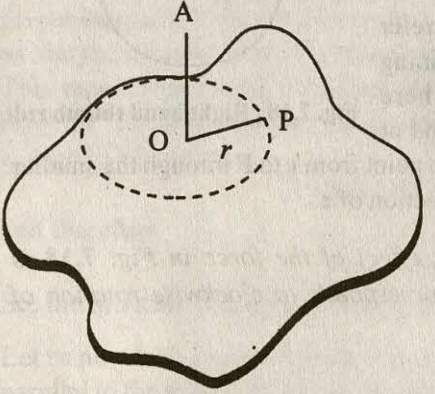


Fig. 7.21 : A rigid body rotating about on axis

You can verify that the turning effect of these torques are in the opposite directions. Therefore, the magnitude of the net turning effect on the body is in the direction of the larger torque, which in this case is τ_1 :

$$\tau = \tau_1 - \tau_2 = bF \qquad (7.21)$$

We may therefore conclude that ***two equal and opposite forces having different lines of action are said to form a couple whose torque is equal to the product of one of the forces and the perpendicular distance between them.***

There is another useful expression for torque which clarifies its correspondence with force in linear motion. Consider a rigid body rotating about an axis passing through a point O (Fig. 7.21). Obviously, a particle like P is rotating about the axis in a circle of radius r. If the circular motion is non–uniform, the particle experiences forces in the radial direction as well as in the tangential direction. The radial force is the centripetal force $m\,\omega^2 r$, which keeps the particle in the circular path. The tangential force is required to change the magnitude of v, which at every instant is along the tangent to the circular path. Its magnitude is $m\,a$, where a is the tangential acceleration. ***The radial force does not produce any torque.*** *Do you know why? The tangential force produces a torque of magnitude $m\,a\,r$.* Since $a = r\,\alpha$, where α is the angular acceleration, the magnitude of the torque is $m\,r^2\,\alpha$. If we consider all the particles of the body, we can write

$$\tau = \sum_{i=0}^{i=n} m_i r_i^2 \alpha = \left(\sum_i m_i r_i^2\right)\alpha$$

$$= \mathrm{I}\,\alpha. \qquad (7.22)$$

because α is same for all the particles at a given instant.

The similarity between this equation and **F** = $m\,a$ shows that τ performs the same role in rotational motion as **F** does in linear motion. A list of corresponding quantities in rotational motion and linear motion is given in Table 7.3. With the help of this table, you can write any equation for rotational motion if you know its corresponding equation in linear motion.

Table 7.3 : Corresponding quantities in rotational and translational motions

Translational Motion		**Rotation about a Fixed Axis**	
Displacement	x	Angular displacement	θ
Velocity	$v = \frac{dx}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass	M	Moment of inertia	I
Force	$F = m\,a$	Torque	$\tau = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$\frac{1}{2}M v^2$	Kinetic energy	$(\frac{1}{2})\,I\omega^2$
Power	$P = Fv$	Power	$P = \tau\,\omega$
Linear momentum	$M v$	Angular momentum	$I\omega$

With the help of Eqn.(7.22) we can calculate the angular acceleration produced in a body by a given torque.

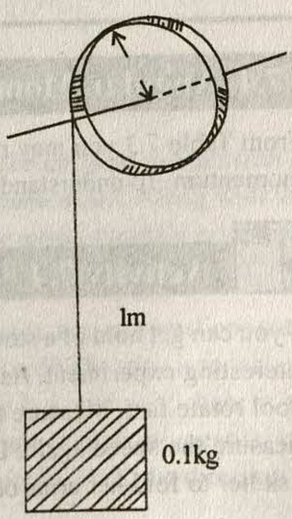


Fig. 7.22

Example 7.6 : A uniform disc of mass 1.0 kg and radius 0.1m can rotate about an axis passing through its centre and normal to its plane without friction. A massless string goes round the rim of the disc and a mass of 0.1 kg hangs at its end (Fig.7.22). Calculate (i) the angular acceleration of the disc, (ii) the angle through which the disc rotates in one second, and (iii) the angular velocity of the disc after one second. Take $g = 10\ \text{ms}^{-2}$

Solution : (i) If R and M denote the radius and mass of the disc, from Table 7.2, we recall that its moment of inertia is given by $I = (\frac{1}{2})\,M R^2$. If F denotes the magnitude of force (= $m\,g$) due to the mass at the end of the string then $\tau = F\,R$. Eqn. (7.22) now gives

$$\alpha = \tau/I = FR/I = 2F/MR$$

$$= \frac{2\times(0.1\text{kg})\times(10\text{ms}^{-2})}{(1.0\text{kg})\times(0.1\,\text{m})} = 20\ \text{rad s}^{-2}.$$

(ii) For angle θ through which the disc rotates, we use Eqn.(7.16). Since the initial angular velocity is zero, we have

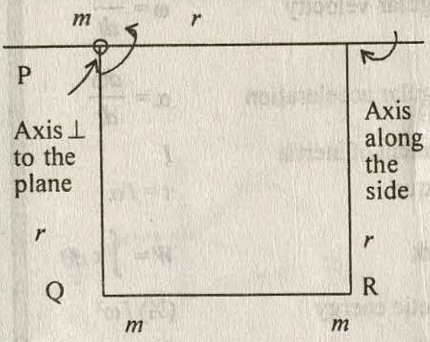
$$\theta = (\tfrac{1}{2}) \times 20 \times 1.0 = 10 \text{ rad}$$

(iii) For the velocity after one second, we have

$$\omega = \alpha\, t = 20 \times 1.0 = 20 \text{ rad s}^{-1}$$

Now, you may like to check your progress. Try the following questions.

Intext Questions 7.3



1. Four particles, each of mass **m**, are fixed at the corners of a square whose each side is of length *r*. Calculate the moment of inertia about an axis passing through one of the corners and perpendicular to the plane of the square. Calculate also the moment of inertia about an axis which is along one of the sides. Verify your result by using the theorem of perpendicular axes.

....

2. Calculate the radius of gyration of a solid sphere if the axis is a tangent to the sphere. (You may use Table 7.2)

....

7.4 Angular Momentum

From Table 7.3 you may recall that rotational analogue of linear momentum is angular momentum. To understand its physical significance, we would like you to do an activity.

Activity 7.4

If you can get hold of a stool which can rotate without much friction, you can perform an interesting experiment. Ask a friend to sit on the stool with her arms folded. Make the stool rotate fast. Measure the speed of rotation. Ask your friend to stretch her arms and measure the speed again. Do you note any change in the speed of rotation of the stool? Ask her to fold her arms once again and observe the change in the speed of the stool.

Let us try to understand why we expect a change in the speed of rotation of the stool in two cases : sitting with folded and stretched hands. For this, let us again consider a rigid body rotating about an axis, say *z*–axis through a fixed point O in the body. All the points of the body describe circular paths about the axis of rotation with the centres of the paths on the axis and have angular velocity ω. Consider a particle like P at distance r_i from the axis

(Fig. 7.20). Its linear velocity is $\upsilon_i = r_i\omega$ and its momentum is therefore $m_i\, r_i\, \omega$. ***The product of linear momentum and the distance from the axis is called angular momentum, denoted by L.*** If we sum this product for all the particles of the body, we get

$$L = \sum_i m_i \omega\, r_i\, r_i = \left(\sum_i m_i\, r_i^2\right)\omega$$

$$= I\,\omega \qquad (7.23)$$

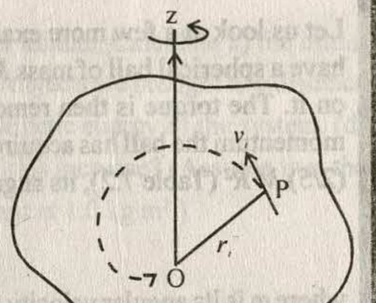


Fig.7.23 : A rigid body rotating about an axis through 'O'

Remember that the angular velocity is the same for all the particles and the term within brackets is the moment of inertia. Like the linear momentum, the angular momentum is also a vector quantity. Eqn. (7.23) gives only the component of the vector **L** along the axis of rotation. It is important to remember that I must refer to the same axis. The unit of angular momentum is kg m^2 s^{-1}

Recall now that the rate of change of ω is α and $I\,\alpha = \tau$. Therefore, ***the rate of change of angular momentum is equal to torque.*** In vector notation, we write the equation of motion of a rotating body as

$$\frac{d\mathbf{L}}{dt} = \tau = I\frac{d\omega}{dt} = I\,\alpha \qquad (7.24)$$

7.4.1 Conservation of angular momentum

Eqn. (7.24) shows that ***if there is no net torque acting on the body,*** $\frac{d\mathbf{L}}{dt} = 0$. ***This means that there is no change in angular momentum, i.e. the angular momentum is constant. This is the principle of conservation of angular momentum.*** Along with the conservation of energy and linear momentum, this is one of the most important principles of physics.

The principle of conservation of angular momentum allows us to answer questions such as : How the direction of toy umbrella floating in air remains fixed? The trick is to make it rotate and thereby impart it some angular momentum. Once it goes in air, there is no torque acting on it. Its angular momentum is then constant. Since angular momentum is a vector quantity, its constancy implies fixed direction and magnitude. Thus, the direction of the toy umbrella remains fixed while it is in air.

In the case of your friend on the rotating stool; when no net torque acts on the stool, the angular momentum of the stool and the person on it must be conserved. When the arms are stretched, she causes the moment of inertia of the system to increase. Eqn. (7.23) then implies that the angular velocity must decrease. Similarly, when she folds her arms, the moment of inertia of the system decreases. This causes the angular velocity to increase. Note that the change is basically caused by the change in the moment of inertia due to change in distance of particles from the axis of rotation.

Let us look at a few more examples of conservation of angular momentum. Suppose we have a spherical ball of mass M and radius R. The ball is set rotating by applying a torque on it. The torque is then removed. When there is no external torque, whatever angular momentum the ball has acquired must be conserved. Since moment of inertia of the ball is (2/5) MR^2 (Table 7.2), its angular momentum is given by

$$L = \frac{2}{5} M R^2 \omega \qquad (7.25)$$

where ω is its angular velocity. Imagine now that the radius of the ball somehow decreases. To conserve its angular momentum, ω must increase and the ball must rotate faster. This is what really happens to some stars, such as those which become pulsars (see Box on page 176).

What would happen if the radius of the ball were to increase suddenly?

You can again use Eqn.(7.25) to show that if R increases, ω must decrease to conserve angular momentum. If instead of radius, the moment of inertia of the system changes some how, ω will change again. For an interesting effect of this kind see Box below

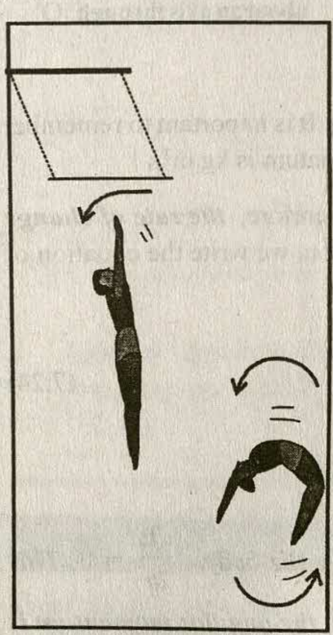


Fig.7.24 : Diver, Sommer saulting after jumping off the diving boards.

The length of the day is not constant

Scientists have observed very small and irregular variations in the period of rotation of the earth about its axis, i.e. the length of the day. One of the causes that they have identified is weather. Due to changes in weather, large scale movement in the air of the earth's atmosphere takes place. This causes a change in the mass distribution around the axis of the earth, resulting in a change in the moment of inertia of the earth. Since the angular momentum of the earth $L = I\,\omega$ must be conserved, a change in I means a change in rotational speed of the earth, or in the length of the day.

Acrobats, skaters, divers and other sports persons make excellent use of the principle of conservation of angular momentum to show off their feats. You must have seen divers jumping off the diving boards during swimming events in national or international events such as Asian Games, Olympics or National meets. At the time of jumping, the diver gives herself a slight rotation, by which she acquires some angular momentum. When she is in air, there is no torque acting on her and therefore her angular momentum must be conserved. If she folds her body to decrease her moment of inertia (Fig. 7.24) her rotation must become faster. If she unfolds her body, her moment of inertia increases and she must rotate slowly. In this way, by controlling the shape of her body, the diver is able to demonstrate her feat before entering into pool of water.

Example 7.7 : Shiela stands at the centre of a rotating platform that has frictionless bearings. She holds a 2.0 kg object in each hand at 1.0 m from the axis of rotation of the system. The system is initially rotating at 10 rotations per minute. Calculate a) the initial angular velocity in rad s^{-1}, b) the angular velocity after the objects are brought to a distance of 0.2 m from the axis of rotation, and (c) change in the kinetic energy of the system. (d) If the kinetic energy has increased, what is the cause of this increase? (Assume that the moment of inertia of Shiela and platform I_{SP} stays constant at 1.0 kg m^2.)

Solution : (a) 1 rotation = 2π radian

$$\therefore \text{ initial angular velocity } \omega = \frac{10 \times 2\pi \text{ radian}}{60 \text{ s}} = 1.05 \text{ rad s}^{-1}$$

(b) The key idea here is to use the law of conservation of angular momentum. The initial moment of inertia

$$I_i = I_{SP} + m r_i^2 + m r_i^2$$

$$= 1.0 \text{ kg m}^2 + (2.0 \text{ kg}) \times (1 \text{ m}^2) + (2.0 \text{ kg}) \times (1 \text{ m}^2)$$

$$= 5.0 \text{ kg m}^2.$$

After the objects are brought to a distance of 0.2 m, final moment of inertia.

$$I_i = I_{SP} + mr_f^2 + mr_f^2$$

$$= 1.0 \text{ kg m}^2 + 2.0 \text{ kg} \times (0.2)^2 \text{ m}^2 + 2.0 \text{ kg} \times (0.2)^2 \text{ m}^2$$

$$= 1.16 \text{ kg m}^2.$$

Conservation of angular momentum requires that

$$I_i\omega_i = I_f\omega_f$$

or

$$\omega_f = \frac{I_i\omega_i}{I_f}$$

$$= \frac{(5.0 \text{ kg m}^2) \times 1.05 \text{ rad s}^{-1}}{1.16 \text{ kg m}^2}$$

$$= 4.5 \text{ rad s}^{-1}$$

Suppose the change in kinetic energy of rotation is ΔE. Then

$$\Delta E = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2$$

$$= \frac{1}{2} \times 1.16 \text{ kg m}^2 \times (4.5)^2 (\text{rad s}^{-1})^2 - \frac{1}{2} \times 5.0 \text{ kg m}^2 \times (1.05)^2 (\text{rad s}^{-1})^2$$

$$= 9.05 \text{J}$$

Since final kinetic energy is higher than the initial kinetic energy, there is an increase in the kinetic energy of the system.

(d) When Shiela pulls the objects towards the axis, she does work on the system. This work goes into the system and increases its kinetic energy.

Intext Questions 7.4

1. A hydrogen molecule consists of two identical atoms, each of mass m and separated by a fixed distance d. The molecule rotates about an axis which is halfway between the two atoms, with angular speed ω. Calculate the angular momentum of the molecule.

....

2. A uniform circular disc of mass 2.0 kg and radius 20 cm is rotated about one of its diameters at an angular speed of 10 rad s^{-1}. Calculate its angular momentum about the axis of rotation.

....

3. A wheel is rotating at an angular speed ω about its axis which is kept vertical. Another wheel of the same radius but half the mass, initially at rest, is slipped on the same axle gently. These two wheels then rotate with a common speed. Calculate the common angular speed.

....

4. It is said that the earth was formed from a contracting gas cloud. Suppose some time in the past, the radius of the earth was 25 times its present radius. What was then its period of rotation on its own axis?

....

7.5 Simultaneous Rotational and Translational Motions

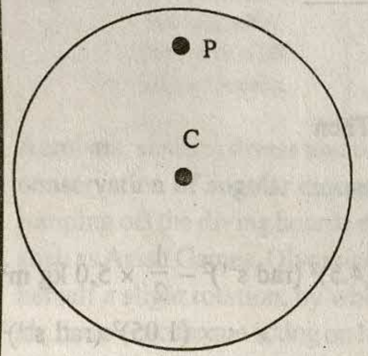


Fig.7.25

We have already noted that if a point in a rigid body is not fixed, it can possess rotational motion as well as translational motion. The general motion of a rigid body consists of both these motions. Imagine the motion of an automobile wheel on a plane horizontal surface. Suppose you are observing the circular face (Fig.7.25). Fix your attention at a point P and at the centre C of the circular face. Remember that the centre of mass of the wheel lies at the centre of its axis and C is the end point of the axis. As it rolls, you would notice that point P rotates round the point C. The point C itself gets translated in the direction of motion. So the wheel has both the rotational and translational motions. If point C or the centre of mass gets translated with velocity υ_{cm}, the kinetic energy of translation is

$$(KE)_{tr} = \frac{1}{2} M \upsilon_{cm}^2 \qquad (7.26)$$

where M is the mass. And if ω is the angular speed of rotation, the kinetic energy of rotation is

$$(\mathrm{KE})_{rot} = \frac{1}{2} I \omega^2 \tag{7.27}$$

where I is the moment of inertia. The total energy of the body due to translation and rotation is the sum of these two kinetic energies. An interesting case, where both translational and rotational motion are involved, is the motion of a body on an inclined plane.

Example 7.8 : Suppose a rigid body has mass M, radius R and moment of inertia I. It is rolling down an inclined plane of height h (Fig.7.26). At the end of its journey, it has acquired a linear speed υ and an angular speed ω. Assume that the loss of energy due to friction is small and can be neglected. Obtain the value of υ in terms of h.

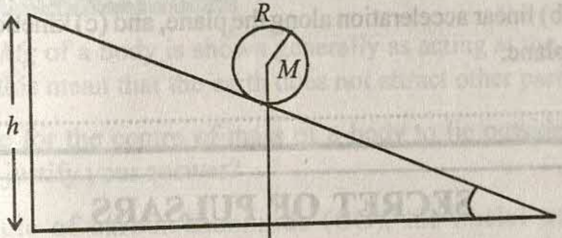


Fig.7.26 : Motion of a rigid body on an inclined plane

Solution : The principle of conservation of energy implies that the sum of the kinetic energies due to translation and rotation at the foot of the inclined plane must be equal to the potential energy that the body had at the top of the inclined plane. Therefore,

$$(½)\, M\upsilon^2 + ½\, \mathrm{I}\, \omega^2 = M\, g\, h \tag{7.28}$$

If the motion is pure rolling and there is no slipping, we can write $\upsilon = R\,\omega$. Inserting this expression is Eqn. (7.28), we get

$$\frac{1}{2} M \upsilon^2 + \frac{1}{2} I \frac{\upsilon^2}{R^2} = M\, g\, h \tag{7.29}$$

To take a simple example, let the body be a hoop. Table 7.2 shows that its moment of inertia about its own axis is MR^2. Eqn.(7.29) then gives

$$\frac{1}{2} M \upsilon^2 + \frac{1}{2} \frac{M R^2 \upsilon^2}{R^2} = M\, g\, h$$

or

$$\upsilon = \sqrt{g\, h} \tag{7.30}$$

Do you notice any thing interesting in this equation? ***The linear velocity is independent of mass and radius of the hoop. Its means that a hoop of any material and any radius rolls down with the same speed on the inclined plane.***

Intext Questions 7.5

1. A solid sphere rolls down a slope without slipping. What will be its velocity in terms of the height of the slope?

....

2. A solid cylinder rolls down an inclined plane without slipping. What fraction of its kinetic energy is translational? What is the magnitude of its velocity after falling through a height *h*?

....

3. A uniform sphere of mass 2 kg and radius 10cm is released from rest on an inclined plane which makes an angle of 30^0 with the horizontal. Deduce its (a) angular acceleration, (b) linear acceleration along the plane, and (c) kinetic energy as it travels 2m along the plane.

....

SECRET OF PULSARS

An interesting example of the conservation of angular momentum is provided by pulsating stars. These are called pulsars. These stars send pulses of radiation of great intensity towards us. The pulses are periodic and the periodicity is extremely precise. The time periods range between a few milliseconds to a few seconds. Such short time periods show that the stars are rotating very fast. Most of the matter of these stars is in the form of neutrons. (The neutrons and protons are the building blocks of the atomic nuclei.) These stars are also called neutron stars. These stars represent the last stage in their life. The secret of their fast rotation is their tiny size. The radius of a typical neutron star is only 10 km. Compare this with the radius of the Sun, which is about 7×10^5km. The Sun rotates on its axis with a period of about 25 days. Imagine that the Sun suddenly shrinks to the size of a neutron star without any change in its mass. In order to conserve its angular momentum, the Sun will have to rotate with a period as short as the fraction of a millisecond.

What You Have Learnt

- A rigid body can have rotational as well as translational motion.
- The equation of translational motion far a rigid body may be written in the same form as for a single particle in terms of the motion of its centre of mass.
- If a point in the rigid body is fixed, then it can possess only rotational motion.
- The moment of inertia about an axis of rotation is defined as $\sum_i m_i \, r_i^2$.
- The moment of inertia plays the same role in rotational motion as does the mass in linear motion.

- The turning effect of a force **F** on a rigid body is given by the torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$.
- Two equal and opposite forces constitute a couple. The magnitude of turning effect of torque is equal to the product of one of the forces and the perpendicular distance between the line of action of forces.
- The application of an external torque changes the angular momentum of the body.
- When no net torque acts on a body, the angular momentum of the body remains constant.
- When a cylindrical or a spherical body rolls down an inclined plane without slipping, its speed is independent of its mass and radius.

Terminal Exercise

1. The weight Mg of a body is shown generally as acting at the centre of mass of the body. Does this mean that the earth does not attract other particles?
2. Is it possible for the centre of mass of a body to lie outside the body? Give two examples to justify your answer?
3. In a molecule of carbon monoxide (CO), the nuclei of the two atoms are 1.13×10^{-10}m apart. Locate of the centre of mass of the molecule.
4. A grinding wheel of mass 5.0 kg and diameter 0.20 m is rotating with an angular speed of 100 rad s^{-1}. Calculate its kinetic energy. Through what distance would it have to be dropped in free fall to acquire this kinetic energy? (Take $g = 10.0$ m s^{-2}).
5. A wheel of diameter 1.0 m is rotating about a fixed axis with an initial angular speed of 2rev s^{-1}. The angular acceleration is 3 rev s^{-2}.

   (a) Compute the angular velocity after 2 seconds.

   (b) Through what angle would the wheel turned during this time?

   (c) What is the tangential velocity of a point on the rim of the wheel at $t = 2$ s?

   (d) What is the centripetal acceleration of a point on the rim of the wheel at $t = 2$ s?
6. A wheel rotating at an angular speed of 20 rads^{-1} is brought to rest by a constant torque in 4.0 seconds. If the moment of inertia of the wheel about the axis of rotation is 0.20 kg m^2, calculate the work done by the torque in the first two seconds.
7. Two wheels are mounted on the same axle. The moment of inertia of wheel A is 5×10^{-2} kg m^2, and that of wheel B is 0.2 kg m^2. Wheel A is set spinning at 600 rev min^{-1}. while wheel B is stationary. A clutch now acts to join A and B so that they must spin together.

   (a) At what speed will they rotate?

   (b) How does the rotational kinetic energy before joining compare with the kinetic energy after joining?

   (c) What torque does the clutch deliver if A makes 10 revolutions during the operation of the clutch?

8. You are given two identically looking spheres and told that one of them is hollow. Suggest a method to detect the hollow one.

9. The moment of inertia of a wheel is 1000 kg m^2. Its rotation is uniformly accelerated. At some instant of time, its angular speed is 10 rad s^{-1}. After the wheel has rotated through an angle of 100 radians, the angular velocity of the wheel becomes 100 rad s^{-1}. Calculate the torque applied to the wheel and the change in its kinetic energy.

10. A disc of radius 10 cm and mass 1kg is rotating about its own axis. It is accelerated uniformly from rest. During the first second it rotates through 2.5 radians. Find the angle rotated during the next second. What is the magnitude of the torque acting on the disc?

Answers to Intext Questions

7.1

1. Yes, because the distances between points on the frame cannot change.

2. No. Any disturbance can change the distance between sand particles. So, a heap of sand cannot be considered a rigid body.

7.2

1. The coordinates of given five masses are A (–1, –1), B (–5, –1), C (6, 3), D (2, 6) and E (–3, 0) and their masses are 1 kg, 2kg, 3kg, 4kg and 5kg respectively.

   Hence, coordinates of centre of mass of the system are

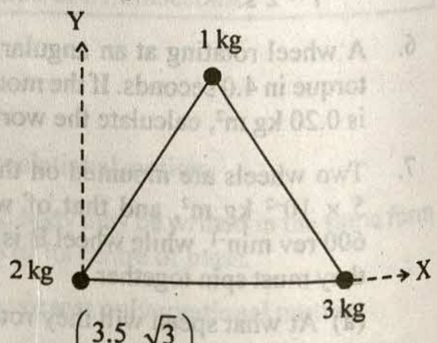
$$x = \frac{-1\times1-5\times2+6\times3+2\times4-3\times5}{1+2+3+4+5} = 0$$

$$y = \frac{-1\times1-1\times2+3\times3+4\times6+0\times5}{1+2+3+4+5} = \frac{30}{15} = 2.0$$

2. Let the three particle system be as shown in the figure here. Consider axes to be as shown with 2 kg mass at the origin.

$$x = \frac{2\times0+1\times0.5+3\times1}{1+2+3} = \frac{3.5}{6}\text{ m} = 0.5\text{ m}$$

$$y = \frac{2\times0+1\times\frac{\sqrt{3}}{2}+3\times0}{1+2+3} = \frac{\sqrt{3}}{12}\text{ m}$$



   Hence, the co-ordinates of the centre of mass are $\left(\frac{3.5}{6}, \frac{\sqrt{3}}{12}\right)$

3. Let the two particles be along the x-axis and let their x-coordinates be o and x. The

coordinate of CM is

$$X = \frac{m_1 \times 0 + m_2 \times x}{m_1 + m_2} = \frac{m_2 x}{m_1 + m_2}, \; Y = 0$$

X is also the distance of m_1 from the CM. The distance of m_2 from CM is

$$x - X = x - \frac{m_2 x}{m_1 + m_2} = \frac{m_1 x}{m_1 + m_2}$$

$$\therefore \; \frac{X}{x + X} = \frac{m_2}{m_1}$$

Thus, the distances from the CM are inversely proportional to their masses.

7.3

1. Moment of inertia of the system about an axis perpendicular to the plane passing through one of the corners and perpendicular to the plane of the square,

   $= m r^2 + m\,(2 r^2) + m r^2 = 4 m r^2$

   M.I. about the axis along the side $= m r^2 + m r^2 = 2 m r^2$

   Verification : Moment of inertia about the axis QP $= m r^2 + m r^2 + 2 m r^2$. Now, according to the theorem of perpendicular axes, MI about SP ($2mr^2$) + MI about QP $2 m r^2$ should be equal to MI about the axis through P and perpendicular to the plane of the square ($4 m r^2$). Since it is true, the results are verified.

2. M.I. of solid sphere about an axis tangential to the sphere

   $= \frac{2}{5} M R^2 + M R^2 = \frac{7}{5} M R^2$ according to the theorem of parallel axes.

   If radius fo gyration is K, then $M K^2 = \frac{7}{5} M R^2$. So,

   Radius of gyration $K = R\sqrt{\frac{7}{5}}$

7.4

1. Angular momentum

   $$L = \left(m\frac{d^2}{4} + m\frac{d^2}{4} \right) \omega$$

   $$L = \frac{m d^2 \omega}{2}$$

2. Angular momentum about an axis of rotation (diameter).

   $$L = I\omega = m\,\frac{r^2}{4} \times \omega$$

as M.I about a diameter = $\frac{m r^2}{4}$

$\therefore L = 20\text{ kg} \times \frac{(0.2)^2 m^2}{4} \times 10\text{ rad s}^{-1} = 0.2\text{ kg m}^2\text{s}^{-1}$.

3. According to conservation of angular momentum

$I_1\omega = (I_1 + I_2)\,\omega_1$

where I_1 is M.I. of the original wheel and I_2 that of the other wheel, ω is the initial angular speed and ω_1 is the common final angular speed.

$$m r^2 \omega = \left(m r^2 + \frac{m}{2} r^2\right)\omega_1$$

$$\omega = \frac{3}{2}\omega_1 \Rightarrow \omega_1 = \frac{2}{3}\,\omega$$

4. Let the present period of revolution of earth be T and earlier be T_0. According to the conservation of angular momentum.

$$\frac{2}{5} M (25\,R)^2 \times \left(\frac{2\pi}{T_0}\right) = \frac{2}{5}\,M R^2 \times \left(\frac{2\pi}{T}\right)$$

$$= \frac{2}{5} M R^2 \times \left(\frac{2\pi}{T}\right)$$

It gives, $T_0 = 6.25\ T$

Thus, period of revolution of earth in the past $T_0 = 6.25$ times the present time period.

7.5

1. Using ($I = \frac{2}{5} M R^2$), Eqn. (7.29) for a solid sphere

$$\frac{1}{2} m v^2 + \frac{1}{2} I\,\omega^2 = m\,g\,h$$

or, $$\frac{1}{2} m v^2 + \frac{1}{2} \times \frac{2}{5} m\,r^2 \,.\, \frac{v^2}{r^2} = m\,g\,h$$

$\because \omega = v/r.$

It gives $v = \sqrt{\frac{10}{7}.g\,h}$

2. For a solid cylinder, $I = \frac{m R^2}{2}$

$\therefore$ Total K.E $\frac{1}{2} m\,v^2 + \frac{1}{2} I\,\omega^2 = \frac{1}{2} m\,v^2 + \frac{1}{2}\,\frac{m R^2}{2}\,.\,\frac{v^2}{R^2} = \frac{3}{4} m\,v^2$

$\because \omega = v/r$

Hence, fraction of translational K.E. $= \dfrac{\frac{1}{2}mv^2}{\frac{3}{4}mv^2} = \dfrac{2}{3}$

Proceeding as in Q.1 above : $v = \sqrt{\frac{4}{3}gh}$

Answers to Terminal Problems

3. At a distance 0.64 Å from carbon atom.
4. 125 J, 2.5 m
5. (a) $16\ \pi$ rad s^{-1} (b) $20\ \pi$ rad (c) 25 m s^{-1} (d) 1280 m s^{-2}
6. 30 J
7. (a) $4\ \pi$ rad s^{-1} (b) $E_i = 5\ E_f$ (c) $49\ \pi$ N m
9. $T = 5 \times 10^4$ N m, KE $= 5 \times 10^6$ J
10. 7.5 rad, $\tau = 5 \times 10^{-2}$ J

SENIOR SECONDARY COURSE

PHYSICS

STUDENT'S ASSIGNMENT – 1

Maximum Marks: 50 **Time : $1\frac{1}{2}$ Hours**

INSTRUCTIONS

- Answer All the questions on a seperate sheet of paper
- Give the following information on your answer sheet:
  - Name
  - Enrolment Number
  - Subject
  - Assignment Number
  - Address
- Get your assignment checked by the subject teacher at your study centre so that you get positive feedback about your performance.

Do not send your assignment to NIOS

1. Give an example to show that the average velocity of a moving particle may be zero, but its average speed can not be zero. (1)
2. Why does the direction of the projectile motion become horizontal at the highest point? (1)
3. Can the law of conservation of linear momentum be applied for a body falling under gravity ? Explain. (1)
4. Why is the handle on a door provided at the largest possible distances form the hinges ? (1)
5. Why does moon have no atmosphere ? (1)
6. Draw velocity time graph of a body moving in a straight line under a constant force. (1)
7. What is the radius of gyration of a disc of radius 20 cm, rotating about an axis passing through its center and normal to its plane? (2)
8. A light and a heavy mass have the same kinetic energy, Which one has more momentum? (1)
9. A vector **A** of magnitude 10 units and another vector **B** of magnitude 6 units make an angle of 60^0 with each other.

   Find the scalar product and the magnitude of the vector products of these two vectors. (2)
10. A footballer can kick a 0.5 kg ball with a maximum speed of 10m s^{-1}. What is the maximum horizontal distance to which he can kick the ball? (2)
11. The displacement of a particle is given by $y = at + bt^2$, where a and b are constants and t is time. Find the dimensional formula of b/a.
12. The length of the second's hand of a clock is 10 cm. Calculate the speed of its tip. (2)

13. If by some freak of nature the earth collapses to 1/8th of its present volume, what would be the duration of a day ?. Explain. (4)

14. Calculate the mean distance of a hypothetical planet from the sun which has a period of revolution of 100 years. You may take the distance between the sun and the earth as 1.5

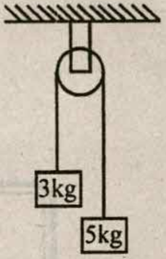
15. A block of mass 2 kg is placed on plane surface. Its inclination from the horizontal may be changed. The block is just at the verge of sliding when the inclination of the plans is 30°, calculate the acceleration with which the bock will slide down when the inclination of the plane is 45°. (4)

16. A constant force of 20 N acts for 2s on a body of mass 2 kg initially at rest. How much distance will this body move in 3s from start? (4)

17. Draw a load-extension graph for a spring. How will you use this graph to calculate (i) force constant of the spring?

    (ii) work done in compressing the spring by a distance x ?

18. Two masses of 3 kg and 5 kg one attached to a massless string and the string is passed over a frictionless pulley as shown in fig. Calculate the tension in the string and acceleration of the 3 kg block. (4)



19. Three rods each of mass per unit length 1 kg m^{-1} and length 20 cm form an equilateral triangle. Determine (i) The center of mass of the system. (ii) Moment of inertia of the system about an axis passing through, the centre of mass and normal to its plane. (5)

20. A body of mass m at rest is hit head-on elastically by a body of mass m kg moving with a speed of u. Find the magnitude and direction of motion of each body after collision. (5)

MODULE - II
MECHANICS OF SOLIDS AND FLUIDS

8. Elastic Properties of Solids
9. Properties of Fluids

8

ELASTIC PROPERTIES OF SOLIDS

In the previous lessons you have studied the effect of force on a body to produce displacement. The force applied on an object may also change its shape or size. For example, when a suitable force is applied on a spring, you will find that its shape as well as size changes. But when you remove the force, it will regain original position. Now apply a force on some objects like wet modelling clay or molten wax. Do they regain their original position after the force has been removed? They do not regain their original shape and size. Thus some objects regain their original shape and size whereas others do not. Such a behaviour of objects depends on a property of matter called **elasticity**.

The elastic property of materials is of vital importance in our daily life. It is used to help us determine the strength of cables to support the weight of bodies such as in cable cars, cranes, lifts etc. We use this property to find the strength of beams for construction of buildings and bridges. In this unit you will learn about nature of changes and the manner in which these can be described.

Objectives

After studying this lesson, you should be able to :

- *distinguish between three states of matter on the basis of molecular theory;*
- *distinguish between elastic and plastic bodies;*
- *distinguish between stress and pressure;*
- *study stress-strain curve for an elastic solid ; and*
- *define Young's modulus, bulk modulus, modulus of rigidity and Poisson's ratio.*

8.1 Molecular Theory of Matter : Inter-Molecular Forces

We know that matter is made up of atoms and molecules. The forces which act between them are responsible for the structure of matter. The interaction forces between molecules are known as **inter-molecular forces**.

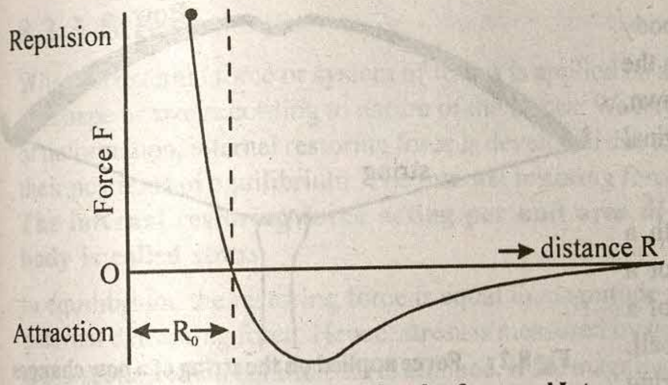


Fig. 8.1 : **Graph between inter-molecular force and Inter molecular separation.**

The variation of inter molecular forces with inter molecular separation is shown in Fig. 8.1.

When the separation is large, the force between two molecules is attractive and weak. As the separation decreases, the net force of attraction increases up to a particular value and beyond this, the force becomes repulsive. At a distance $R = R_0$ the net force between the molecules is zero. This separation is called **equilibrium separation**. Thus, if inter-molecular separation $R > R_0$ there will be an attractive force between molecules. When $R < R_0$, a repulsive force will act between them.

In **solids**, molecules are very close to each other at their equilibrium separation ($\sim 10^{-10}$ m). Due to high intermolecular forces, they are almost fixed at their positions. You may now appreciate why a solid has a definite shape.

In **liquids**, the average separation between the molecules is somewhat larger ($\sim 10^{-8}$ m). The attractive force is weak and the molecules are comparatively free to move inside the whole mass of the liquid. You can understand now why a liquid does not have fixed shape. It takes the shape of the vessel in which it is filled.

In **gases**, the intermolecular separation is significantly larger and the molecular force is very weak (almost negligible). Molecules of a gas are almost free to move inside a container. That is why gases do not have fixed shape and size.

Ancient Indian view about Atom

Kanada was the first expounder of the atomic concept in the world. He lived around 6th century B.C. He resided at Prabhasa (near Allahabad).

According to him, everything in the universe is made up of **Parmanu** or Atom. They are eternal and indestructible. Atoms combine to form different molecules. If two atoms combine to form a molecule, it is called **duyanuka** and a triatomic molecule is called triyanuka. He was the author of "Vaisesika Sutra".

The size of atom was also estimated. In the biography of Buddha (Lalitavistara), the estimate of atomic size is recorded to be of the order 10^{-10} m, which is very close to the modern estimate of atomic size.

8.2 Elasticity

You would have noticed that when an external force is applied on an object, its shape or size (or both) change, i.e. deformation takes place. The extent of deformation depends on the material and shape of the body and the external force. When the deforming forces are withdrawn, the body tries to regain its original shape and size.

You may compare this with a spring loaded with a mass or a force applied on the string of a bow or pressing of a rubber ball. If you apply a force on the string of the bow to pull it (Fig 8.2), you will observe that its shape changes. But on releasing the string, the bow regains its original shape and size.

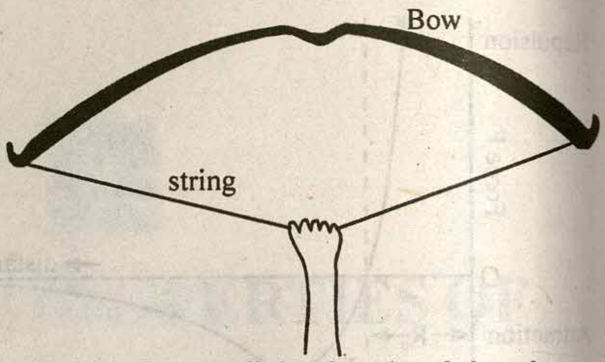


Fig 8.2 : Force applied on the string of a bow changes it shape

The property of matter to regain its original shape and size after removal of the deforming forces is called **elasticity**.

8.2.1 Elastic and Plastic Bodies

A body which regains its original state completely on removal of the deforming force is called **perfectly elastic**. On the other hand, if it completely retains its modified form even on removing the deforming force, i.e. shows no tendency to recover the deformation, it is said to be **perfectly plastic**. However, in practice the behaviour of all bodies is in between these two limits. There exists no perfectly elastic or perfectly plastic body in nature. The nearest approach to a perfectly elastic body is quartz fiber and to the perfectly plastic is ordinary putty. Here it can be added that the object which opposes the deformation more is more elastic. No doubt elastic deformations are very important in science and technology, but plastic deformations are also important in mechanical processes. You might have seen the processes such as stamping, bending and hammering of metal pieces. These are possible only due to plastic deformations.

The phenomenon of elasticity can be explained in terms of inter-molecular forces.

8.2.2 Molecular Theory of Elasticity

You are aware that a solid is composed of a large number of atoms arranged in a definite order. Each atom is acted upon by forces due to neighbouring atoms. Due to inter-atomic forces, solid takes such a shape that each atom remains in a stable equilibrium. When the body is deformed, the atoms are displaced from their original positions and the inter-atomic distances change. If in deformation, the separation increases beyond their equilibrium separation (i.e., $R > R_0$), strong attractive forces are developed. However, if inter–atomic separation decreases (i.e. $R < R_0$), strong repulsive forces develop. These forces, called **restoring forces,** drive atoms to their original positions. The behaviour of atoms in a solid can be compared to a system in which balls are connected with springs.

Now, let us learn how forces are applied to deform a body.

8.2.3 Stress

When an external force or system of forces is applied on a body, it undergoes a change in the shape or size according to nature of the forces. We have explained that in the process of deformation, internal restoring force is developed due to molecular displacements from their positions of equilibrium. The internal restoring force opposes the deforming force. **The internal restoring force acting per unit area of cross-section of a deformed body is called *stress*.**

In equilibrium, the restoring force is equal in magnitude and opposite in direction to the external deforming force. Hence, stress is measured by the external force per unit area of cross-section when equilibrium is attained. If the magnitude of deforming force is *F* and it acts on area *A*, we can write

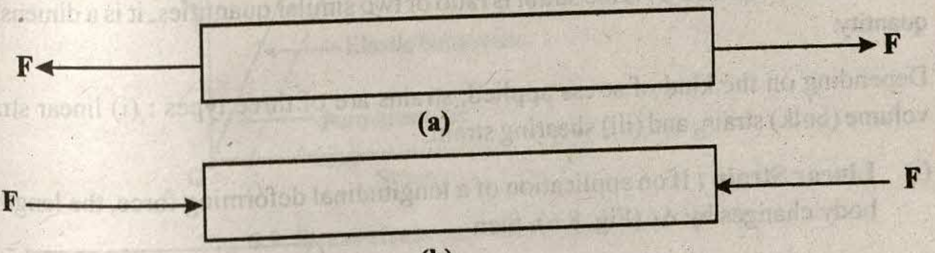
$$\text{Stress} = \frac{\text{restoring force}}{\text{area}} = \frac{\text{deforming force }(F)}{\text{area }(A)}$$

or

$$\text{Stress} = \frac{F}{A} \tag{8.1}$$

The unit of stress is Nm^{-2}. The stress may be longitudinal, normal or shearing. Let us study them one by one.

(i) **Longitudinal Stress :** If the deforming forces are along the length of the body, we call the stress produced as **longitudinal stress,** as shown in its two forms in Fig 8.3 (a) and Fig 8.3 (b).

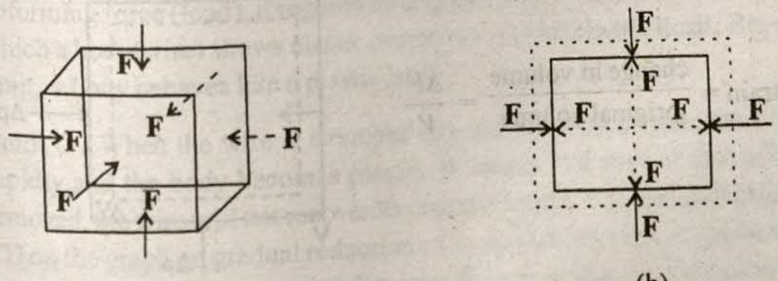


(a)

(b)

Fig 8.3 (a) : Tensile stress; (b) Compressive stress

(ii) **Normal Stress :** If the deforming forces are applied uniformly and normally all over the surface of the body so that the change in its volume occurs without change in shape (Fig. 8.4), we call the stress produced as normal stress. You may produce normal stress by applying force uniformly over the entire surface of the body. Deforming force per unit area normal to the surface is called pressure while restoring force developed inside the body per unit area normal to the surface is known as **stress.**



(a)

(b)

Fig. 8.4 : Normal stress

(iii) **Shearing Stress :** If the deforming forces act tangentially or parallel to the surface (Fig 8.5a) so that shape of the body changes without change in volume, the stress is called **shearing stress**. An example of shearing stress is shown in Fig 8.5 (b) in which a book is pushed side ways. Its opposite face is held fixed by the force of friction.

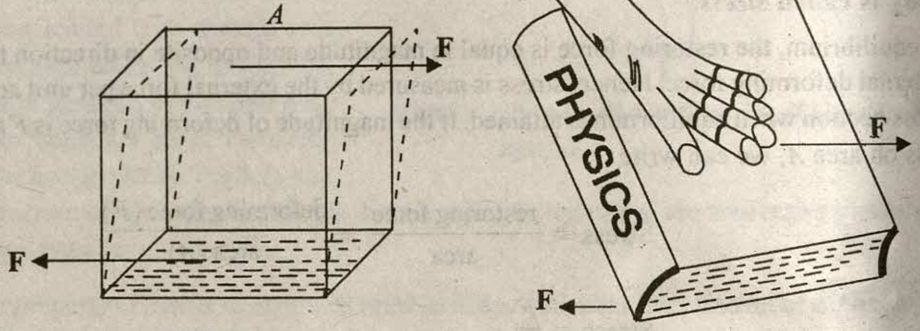


Fig. 8.5: (a) Shearing stress; (b) Pushing a book side ways

8.2.4 Strain

Deforming forces produce changes in the dimensions of the body. In general, the **strain is defined as the change in dimension (e.g. length, shape or volume) per unit dimension of the body.** As the strain is ratio of two similar quantities, it is a dimensionless quantity.

Depending on the kind of stress applied, strains are of three types : (i) linear strain,(ii) volume (bulk) strain, and (iii) shearing strain.

(i) **Linear Strain :** If on application of a longitudinal deforming force, the length ℓ of a body changes by $\Delta\ell$ (Fig. 8.6), then

$$\text{linear strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta\ell}{\ell}$$

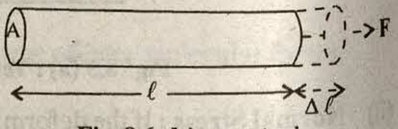


Fig. 8.6: Linear strain

(ii) **Volume Strain :** If on application of a uniform pressure Δp, the volume V of the body changes by ΔV (Fig 8.7) without change of shape of the body, then

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

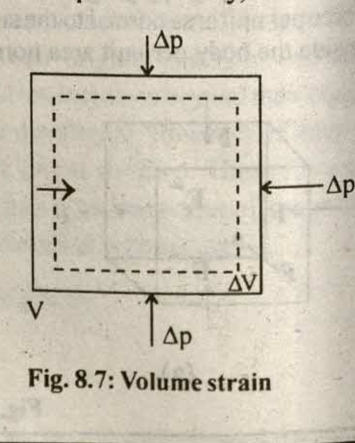


Fig. 8.7: Volume strain

(i) **Shearing strain:** When the deforming forces are tangential (Fig 8.8), the shearing strain is given by the angle θ through which a line perpendicular to the fixed plane is turned due to deformation. (The angle θ is usually very small.) Then we can write

$$\theta = \frac{\Delta x}{y}$$

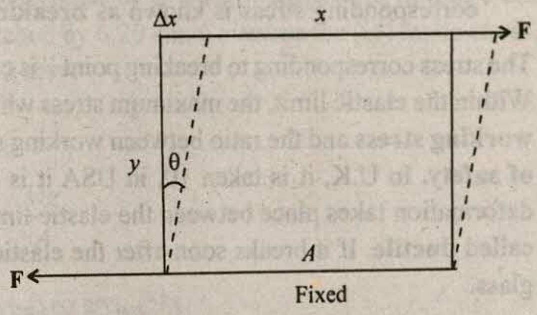


Fig. 8.8 : Shearing strain

8.2.5 Stress-strain Curve for a Metallic Wire

Refer to Fig. 8.9 which shows variation of stress with strain when a metallic wire of uniform cross-section is subjected to an increasing load. Let us study the regions and points on this curve that are of particular importance.

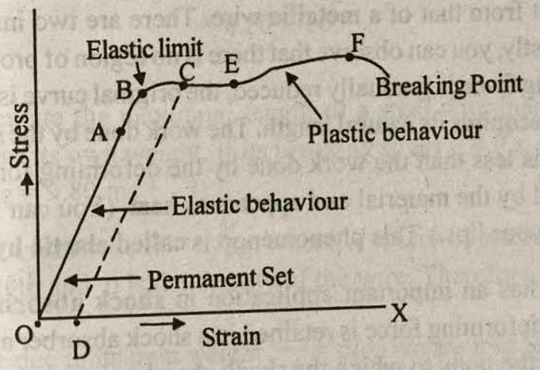


Fig. 8.9: Stress-strain curve for a steel wire

(i) **Region of Proportionality** OA is a straight line which indicates that in this region, stress is linearly proportional to strain and the body behaves like a perfectly elastic body.

(ii) **Elastic Limit** : If we increase the strain a little beyond A, the stress is not linearly proportional to strain. However, the wire still remains elastic, i.e. after removing the deforming force (load), it regains its original state. The maximum value of strain for which a body(wire) shows elastic property is called **elastic limit.** Beyond the elastic limit, a body behaves like a plastic body.

(iii) **Point C** : When the wire is stretched beyond the limit B, the strain increases more rapidly and the body becomes plastic. It means that even if the deforming load is removed, the wire will not recover its original length. The material follows dotted line CD on the graph on gradual reduction of load. The left over strain on zero load strain is known as a **permanent set.** After point E on the curve, no extension is recoverable.

(iv) **Breaking point** F : Beyond point E, strain increases very rapidly and near point F, the length of the wire increases continuously even without increasing of load. The wire breaks at point F. This is called the **breaking point** or **fracture point** and the corresponding stress is known as **breaking stress.**

The stress corresponding to breaking point F is called **breaking stress** or **tensile strength.** Within the elastic limit, the maximum stress which an object can be subjected to is called **working stress** and the ratio between working stress and breaking stress is called **factor of safety.** In U.K, it is taken 10, in USA it is 5. We have adopted UK norms. If large deformation takes place between the elastic limit and the breaking point, the material is called **ductile.** If it breaks soon after the elastic limit is crossed, it is called **brittle e.g.** glass.

8.2.6 Stress-Strain Curve for Rubber

When we stretch a rubber cord to a few times its natural length, it returns to its original length after removal of the forces. That is, the elastic region is large and there is no well defined plastic flow region. Substances having large strain are called **elastomers.** This property arises from their molecular arrangements. The stress-strain curve for rubber is distinctly different from that of a metallic wire. There are two important things to note from Fig. 8.10. Firstly, you can observe that there is no region of proportionality. Secondly, when the deforming force is gradually reduced, the original curve is not retraced, although the sample finally acquires its natural length. The work done by the material in returning to its original shape is less than the work done by the deforming force. This difference of energy is absorbed by the material and appears as heat. (You can feel it by touching the rubber band with your lips.) This phenomenon is called **elastic hysteresis.**

Elastic hysteresis has an important application in **shock absorbers.** A part of energy transferred by the deforming force is retained in a shock absorber and only a small part of it is transmitted to the body to which the shock absorber is attached.

8.2.7. Steel is more Elastic than Rubber

A body is said to be more elastic if on applying a large deforming force on it, the strain produced in the body is small. If you take two identical rubber and steel wires and apply equal deforming forces on both of them, you will see that the extension produced in the steel wire is smaller than the extension produced in the rubber wire. But to produce same strain in the two wires, significantly higher stress is required in the steel wire than in rubber wire. Large amount of stress needed for deformation of steel indicates that

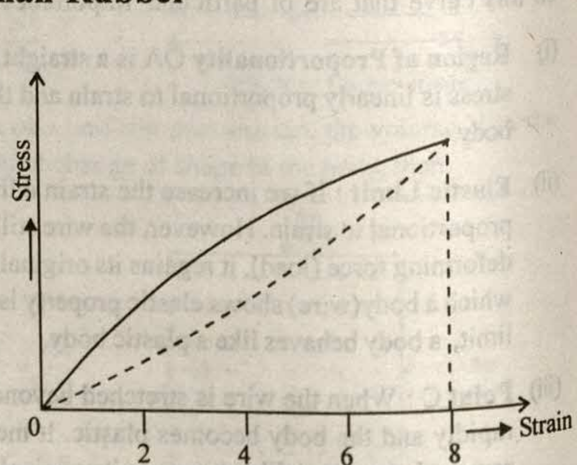


Fig. 8.10: Stress-strain curve for rubber

magnitude of internal restoring force produced in steel is higher than that in rubber. Thus, steel is more elastic than rubber.

Example 8.1 : A load of 100 kg is suspended by a wire of length 1.0 m and cross sectional area 0.10 cm^2. The wire is stretched by 0.20 cm. Calculate the (i) tensile stress, and (ii) strain in the wire. Given, $g = 9.80$ ms^{-2}.

Solution :

$$\text{(i) Tensile stress} = \frac{F}{A} = \frac{Mg}{A}$$

$$= \frac{(100\ \text{kg})(9.80\ \text{ms}^{-2})}{0.10 \times 10^{-4}\ \text{m}^2}$$

$$= 9.8 \times 10^7\ \text{Nm}^{-2}$$

$$\text{(ii) Tensile strain} = \frac{\Delta \ell}{\ell} = \frac{0.20 \times 10^{-2}\ \text{m}}{1.0\ \text{m}}$$

$$= 0.20 \times 10^{-2}$$

Example 8.2 : Calculate the maximum length of a steel wire that can be suspended without breaking under its own weight, if its breaking stress = 4.0 x 10^8 Nm^{-2}, density = 7.9 × 10^3 kg m^{-3} and $g = 9.80$ ms^{-2}

Solution : The weight of the wire $W = A\ell\rho g$, where, A is area of cross section of the wire, ℓ is the maximum length and ρ is the density of the wire. Therefore, the breaking stress developed in the wire due to its own weight $\frac{W}{A} = \rho l g$. We are told that breaking stress is 4.0 x 10^8 Nm^{-2}. Hence

$$\ell = \frac{4.0 \times 10^8\ \text{Nm}^{-2}}{(7.9 \times 10^3\ \text{kg m}^{-3})(9.8\ \text{ms}^{-2})}$$

$$= 0.05 \times 10^5\ \text{m}$$

$$= 5 \times 10^3\ \text{m} = 5\ \text{km.}$$

Now it is time to take a break and check your understanding

Intext Questions 8.1

1. What will be the nature of inter-atomic forces when deforming force applied on an object (i) increases, (ii) decreases the inter-atomic separation?

....

2. If we clamp a rod rigidly at one end and a force is applied normally to its cross section at the other end, name the type of stress and strain?

....

3. The ratio of stress to strain remains constant for small deformation of a metal wire. For large deformations what will be the changes in this ratio?

....

4. Under what conditions, a stress is known as breaking stress ?

....

5. If mass of 4 kg is attached to the end of a vertical wire of length 4 m with a diameter 0.64 mm, the extension is 0.60 mm. Calculate the tensile stress and strain?

....

8.3 Hooke's Law

In 1678, Robert Hooke obtained the stress-strain curve experimentally for a number of solid substances and established a law of elasticity known as Hooke's law. According to this law: **Within elastic limit, stress is directly proportional to corresponding strain.**

i.e. stress α strain

or

$$\frac{\text{stress}}{\text{strain}} = \text{constant } (E) \qquad (8.2)$$

This constant of proportionality E is a measure of elasticity of the substance and is called **modulus of elasticity**. As strain is a dimensionless quantity, the modulus of elasticity has the same dimensions (or units) as stress. Its value is independent of the stress and strain but depends on the nature of the material. To see this, you may like to do the following activity.

Activity 8.1

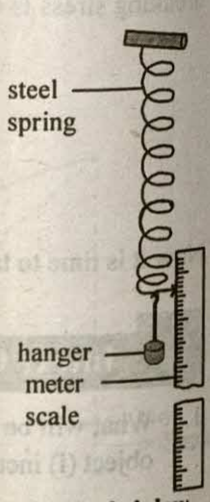


Fig. 8.11: Hooke's law apparatus

Arrange a steel spring with its top fixed with a rigid support on a wall and a metre scale along its side, as shown in the Fig. 8.11.

Add 100 g load at a time on the bottom of the hanger in steps. It means that while putting each 100 g load, you are increasing the stretching force by 1N. Measure the extension. Take the reading upto 500 g and note the extension each time.

Plot a graph between load and extension. What is the shape of the graph? Does it obey Hooke's law?

The graph should be a straight line indicating that the ratio (load/extension) is constant.

Repeat this activity with rubber and other materials.

You should know that the materials which obey Hooke's law are used in spring balances or as force measurer, as shown in the Fig. 8.11. You would have seen that when some object is placed on the pan, the length of the spring increases. This increase in length shown by the pointer on the scale can be treated as a measure of the increase in force (i.e., load applied).

Robert Hooke
(1635 – 1703)

Robert Hooke, experimental genius of seventeenth century, was a contemporary of Sir Isaac Newton. He had varied interests and contributed in the fields of physics, astronomy, chemistry, biology, geology, paleontology, architecture and naval technology. Among other accomplishments he has to his credit the invention of a universal joint, an early proto type of the respirator, the iris diaphragm, anchor escapement and balancing spring for clocks. As chief surveyor, he helped rebuild London after the great fire of 1666. He formulated Hooke's law of eleasticity and correct theory of combustion. He is also credited to invent or improve meteorological instruments such as barometer, anemometer and hygrometer.

8.3.1 Moduli of Elasticity

In previous sections, you have learnt that there are three kinds of strain. It is therefore clear that there should be three modulli of elasticity corresponding to these strains. These are **Young's modulus, Bulk Modulus and Modulus of rigidity** corresponding to linear strain, volume strain and shearing strain, respectively. We now study these one by one.

(i) **Young's Modulus:** The ratio of the longitudinal stress to the longitudinal strain is called Young's modulus for the material of the body.

Suppose that when a wire of length L and area of cross-section A is stretched by a force of magnitude F, the change in its length is equal to ΔL. Then

$$\text{Longitudinal stress} = \frac{F}{A}$$

and

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

Hence,

$$\text{Young's modulus } Y = \frac{F/A}{\Delta L/L} = \frac{F \times L}{A \times \Delta L}$$

If the wire of radius r is suspended vertically with a rigid support and a mass M hangs at its lower end, then $A = \pi r^2$ and $F = Mg$.

$$\therefore \quad Y = \frac{M\,g\,L}{\pi r^2 \Delta L} \qquad (8.3)$$

The SI unit of Y in is N m^{-2}. The values of Young's modulus for a few typical substances are given in Table. 8.1. Note that steel is most elastic.

Table 8.1. Young's modulus of some typical materials

Name of substance	Y (10^9Nm^{-2})
Aluminium	70
Copper	120
Iron	190
Steel	200
Glass	65
Bone	9
Polystyrene	3

(ii) **Bulk Modulus:** The ratio of normal stress to the volume strain is called bulk modulus of the material of the body.

If due to increase in pressure P, volume V of the body decreases by ΔV without change in shape, then

$$\text{Normal stress} = \Delta P$$

$$\text{Volume strain} = \Delta V/V$$

$$\text{Bulk modulus } B = \frac{\Delta P}{\Delta V/V} = V\frac{\Delta P}{\Delta V} \tag{8.4}$$

The reciprocal of bulk modulus of a substance is called compressibility :

$$k = \frac{1}{B} = \frac{1}{V}\frac{\Delta V}{\Delta P} \tag{8.5}$$

Gases being most compressible are least elastic while solids are most elastic or least compressible i.e. $B_{solid} > B_{liquid} > B_{gas}$

(iii) **Modulus of Rigidity or Shear Modulus:** The ratio of the shearing stress to shearing strain is called modulus of rigidity of the material of the body.

If a tangential force *F* acts on an area *A* and θ is the shearing strain, the modulus of rigidity

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} \tag{8.6}$$

You should know that both solid and fluids have bulk modulus. However, fluids do not have Young's modulus and shear modulus because a liquid can not sustain a tensile or shearing stress.

Example 8.3 : Calculate the force required to increase the length of a wire of steel of cross sectional area 0.1 cm² by 50%. Given $Y = 2 \times 10^{11}$ N m^{-2}.

Solution : Increase in the length of wire = 50%. If ΔL is the increase and L is the normal length of wire then $\frac{\Delta L}{L} = \frac{1}{2}$

$$\therefore \qquad Y = \frac{F \times L}{A \times \Delta L}$$

or

$$F = \frac{Y \times A \times \Delta L}{L} = \frac{(2\times10^{11}\,\text{Nm}^{-2})\,(0.1\times10^{-4}\,\text{m}^2)\times 1}{2}$$

$$= 0.1 \times 10^{7}\ \text{N} = 10^{6}\ \text{N}$$

Example 8.4 : When a solid rubber ball is taken from the surface to the bottom of a lake, the reduction in its volume is 0.0012 %. The depth of lake is 360 m, the density of lake water is 10^3 kgm^{-3} and acceleration due to gravity at the place is 10 m s^{-2}. Calculate the bulk modulus of rubber.

Solution :

Increase of pressure on the ball

$$P = h\rho g = 360\text{m} \times 10^3\,\text{kgm}^{-3} \times 10\,\text{ms}^{-3}$$

$$= 3.6 \times 10^6\,\text{Nm}^{-2}$$

$$\text{Volume strain} = \frac{\Delta V}{V} = \frac{0.0012}{100} = 1.2 \times 10^{-5}$$

$$\text{Bulk Modulus } B = \frac{PV}{\Delta V} = \frac{3.6 \times 10^6}{1.2 \times 10^{-5}} = 3.0 \times 10^{11}\,\text{Nm}^{-2}$$

8.3.2 Poisson's Ratio

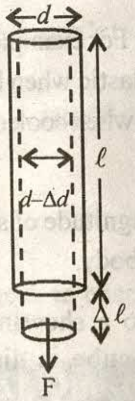


Fig: 8.12 : A stretched rubber tube.

You may have noticed that when a rubber tube is stretched along its length, there is a contraction in its diameter (Fig.8.12). (This is also true for a wire but may not be easily visible.) While the length increases in the direction of forces, a contraction occurs in the perpendicular direction. The strain perpendicular to the applied force is called **lateral strain**. Poisson pointed out that within elastic limit, lateral strain is directly proportional to longitudinal strain i.e. the ratio of lateral strain to longitudinal strain is constant for a material body and is known as **Poisson's ratio**. It is denoted by a Greek letter σ (sigma). If α and β are the longitudinal strain and lateral strain respectively, then Poisson's ratio

$$\sigma = \beta / \alpha.$$

If a wire (rod or tube) of length ℓ and diameter d is elongated by applying a stretching force by an amount $\Delta\ell$ and its diameter decreases by Δd, then longitudinal strain

$$\alpha = \frac{\Delta\ell}{\ell}$$

lateral strain $$\beta = \frac{\Delta d}{d}$$

and Possion's ratio $$\sigma = \frac{\Delta d/d}{\Delta\ell/\ell} = \frac{\ell}{d}\,\frac{\Delta d}{\Delta\ell} \qquad (8.7)$$

Since Poisson's ratio is a ratio of two strains, it is a pure number.

The value of Poisson's ratio depends only on the nature of material and for most of the substances, it lies between 0.2 and 0.4. When a body under tension suffers no change in volume, i.e. the body is perfectly incompressible, the value of Poisson's ratio is maximum i.e. 0.5. Theoretically, the limiting values of Poisson's ratio are –1 and 0.5

Activity 8.2

Take two identical wires. Make one wire to execute torsional vibrations for some time. After some time, set the other wire also in similar vibrations. Observe the rate of decay of vibrations of the two wires.

You will note that the vibrations decay much faster in the wire which was vibrating for longer time. The wire gets tired or fatigued and finds it difficult to continue vibrating. This phenomenon is known as **elastic fatigue**.

Some other facts about elasticity :

1. If we add some suitable impurity to a metal, its elastic properties are modified. For example, if carbon is added to iron or potassium is added to gold, their elasticity increases.
2. The increase in temperature decreases elasticity of materials. For example, carbon, which is highly elastic at ordinary temperature, becomes plastic when heated by a current through it. Similarly, plastic becomes highly elastic when cooled in liquid air.
3. The value of modulus of elasticity is independent of the magnitude of stress and strain. It depends only on the nature of the material of the body.

Example 8.5: A Metal cube of side 20 cm is subjected to a shearing stress of 10^4 Nm^{-2}. Calculate the modulus of rigidity, if top of the cube is displaced by 0.01 cm. with respect to bottom.

Solution : Shearing stress $= 10^4 \text{Nm}^{-2}, \Delta x = 0.01$ cm, and $y = 20$ cm.

$$\therefore \quad \text{Shearing strain} = \frac{\Delta x}{y} = \frac{0.01\,\text{cm}}{20\,\text{cm}}$$

Hence, $= 0.005$

$$\text{Modulus of rigidity } \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{10^4\,\text{Nm}^{-2}}{.0005}$$

$$= 2 \times 10^7\,\text{N m}^{-2}$$

Example 8.6 : A 10 kg mass is attached to one end of a copper wire of length 5 m long and 1 mm in diameter. Calculate the extension and lateral strain, if Poisson's ratio is 0.25. Given Young's modulus of the wire = 11×10^{10} N m^{-2}.

Solution : Here $L = 5$ m, $r = 0.05 \times 10^{-3}$ m, $y = 11 \times 10^{10}$ Nm^{-2} $F = 10 \times 9.8$ N, and $\sigma = 0.25$.

Extension produced in the wire

$$\Delta \ell = \frac{F.\ell}{\pi r^2 Y} = \frac{(10\,\text{kg}) \times (9.8\text{ms}^{-2}) \times (5\text{m})}{3.14\,(0.5 \times 10^{-3}\text{m})^2 \times (11 \times 10^{10}\,\text{Nm}^{-2})}$$

$$= \frac{490}{8.63 \times 10^4}\ \text{m}$$

$$= 5.6 \times 10^{-3}\ \text{m}$$

$$\text{longitudinal streain} = \alpha = \frac{\Delta \ell}{\ell}$$

$$= \frac{5.6 \times 10^{-3}\,\text{m}}{5\text{m}}$$

$$= 1.12 \times 10^{-2}$$

$$\text{Poission's ratio } (\sigma) = \frac{\text{lateral strain}(\beta)}{\text{longitudinal strain}(\alpha)}$$

$$\therefore \quad \text{lateral strain } \beta = \sigma \times \alpha$$

$$= 0.125 \times 1.12 \times 10^{-2}$$

$$= 0.14 \times 10^{-3}.$$

Now take a break to check your progress.

Intext Questions 8.2

1. Is the unit of longitudinal stress same as that of Young's modulus of elasticity? Give reason for your answer.

   ....

2. Solids are more elastic than liquids and gases. Justify

   ....

3. The length of a wire is cut to half. What will be the effect on the increase in its length under a given load?

   ....

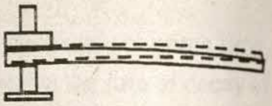
4. Two wires are made of the same metal. The length of the first wire is half that of the second and its diameter is double that of the second wire. If equal loads are applied on both wires, find the ratio of increase in their lengths?

   ....

5. A wire increases by 10^{-3} of its length when a stress of $1 \times 10^8\ \text{Nm}^{-2}$ is applied to it. Calculate Young's modulus of material of the wire.

   ....

Applications of Elastic Behaviour of Materials



Elastic behaviour of materials plays an important role in our day to day life. Pillars and beams are important parts of our structures. A uniform beam clamped at one end and loaded at the other is called a Cantilever [Fig.(i)]. The hanging bridge of Laxman Jhula in Rishkesh and Vidyasagar Sethu in Kolkata are supported on cantilevers.

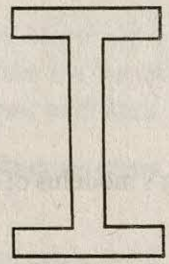
(i)

A cantilever of length l, breadth b and thickness d undergoes a depression δ at its free end when it is loaded by a weight of mass M:

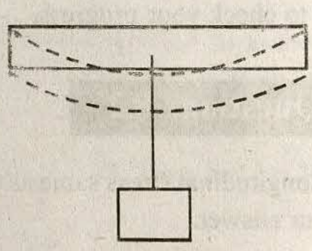
$$\delta = \frac{4M\,g\,\ell^3}{\gamma\,b\,d^3}$$

It is now easy to understand as to why the cross-section of girders and rails is kept I-shaped (Fig. ii). Other factors remaining same, $\delta\ \alpha\ d^{-3}$. Therefore, by increasing thickness, we can decrease depression under the same load more effectively. This considerably saves the material without sacrificing strength for a beam clamped at both ends and loaded in the middle (Fig.iii), the sag in the middle is given by

$$\delta = \frac{M\,g\,\ell^3}{4\,b\,d^3\,\gamma}$$



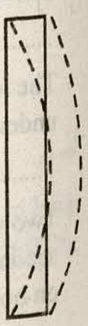
(ii)



(iii)

Thus for a given load, we will select a material with a large Young's modulus Y and again a large thickness to keep δ small. However, a deep beam may have a tendency to buckle (Fig iv). To avoid this, a large load bearing surface is provided. In the form I-shaped cross-section, both these requirements are fulfilled.

In cranes, we use a thick metal rope to lift and move heavy loads from one place to another. To lift a load of 10 metric tons with a steel rope of yield strength 300 mega pascal, it can be shown, that the minimum area of cross section required will be 10 cm or so. A single wire of this radius will practically be a rigid rod. That is why ropes are always made of a large number of turns of thin wires braided together. This provides ease in manufacturing, flexibility and strength.



(iv)

Do you know that the maximum height of a mountain on earth can be ~ 10 km or else the rocks under it will shear under its load.

What You Have Learnt

- A force which causes deformation in a body is called deforming force.
- On deformation, internal restoring force is produced in a body and enables it to regain its original shape and size after removal of deforming force.
- The property of matter to restore its original shape and size after withdrawal of deforming force is called elasticity.
- The body which gains completely its original state on the removal of the deforming forces is called perfectly elastic.
- If a body completely retains its modified form after withdrawal of deforming force, it is said to be perfectly plastic.
- The stress equals the internal restoring force per unit area. Its units is Nm^{-2}
- The strain equals the change in dimension (e.g. length, volum or shape) per unit dimension. Strain has no unit.
- In normal state, the net inter-atomic force on an atom is zero. If the separation between the atoms becomes more than the separation in normal state, the interatomic forces become attractive. However, for smaller separation, these forces become repulsive.
- The maximum value of stress up to which a body shows elastic property is called its elastic limit. A body beyond the elastic limit behaves like a plastic body.
- Hooke's law states that within elastic limit, stress developed in a body is directly proportional to strain.
- Young's modulus is the ratio of longitudinal stress to longitudinal strain.
- Bulk modulus is the ratio of normal stress to volume strain.
- Modulus of rigidity is the ratio of the shearing stress to shearing strain.
- Poisson's ratio is the ratio of lateral strain to longitudinal strain.

Terminal Questions

1. Define the term elasticity. Give examples of elastic and plastic objects.
2. Explain the terms stress, strain and Hooke's Law.
3. Explain elastic properties of matter on the basis of inter-molecular forces.
4. Define Young's modulus, Bulk modulus and modulus of rigidity.
5. Discuss the behaviour of a metallic wire under increasing load with the help of stress-strain graph.
6. Why steel is more elastic than rubber.
7. Why poission's ratio has no units.
8. In the three states of matter i.e., solid, liquid and gas, which is more elastic and why?

9. A metallic wire 4m in length and 1mm in diameter is stretched by putting a mass 4kg. Determine the alongation produced. Given that the Young's modulus of elasticity for the material of the wire is 13.78×10^{10} N m^{-2}.

10. A sphere contracts in volume by 0.02% when taken to the bottom of sea 1km deep. Calculate the bulk modulus of the material of the sphere. You make take density of sea water as 1000 kgm^{-3} and $g = 9.8$ms^{-2}.

11. How much force is required to have an increase of 0.2% in the length of a metallic wire of radius 0.2mm. Given $Y = 9 \times 10^{10}$ N m^{-2}.

12. What are shearing stress, shearing strain and modulus of rigidity?

13. The upper face of the cube of side 10cm is displaced 2mm parallel to itself when a tangential force of 5×10^5N is applied on it, keeping lower face fixed. Find out the strain?

14. Property of elasticity is of vital importance in our lives. How does the plasticity helps us?

15. A wire of length L and radius r is clamped rigidly at one end. When the other end of wire is pulled by a force F, its length increases by x. Another wire of the same material of length $2L$ and radius $2r$, when pulled by a force $2F$, what will be the increase in its length.

Answers to Intext Questions

8.1

1. If $R > R_0$, the nature of force is attractive and if (ii) $R < R_0$ it is repulsive.
2. Longitudinal stress and linear strain.
3. The ratio will decrease.
4. The stress corresponding to breaking point is known as breaking stress.
5. 0.12×10^{10}N m^{-2}.

8.2

1. Both have same units since strain has no unit?
2. As compressibility of liquids and gases is more than solids, the bulk modulus is reciprocal of compressibility. Therefore solids are more elastic than liquid and gases.
3. Half.
4. 1 : 8
5. 1×10^{11}N m^{-2}.

Answers To Terminal Problems

9. 0.15 m.

10. 4.9×10^{-10} N m^{-2}

11. 22.7 N

13. 2×10^{-2}

15. x.

9

PROPERTIES OF FLUIDS

In the previous lesson, you have learnt that interatomic forces in solids are responsible for determining the elastic properties of solids.Does the same hold for liquids and gases? (These are collectively called fluids because of their nature to flow in suitable conditions). Have you ever visited the site of a dam on a river in your area / state/ region? If so, you would have noticed that as we go deeper, the thickness of the walls increases. Did you think of the underlying physical principle? Similarly, can you believe that you can lift a car, truck or an elephant by your own body weight standing on one platform of a hydraulic lift? Have you seen a car on the platform of a hydraulic jack at a service centre? How easily is it lifted? You might have also seen that mosquitoes can sit or walk on still water, but we cannot do so. You can explain all these observations on the basis of properties of liquids like hydrostatic pressure, Pascal's law and surface tension. You will learn about these in this lesson.

Have you experienced that you can walk faster on land than under water? If you pour water and honey in separate funnels you will observe that water comes out more easily than honey. In this lesson we will learn the properties of liquids which cause this difference in their flow.

You may have experienced that when the opening of soft plastic or rubber water pipe is pressed, the stream of water falls at larger distance. Do you know how a cricketer swings the ball? How does an aeroplane take off? These interesting observations can be explained on the basis of Bernoulli's principle. You will learn about it in this lesson.

Objectives

After studying this lesson, you would be able to :

- *calculate the hydrostatic pressure at a certain depth inside a liquid;*
- *describe buoyancy and Archimedes Principle;*
- *state Pascal's law and explain the functioning of hydrostatic press , hydraulic lift and hydraulic brakes.;*
- *explain surface tension and surface energy ;*
- *derive an expression for the rise of water in a capillary tube;*

- *differentiate between streamline and turbulent motion of fluids;*
- *define critical velocity of flow of a liquid and calculate Reynold's number;*
- *define viscosity and explain some daily life phenomena based on viscosity of a liquid; and*
- *state Bernoulli's Principle and apply it to some daily life experiences.*

9.1 Hydrostatic Pressure

While pinning papers, you must have experienced that it is easier to work with a sharp tipped pin than a flatter one. If area is large, you will have to apply greater force. Thus we can say that for the same force, the effect is greater for smaller area. This effect of force on unit area is called *pressure.*

Refer to Fig. 9.1. It shows the shape of the side wall of a dam. Note that it is thicker at the base. Do we use similar shape for the walls of our house. No, the walls of rooms are of uniform thickness. Do you know the basic physical characteristic which makes us to introduce this change?

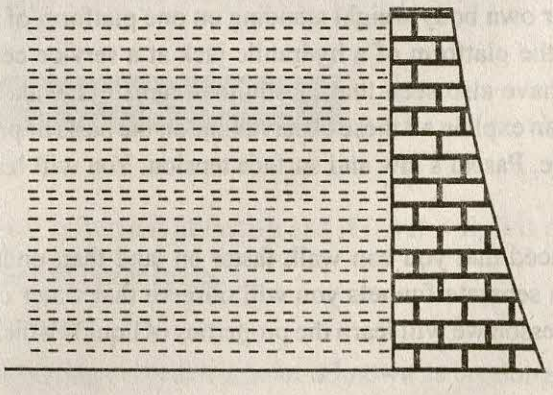


Fig. 9.1 : The structure of side wall of a dam

From the previous lesson you may recall that solids develop shearing stress when deformed by an external force, because the magnitude of inter-atomic forces is very large. But fluids do not have shearing stress and when an object is submerged in a fluid, the force due to the fluid acts normal to the surface of the object (Fig. 9.2). Also, the fluid exerts a force on the container normal to its walls at all points.

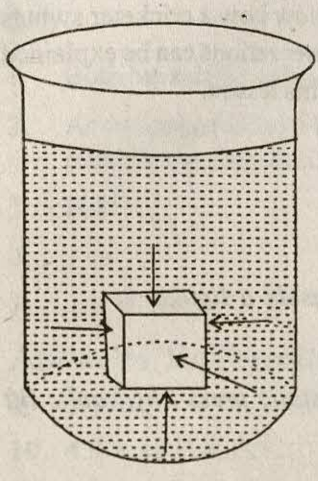


Fig. 9.2 : Force exerted by a fluid on a submerged object

The normal force or thrust per unit area exerted by a fluid is called pressure. We denote it by P :

$$P = \frac{\text{Thrust}}{\text{area}} \tag{9.1}$$

The pressure exerted by a fluid at rest is known as hydrostatic pressure

The SI Unit of pressure is Nm^{-2} and is also called pascal (Pa) in the honour of French scientist Blaise Pascal.

9.1.1 Hydrostatic Pressure at a point in side a liquid

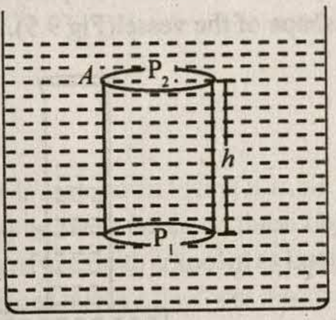


Fig. 9.3 : An imaginary cylinder of height *h* in a liquid.

Consider a liquid in a container and an imaginary right circular cylinder of cross sectional area *A* and height *h*, as shown in Fig. 9.3. Let the pressure exerted by the liquid on the bottom and top faces of the cylinder be P_1, and P_2, respectively. Therefore, the upward force exerted by the liquid on the bottom of the cylinder is P_1A and the downward force on the top of the cylinder is P_2A.

∴ The net force in upward direction is $(P_1A - P_2A)$.

Now mass of the liquid in cylinder = density × volume of the cylinder

$= \rho.\,A.\,h$ where ρ is the density of the liquid.

∴ Weight of the liquid in the cylinder = $\rho.\,g.\,h.\,A$

Since the cylinder is in equilibrium, the resultant force acting or it must be equal to zero, i.e.

$$P_1A - P_2A - \rho g h A = 0$$

$$\Rightarrow \quad P_1 - P_2 = \rho g h \qquad (9.2)$$

So, the pressure P at the bottom of a column of liquid of height *h* is given by

$$P = \rho g h$$

That is, hydrostatic pressure due to a fluid increases linearly with depth. It is for this reason that the thickness of the wall of a dam has to be increased with increase in the depth of the dam.

If we consider the upper face of the cylinder to be at the open surface of the liquid, as shown in Fig.(9.4), then P_2 will have to be replaced by P_{atm} (Atmospheric pressure). If we denote P_1 by P, the absolute pressure at a depth below the surface will be

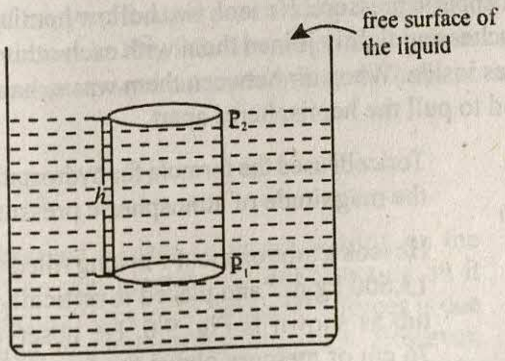


Fig. 9.4 : Cylinder in a liquid with one face at the surface of the liquid

$$P - P_{atm} + \rho g h$$

or

$$P = P_{atm} + \rho g h \qquad (9.3)$$

Note that the expression given in Eqn. (9.3) does not show any term having area of the cylinder It means that pressure in a liquid at a given depth is equal, irrespective of the shape of the vessel(Fig 9.5).

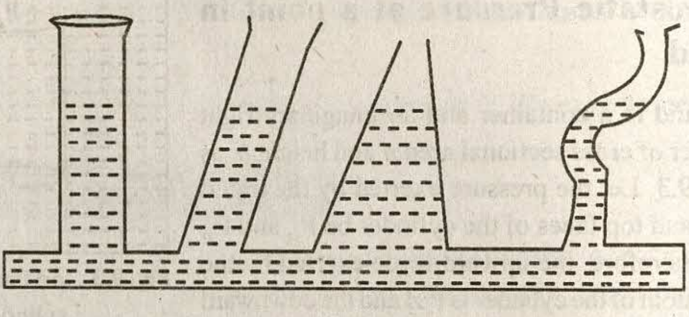


Fig. 9.5 : Pressure does not depend upon shape f the versel.

Example 9.1: A cemented wall of thickness one metre can withstand a side pressure of $10^5\ \mathrm{Nm^{-2}}$. What should be the thickness of the side wall at the bottom of a water dam of depth 100 m. Take density of water = $10^3\ \mathrm{kg\ m^{-3}}$ and $g = 9.8\ \mathrm{ms^{-2}}$.

Solution: The pressure on the side wall of the dam at its bottom is given by

$$\begin{aligned} P &= h d g \\ &= 100 \times 10^3 \times 9.8 \\ &= 9.8 \times 10^5\ \mathrm{Nm^{-2}} \end{aligned}$$

Using unitary method, we can calculate the thickness of the wall, which will withstand pressure of $9.8\times10^5\ \mathrm{Nm^{-2}}$. Therefore thickness of the wall

$$\begin{aligned} t &= \frac{9.8 \times 10^5}{10^5} \frac{\mathrm{Nm^{-2}}}{\mathrm{Nm^{-2}}} \\ &= 9.8\ \mathrm{m} \end{aligned}$$

9.1.2 Atmospheric Pressure

We know that the earth is surrounded by an atmosphere upto a height of about 200 km. The pressure exerted by the atmosphere is known as the *atmospheric pressure*. A German Scientist O.V. Guericke performed an experiment to demonstrate the force exerted on bodies due to the atmospheric pressure. He took two hollow hemispheres made of copper, having diameter 20 inches and tightly joined them with each other. These could easily be separated when air was inside. When air between them was exhausted with an air pump, 8 horses were required to pull the hemispheres apart.

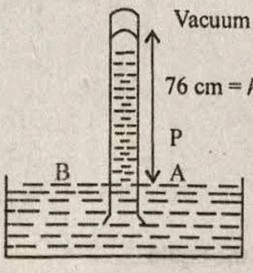


Fig: 9.6 : Toricelli's Barometer

Toricelli used the formula for hydrostatic pressure to determine the magnitude of atmospheric pressure.

He took a tube of about 1 m long filled with mercury of density 13,600 kg $\mathrm{m^{-3}}$ and placed it vertically inverted in a mercury tub as shown is Fig. 9.6. He observed that the column of 76 cm of mercury above the free surface remained filled in the tube.

In equilibrium, atmospheric pressure equals the pressure exerted by the mercury column. Therefore,

$$P_{atm} = h\rho g = 0.76 \times 13600 \times 9.8 \text{ Nm}^{-2}$$

$$= 1.01 \times 10^{5} \text{ Nm}^{-2}$$

$$= 1.01 \times 10^{5} \ Pa$$

9.2 Buoyancy

It is a common experience that lifting an object in water is easier than lifting it in air. It is because of the difference in the upward forces exerted by these fluids on these object. The upward force, which acts on an object when submerged in a fluid, is known as **buoyant force**. The nature of buoyant force that acts on objects placed inside a fluid was discovered by. Archimedes Based on his observations, he enunciated a law now known as Archimedes principle. It state that *when an object is submerged partially or fully in a fluid, the magnitude of the buoyant force on it is always equal to the weight of the fluid displaced by the object.*

The different conditions of an object under buoyant force is shown in Fig 9.7.

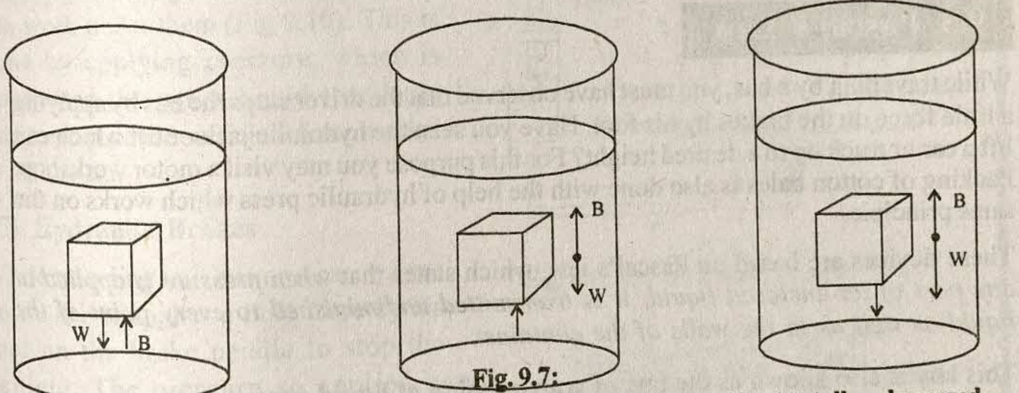


Fig. 9.7:

(a) : The magnitude of buoyant force B on the object is exactly equal to its weight in equilibrium.

(b) : A totally submerged object of density less than that of the fluid experiences a net upward force.

(c) : A totally submerged object denser than the fluid sinks.

Another example of buoyant force is provided by the motion of hot air balloon shown in Fig. 9.8. Since hot air has less density than cold air, a net upward buoyant force on the balloon makes it to float.

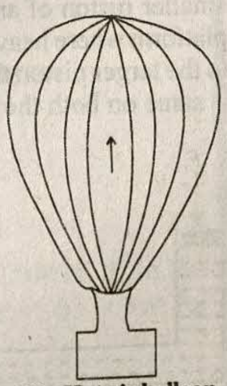
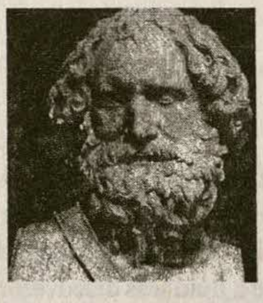


Fig. 9.8: Hot air balloon floating in air

Floating objects

You must have observed a piece of wood floating on the surface of water. Can you identify the forces acting on it when it is in equilibrium? Obviously, one of the forces is due to gravitational force, which pulls it downwards. However, the displaced water exerts buoyant force which acts upwards. These forces balance each other in equilibrium state and the object is then said to be floating on water. It means that a floating body displaces the fluid equal to its own weight.

Archimedes
(287- 212 B.C)



A Greek physicist, engineer and mathematician was perhaps the greatest scientist of his time. He is well known for discovering the nature of buoyant forces acting on objects. The Archimedes screw is used even today. It is an inclined rotating coiled tube used originally to lift water from the hold of ships. He also invented the catapult and devised the system of levers and pulleys.

Once Archimedes was asked by king Hieron of his native city Syracuse to determine whether his crown was made up of pure gold or alloyed with other metals without damaging the crown. While taking bath, he got a solution, noting a partial loss of weight when submerging his arm and legs in water. He was so excited about his discovery that he ran undressed through the streets of city shouting " Eureka, Eureka", meaning I have found it.

9.3 Pascal's Law

While travelling by a bus, you must have observed that the driver stops the bus by applying a little force on the brakes by his foot. Have you seen the hydraulic jack or lift which can lift a car or truck up to a desired height? For this purpose you may visit a motor workshop. Packing of cotton bales is also done with the help of hydraulic press which works on the same principle.

These devices are based on Pascal's law, which states that *when pressure is applied at any part of an enclosed liquid, it is transmitted undiminished to every point of the liquid as well as to the walls of the container.*

This law is also known as the **law of transmission of liquid pressure.**

9.3.1 Applications of Pascal's Law

(A) Hydraulic Press/Balance/Jack/Lift

It is a simple device based on Pascal's law and is used to lift heavy loads by applying a small force. The basic arrangement is shown in Fig.9.9. Let a force F_1 be applied to the smaller piston of area A_1. On the other side, the piston of large area A_2 is attached to a platform where heavy load may be placed. The pressure on the smaller piston is transmitted to the larger piston through the liquid filled in-between the two pistons. Since the pressure is same on both the sides, we have

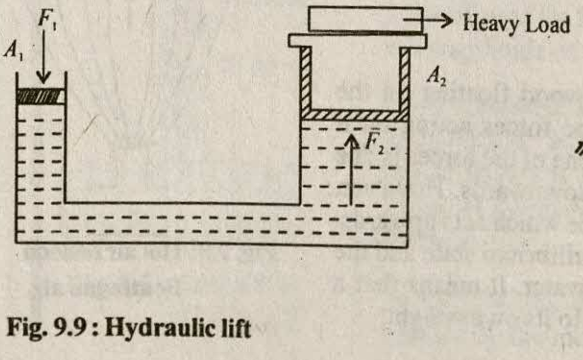


Fig. 9.9 : Hydraulic lift

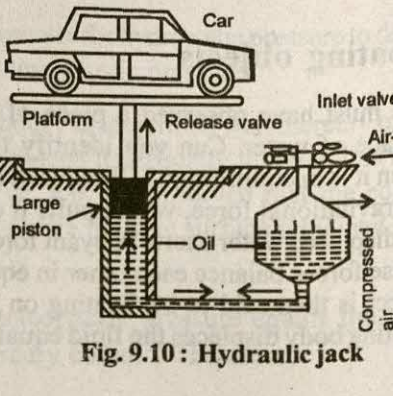


Fig. 9.10 : Hydraulic jack

Pressure on the smaller piston, $P = \dfrac{\text{force}}{\text{area}} = \dfrac{F_1}{A_1}$

According to Pascal's law, the same pressure is transmitted to the larger cylinder of area A_2. Hence the force acting on the larger piston

$$F_2 = \text{pressure} \times \text{area} = \frac{F_1}{A_1} \times A_2 \tag{9.4}$$

It is clear from Eqn. (9.4) that force $F_2 > F_1$ by an amount equal to the ratio (A_2/A_1)

With slight modifications, the same arrangement is used in hydraulic press, hydraulic balance, and hydraulic Jack, etc.

(B) Hydraulic Jack or Car Lifts

At automobile service stations, you would see that cars, buses and trucks are raised to the desired heights so that a mechanic can work under them (Fig 9.10). This is done by applying pressure, which is transmited through a liquid to a large surface to produce sufficient force needed to lift the car.

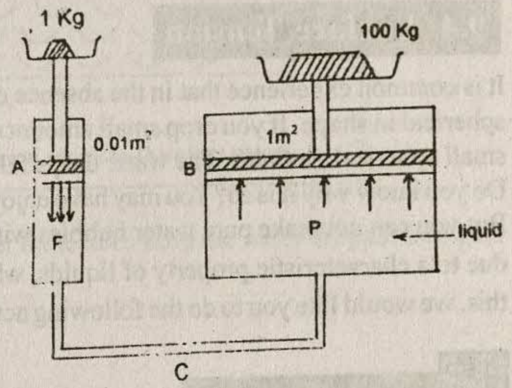


Fig. 9.11(a) : Hydraulic balance

(C) Hydraulic Brakes

While traveling in a bus or a car, we see how a driver applies a little force by his foot on the brake paddle to stop the vehicle. The pressure so applied gets transmitted through the brake oil to the piston of slave cylinders, which, in turn, pushes the break shoes against the break drum in all four wheels, simultaneously. The wheels stop rotating at the same time and the vehicle comes to stop instantaneously.

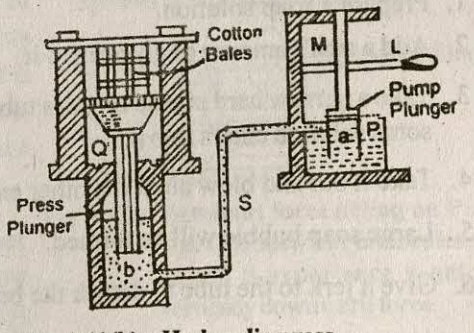


Fig. 9.11(b) : Hydraulic press

Intext Questions 9.1

1. Why are the shoes used for skiing on snow made big in size?

....

2 Calculate the pressure at the bottom of an ocean at a depth of 1500 m. Take the density of sea water 1.024×10^3 kg m^{-3}, atmospheric pressure=1.01×10^5 Pa and $g = 9.80$ ms^{-2}.

....

3. An elephant of weight 5000 kg f is standing on the bigger piston of area 10 m² of a

hydraulic lift. Can a boy of 25 kg wt standing on the smaller piston of area $0.05 m^2$ balance or lift the elephant?

....

4. If a pointed needle is pressed against your skin, you are hurt but if the same force is applied by a rod on your skin nothing may happen. Why?

....

5. A body of 50 kg f is put on the smaller piston of area $0.1 m^2$ of a big hydraulic lift. Calculate the maximum weight that can be balanced on the bigger piston of area $10 m^2$ of this hydraulic lift.

....

9.4 Surface Tension

It is common experience that in the absence of external forces, drops of liquid are always spherical in shape. If you drop small amount of mercury from a small height, it spreads in small spherical globules. The water drops falling from a tap or shower are also spherical. Do you know why it is so? You may have enjoyed the soap bubble game in your childhood. But you can not make pure water bubbles with same case? All the above experiences are due to a characteristic property of liquids, which we call **surface tension**. To appreciate this, we would like you to do the following activity.

Activity 9.1

1. Prepare a soap solution.
2. Add a small amount of glycerin to it.
3. Take a narrow hard plastic or glass tube. Dip its one end in the soap solution so that some solution enters into it.
4. Take it out and blow air at the other end with your mouth.
5. Large soap bubble will be formed.
6. Give a jerk to the tube to detach the bubble which then floats in the air.

To understand as to how surface tension arises, let us refresh our knowledge of intermolecular forces. In the previous lesson, you have studied the variation of intermolecular forces with distance between the centres of molecules/atoms.

The intermolecular forces are of two types: **cohesive** and **adhesive**. Cohesive forces characterise attraction between the molecules of the same substance, whereas force of adhesion is the attractive force between the molecules of two different substances. It is the force of adhesion which makes it possible for us to write on this paper. Gum, Fevicol etc. show strong adhesion.

We hope that now you can explain why water wets glass while mercury does not.

Activity 9.2

To show adhesive forces between glass and water molecule.

1. Take a clean sheet of glass
2. Put a few drops of water on it
3. Hold water containing side downward.
4. Observe the water drops.

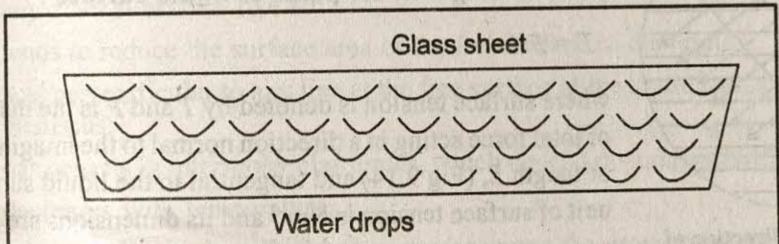


Fig. 9.12 Water drops remain stuck to the glass sheet

The Adhesive forces between glass and water molecules keep the water drops sticking on the glass sheet, as shown in Fig. 9.12.

9.4.1 Surface Energy

The surface layer of a liquid in a container exhibits a property different from the rest of the liquid. In Fig. 9.13, molecules are shown at different heights in a liquid. A molecule, say P, well inside the liquid is attracted by other molecules from all sides. However, it is not the case for the molecules at the surface.

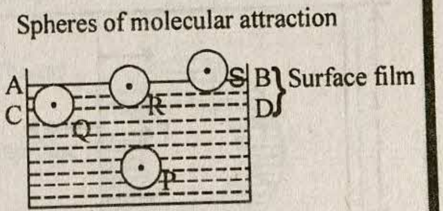


Fig 9.13 : Resultant force acting on P and Q is zero but molecules R and S experience a net vertically downward force.

Molecules S and R, which lie on the surface layer, experience a net resultant force downward because the number of molecules in the upper half of sphere of influence attracting these molecules is less than those in the lower half. If we consider the molecules of liquid on the upper half of the surface of the liquid or liquid-air interface, even then the molecules will experience a net downward force because of less number of molecules of liquid. Therefore, if any liquid molecule is brought to the surface layer, work has to be done against the net inward force, which increases their potential energy. This means that surface layer possesses an additional energy, which is termed as *surface energy*.

For a system to be in equilibrium, its potential energy must be minimum. Therefore, the area of surface must be minimum. That is why free surface of a liquid at rest tends to attain minimum surface area. This produces a tension in the surface, called **surface tension.**

Surface tension is a property of the liquid surface due to which it has the tendency to decrease its surface area. As a result, the surface of a liquid acts like a stretched membrane You can visualise its existence easily by placing a needle gently on water surface and see it float.

Let us now understand this physically. Consider an imaginary line AB drawn at the surface of a liquid at rest, as shown in Fig 9.14. The surface on either side of this line exerts a pulling force on the surface on the other side.

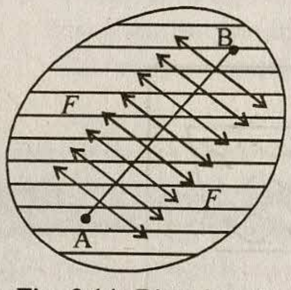


Fig. 9.14 : Direction of surface tension on a liquid surface

The **surface tension of a liquid can be defined as the force per unit length in the plane of liquid surface** :

$$T = F/L \tag{9.5}$$

where surface tension is denoted by T and F is the magnitude of total force acting in a direction normal to the imaginary line of length L, (Fig 9.14) and tangential to the liquid surface. SI unit of surface tension is Nm^{-1} and its dimensions are $[\text{MT}^{-2}]$.

Let us take a rectangular frame, as shown in Fig. 9.15 having a sliding wire on one of its arms. Dip the frame in a soap solution and take out. A soap film will be formed on the frame and have two surfaces. Both the surfaces are in contact with the sliding wire, So we can say that surface tension acts on the wire due to both these surfaces.

Let T be the surface tension of the soap solution and L be the length of the wire.

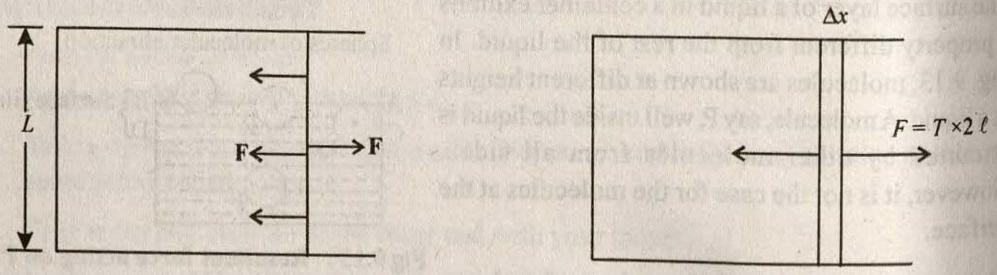


Fig.9.15 : A Film in equilibirum

The force exerted by each surface on the wire will be equal to $T \times L$. Therefore, the total force F on the wire = $2TL$.

Suppose that the surfaces tend to contract say, by Δx. To keep the wire in equilibrium we will have to apply an external uniform force equal to F. If we increase the surface area of the film by pulling the wire with a constant speed through a distance Δx, as shown in Fig. 9.15b, the work done on the film is given by

$$W = F \times \Delta x = T \times 2L \times \Delta x$$

where $2L \times \Delta x$ is the total increase in the area of both the surfaces of the film. Let us denote it by A. Then, the expressopm for work done on the film simplifies to

$$W = T \times A$$

This work done by the external force is stored as the potential energy of the new surface and is called as surface energy. By rearranging terms, we get the required expresion for surface tension :

$$T = W/A \qquad (9.6)$$

Thus, we see that **surface tension of a liquid is equal to the work done in increasing the surface area of its free surface by one unit.** We can also say that **surface tension is equal to the surface energy per unit area.**

We may now conclude that surface tension

- is a property of the surface layer of the liquid or the interface between a liquid and any other substance like air;
- tends to reduce the surface area of the free surface of the liquid;
- acts perpendicular to any line at the free surface of the liquid and is tangential to its meniscus;
- has genesis in intermolecular forces, which depend on temperature; and
- decreases with temperature.

A simple experiment described below demonstrates the property of surface tension of liquid surfaces.

Activity 9.3

Take a thin circular frame of wire and dip it in a soap solution. You will find that a soap film is formed on it. Now take a small circular loop of cotton thread and put it gently on the soap film. The loop stays on the film in an irregular shape as shown in Fig. 9.16(a). Now take a needle and touch its tip to the soap film inside the loop. What do you observe?

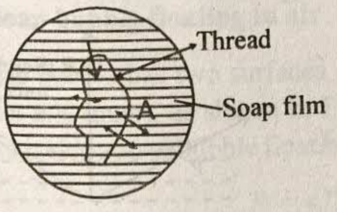


Fig 9.16 (a) : A soap film with closed loop of thread

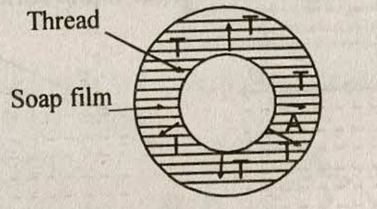


Fig. 9.16 (b) : The shape of the thread without inner soap film

You will find that the loop of cotton thread takes a circular shape as shown in Fig 9.16(b). Initially there was soap film on both sides of the thread. The surface on both sides pulled it and net forces of surface tension were zero. When inner side was punctured by the needle, the outside surface pulled the thread to bring it into the circular shape, so that it may acquire minimum area.

9.4.2 Applications of Surface Tension

(a) Mosquitoes sitting on water

In rainy reason, we witness spread of diseases like dengue, malaria and chickungunya by mosquito breeding on fresh stagnant water. Have you seen mosquitoes sitting on water surface? They do not sink in water due to surface tension. At the points where the legs of the mosquito touch the liquid surface, the surface becomes concave due to the weight of the mosquito. The surface tension acting tangentially on the free surface, therefore, acts at a certain angle to the horizontal. Its vertical component acts upwards. The total force acting vertically upwards all along the line of contact of certain length balances the weight of the mosquito acting vertically downward, as shown in Fig 9.17.

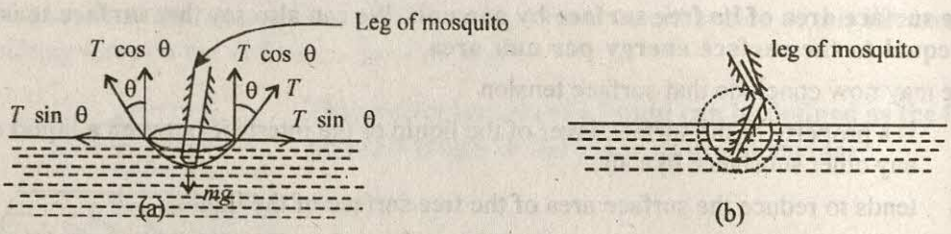


Fig. 9.17 : The weight of a mosquito is balanced by the force of surface tension = $2\pi rT\cos\theta$ (a) Dip in the level to form concave surface, and (b) magnified image

(b) Excess pressure on concave side of a spherical surface

Consider a small surface element with a line PQ of unit length on it, as shown in Fig. 9.18. If the surface is plane, i.e. $\theta = 90^0$, the surface tension on the two sides tangential to the surface balances and the resultant tangential force is zero [Fig. 9.18 (a)]. If, however, the surface is convex, [Fig. (9.18 (b)] or concave [Fig. 9.18 (c)], the forces due to surface tension acting across the sides of the line PQ will have resultant force **R** towards the center of curvature of the surface.

Thus, whenever the surface is curved, the surface tension gives rise to a pressure directed towards the center of curvature of the surface. This pressure is balanced by an equal and opposite pressure acting on the surface. Therefore, there is always an excess pressure on the concave side of the curved liquid surface [Fig. (9.18 b)].

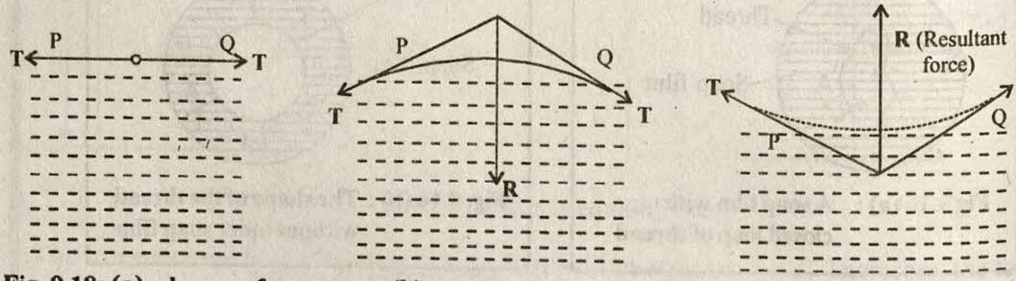


Fig. 9.18: (a) plane surface (b) convex surface (c) concave surface

(i) Spherical drop

A liquid drop has only one surface i.e. the outer surface. (The liquid area in contact with air is called the surface of the liquid.) Let r be the radius of a small spherical liquid drop and P be excess pressure inside the drop (which is concave on the inner side, but convex on the outside). Then

$$P = (P_i - P_0)$$

where P_i and P_0 are the inside and outside pressures of the drop, respectively (Fig 9.19a)

If the radius of the drop increases by Δr due to this constant excess pressure P, then increase in surface area of the spherical drop is given by

$$\Delta A = 4\pi\,(r + \Delta r)^2 - 4\pi r^2$$

$$= 8\pi\, r\, \Delta r$$

where we have neglected the term containing second power of Δr.

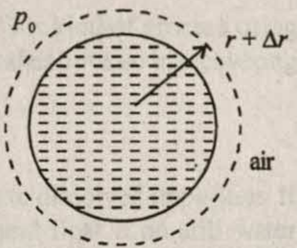


Fig. 9.19 (a) : A spherical drop

The work done on the drop for this increase in area is given by

$$\text{W} = \text{Extra surface energy} = T\Delta A = T.\ 8\pi\, r\, \Delta r \qquad (9.7)$$

If the drop is in equilibrium, this extra surface energy is equal to the work done due to expansion under the pressure difference or excess pressure P:

$$\text{Work done} = P\,\Delta \text{V} = P.\ 4\pi\, r^2 \Delta r \qquad (9.8)$$

On combining Eqns. (9.7) and (9.8), we get

$$P.\ 4\pi\, r^2 \Delta r = T.8\,\pi\, r\, \Delta r$$

Or

$$P = 2\,T/\,r \qquad (9.9)$$

(ii) Air Bubble in water

An air bubble also has a single surface, which is the inner surface (Fig. 9.19b). Hence, the excess of pressure P inside an air bubble of radius r in a liquid of surface tension T is given by

$$P = 2T/\,r \qquad (9.10)$$

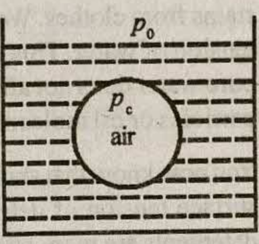


Fig. 9.19 b : Air Bubble

(iii) Soap bubble floating in air

The soap bubble has two surfaces of equal surface area (i.e. the outer and inner), as shown in Fig. 9.19(c). Hence, excess pressure inside a soap bubble floating in air is given by

$$P = 4T/\,r \qquad (9.11)$$

where T is suface tension of soap solution.

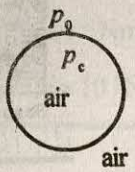


Fig.9.19 (c)

This is twice that inside a spherical drop of same radius or an air bubble in water. Now you can understand why a little extra pressure is needed to form a soap bubble.

Example 9.3: Calculate the difference of pressure between inside and outside of a (i) spherical soap bubble in air, (ii) air bubble in water, and (iii) spherical drop of water, each of radius 1 mm. Given surface tension of water = 7.2×10^{-2} Nm^{-1} and surface tension of soap solution = 2.5×10^{-2} Nm^{-1}.

Solution:

(i) Excess pressure inside a soap bubble of radius r is

$$P = 4T/r$$

$$= \frac{4 \times 2.5 \times 10^{-2}}{1 \times 10^{-3}\,\text{m}}\ \text{Nm}^{-1}$$

$$= 100\ \text{Nm}^{-2}$$

(ii) Excess pressure inside an air bubble in water

$$= 2T'/r$$

$$= \frac{2 \times 7.2 \times 10^{-2}\,\text{Nm}^{-1}}{1 \times 10^{-3}\,\text{m}}$$

$$= 144\ \text{Nm}^{-2}$$

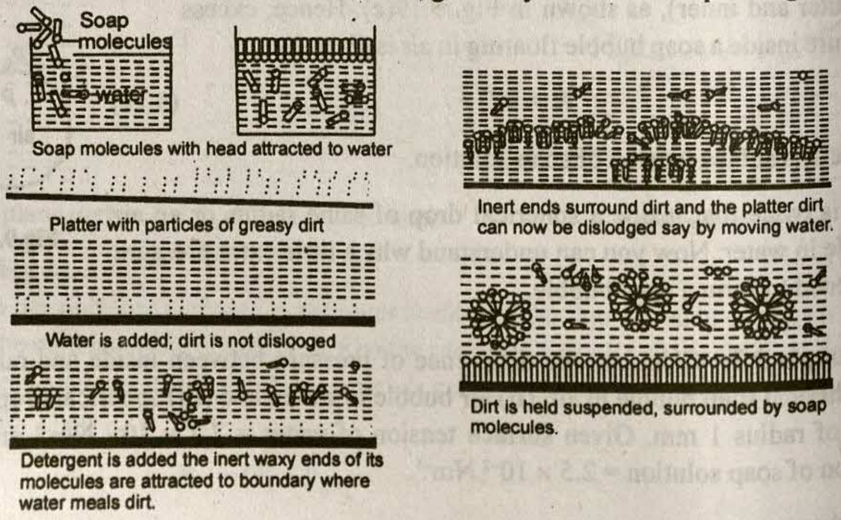
(iii) Excess pressure inside a spherical drop of water $=2T'/r$

$$= 144\ \text{Nm}^{-2}$$

(c) Detergents and surface tension

You may have seen different advertisements highlighting that detergents can remove oil stains from clothes. Water is used as cleaning agent. Soap and detergents lower the surface tension of water. This is desirable for washing and cleaning since high surface tension of pure water does not allow it to penetrate easily between the fibers of materials, where dirt particles or oil molecules are held up.

You now know that surface tension of soap solution is smaller than that of pure water but the surface tension of detergent solutions is smaller than that of soap solution. That is why detergents are more effective than soap. A detergent dissolved in water weakens the hold of dirt particles on the cloth fibers which therefore, get easily detached on squeezing the cloth.



Soap molecules with head attracted to water

Platter with particles of greasy dirt

Water is added; dirt is not dislooged

Detergent is added the inert waxy ends of its molecules are attracted to boundary where water meals dirt.

Inert ends surround dirt and the platter dirt can now be dislodged say by moving water.

Dirt is held suspended, surrounded by soap molecules.

Fig: 9.20 : Detergent action

The addition of detergent, whose molecules attract water as well as oil, drastically reduces the surface tension (T) of water-oil. It may even become favourable to form such interfaces, i.e. globes of dirt surrounded by detergent and then by water. This kind of process using surface active detergents is important for not only cleaning the clothes but also in recovering oil, mineral ores etc.

(d) Wax-Duck floating on water

You have learnt that the surface tension of liquids decreases due to dissolved impurities. If you stick a tablet of camphor to the bottom of a wax-duck and float it on still water surface, you will observe that it begins to move randomly after a minute or two. This is because camphor dissolves in water and the surface tension of water just below the duck becomes smaller than the surrounding liquid. This creates a net difference of force of surface tension which makes the duck to move.

Now, it is time for you to check how much you have learnt. Therefore, answer the following questions.

Intext Questions 9.2

1. What is the difference between force of cohesion and force of adhesion?

   ....

2. Why do small liquid drops assume a spherical shape.

   ....

3. Do solids also show the property of surface tension? Why?

   ....

4. Why does mercury collect into globules when poured on plane surface?

   ....

5. Which of the following has more excess pressure?

   (i) An air bubble in water of radius 2 cm. Surface tension of water is 727×10^{-3} Nm^{-1} or

   (ii) A soap bubble in air of radius 4 cm. Surface tension of soap solution is $25 \times 10^{-3} Nm^{-1}$.

9.5 Angle of Contact

You can observe that the free surface of a liquid kept in a container is curved. For example, when water is filled in a glass jar, it becomes concave but if we fill water in a paraffin wax container, the surface of water becomes convex. Similarly, when mercury is filled in a glass jar, its surface become convex. Thus, we see that shape of the liquid surface in a container depends on the nature of the liquid, material of container and the medium above free surface of the liquid. To characterize it, we introduce the concept of angle of contact.

It is the angle **that the tangential plane to the liquid surface makes with the tangential plane to the wall of the container, to the point of contact, as measured from within the liquid, is known as angle of contact.**

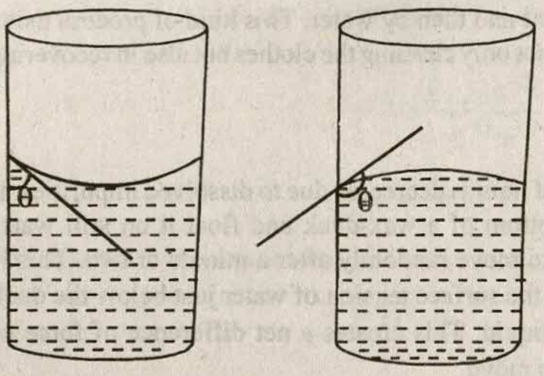
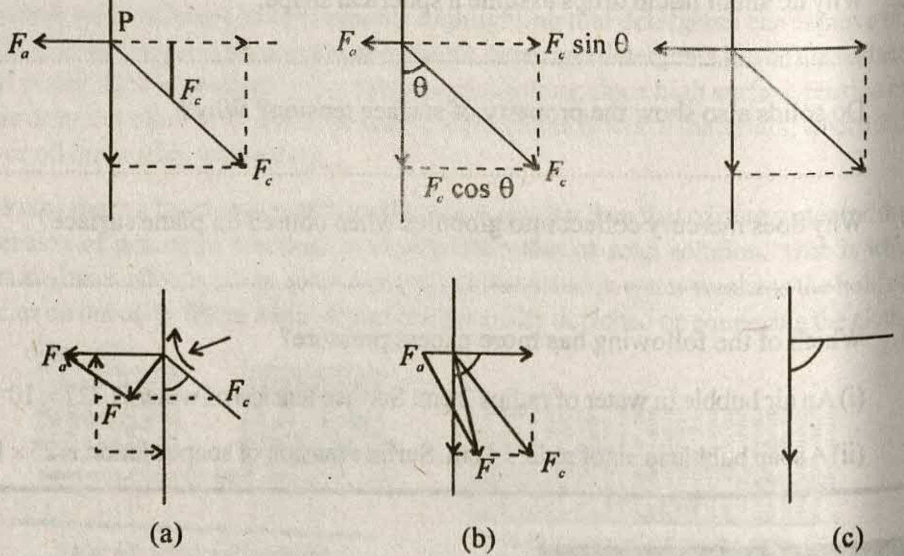


Fig 9.21 : Nature of free surface when water is filled in (a) glass jar, and (b) paraffin wax jar

Fig. 9.21 shows the angles of contact for water in a glass jar and paraffin jar. The angle of contact is acute for concave spherical meniscus, e.g. water with glass and obtuse (or greater than 90^0) for convex spherical meniscus e.g. water in paraffin or mercury in glass tube.

Various forces act on a molecule in the surface of a liquid contained in a vessel near the boundary of the menisus. As the liquid is present only in the lower quadrant, the resultant cohesive force acts on the molecule at P symmetrically, as shown in the Fig.9.22(a). Similarly due to symmetry, the resultant adhesive force F_a acts outwards at right angles to the walls of the container vessel . The force F_c can be resolved into two mutually perpendicular components F_c cos θ acting vertically downwards and F_c sin θ acting at right angled to the boundary, The value of the angle of contact depends upon the relative values of F_c and F_a.



(a) (b) (c)

Fig. 9.22 : Different shapes of liquid meniscuses

CASE 1: If $F_a > F_c$ sin θ, the net horizontal force is outward and the resultant of ($F_a - F_c$ sin θ) and F_c cos θ lies outside the wall. Since liquids can not sustain constant shear, the liquid surface and hence all the molecules in it near the boundary adjust themselves at right angles to F_c so that no component of F acts tangential to the liquid surface. Obviously such a surface at the boundary is concave spherical (Since radius of a circle is perpendicular to the circumference at every point.) This is true in the case of water filled in a glass tube.

Case 2 : If $F_a < F_c$ sin θ the resultant F of (F_c sin θ – F_a) acting horizontally and F_c cos θ acting vertically down wards is in the lower quadrant acting into the liquid. The liquid surface at the boundary, therefore, adjusts itself at right angles to this and hence becomes convex spherical. This is true for the case of mercury filled in the glass tube.

Case 3 : When $F_a = F_c \sin\theta$, the resultant force acts vertically downwards and hence the liquid surface near the boundary becomes horizontal or plane.

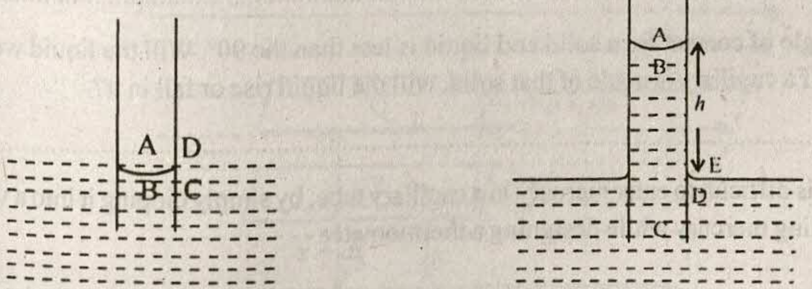
9.6 Capillary Action

You might have used blotting paper to absorb extra ink from your notebook. The ink rises in the narrow air gaps in the blotting paper. Similarly, if the lower end of a cloth gets wet, water slowly rises upward. Also water given to the fields rises in the innumerable capillaries in the stems of plants and trees and reaches the branches and leaves. Do you know that farmers plough their fields only after rains so that the capillaries formed in the upper layers of the soil are broken. Thus, water trapped in the soil is taken up by the plants. On the other hand, we find that when a capillary tube is dipped into mercury, the level of mercury inside it is below the outside level. Such an important phenomenon of the elevation or depression of a liquid in an open tube of small cross- section (i.e., capillary tube) is basically due to surface tension and is known as capillary action.

The phenomenon of rise or depression of liquids in capillary tubes is known as capillary action or capillarity.

9.6.1 Rise of a Liquid in a Capillary Tube

Let us take a capillary tube dipped in a liquid, say water. The meniscus inside the tube will be concave, as shown in Fig. 9.23 (a). This is essentially because the forces of adhesion between glass and water are greater than cohesive forces.



(a) (b)

Fig. 9.23 : Capillary action

Let us consider four points A, B, C and D near the liquid-air interface Fig. 9.23(a). We know that pressure just below the meniscus is less than the pressure just above it by $2T/R$, i.e.

$$P_B = P_A - 2T/R \qquad (9.12)$$

where T is surface tension at liquid-air interface and R is the radius of concave surface.

But pressure at A is equal to the pressure at D and is equal to the atmospheric pressure

P (say). And pressure at D is equal to pressure at C . Therefore, pressure at B is less than pressure at D. But we know that the pressure at all points at the same level in a liquid must be same. That's why water begins to flow from the outside region into the tube to make up the deficiency of pressure at point B.

Thus liquid begins to rise in the capillary tube to a certain height h (Fig 9.23 b) till the pressure of liquid column of height h becomes equal to $2T/R$. Thereafter, water stops rising. In this condition

$$h \rho g = 2\, T/R \qquad (9.13)$$

where ρ is the density of the liquid and g is the acceleration due to gravity. If r be radius of capillary tube and θ be the angle of contact, then from Fig. 9.24, we can write

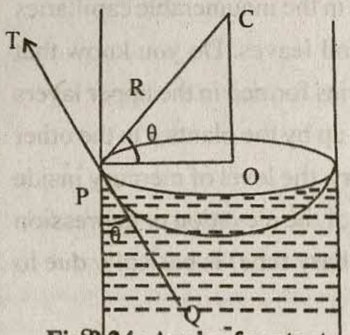


Fig 9.24 : Angle of contact

$$R = r/\cos\theta$$

Substituting this value of R in Equation (9.13)

$$h \rho g = 2T/\, r/\cos\theta$$

or

$$h = 2T\ \cos\theta\ /\ r \rho g \qquad (9.14)$$

It is clear from the above expression that if the radius of tube is less (i.e. in a very fine bore capillary), liquid rise will be high.

Intext Questions 9.3

1. Does the value of angle of contact depend on the surface tension of the liquid?

....

2. The angle of contact for a solid and liquid is less than the 90°. Will the liquid wet the solid? If a capillary is made of that solid, will the liquid rise or fall in it?

....

3. Why it is difficult to enter mercury in a capillary tube, by simply dipping it into a vessel containing mercury while designing a thermometer.

....

4. Calculate the radius of a capillary to have a rise of 3 cm when dipped in a vessel containing water of surface tension 7.2×10^{-2} N m^{-1}. The density of water is 1000 kg m^{-3}, angle of contact is zero, and $g = 10$ m s^{-2}.

....

5. How does kerosene oil rise in the wick of a lantern?

....

9.7 Viscosity

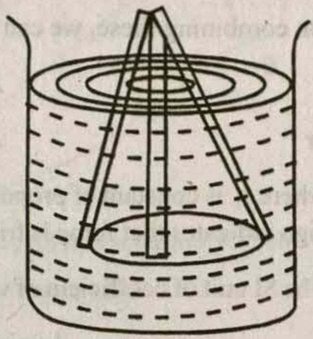


Fig. 9.25: Water being stirred with a glass rod

If you stir a liquid taken in a beaker with a glass rod in the middle, you will note that the motion of the liquid near the walls and in the middle is not same (Fig.9.25). Next watch the flow of two liquids (e.g. glycerin and water) through identical pipes. You will find that water flows rapidly out of the vessel whereas glycerine flows slowly. Drop a steel ball through each liquid. The ball falls more slowly in glycerin than in water. These observations indicate a characteristic property of the liquid that determines their motion. This property is known as ***viscosity***. Let us now learn how it arises.

9.7.1 Viscosity

We know that when one body slides over the other, a frictional force acts between them. Similarly, whenever a fluid flows, two adjacent layers of the fluid exert a tangential force on each other; this force acts as a drag and opposes the relative motion between them. *The property of a fluid by virtue of which it opposes the relative motion in its adjacent layers is known as viscosity.*

Fig. 9.26 shows a liquid flowing through a tube. The layer of the liquid in touch with the wall of the tube can be assumed to be stationary due to friction between the solid wall and the liquid. Other layers are in motion and have different velocities Let v be the velocity of the layer at a distance x from the surface and $v + dv$ be the velocity at a distance $x + dx$.

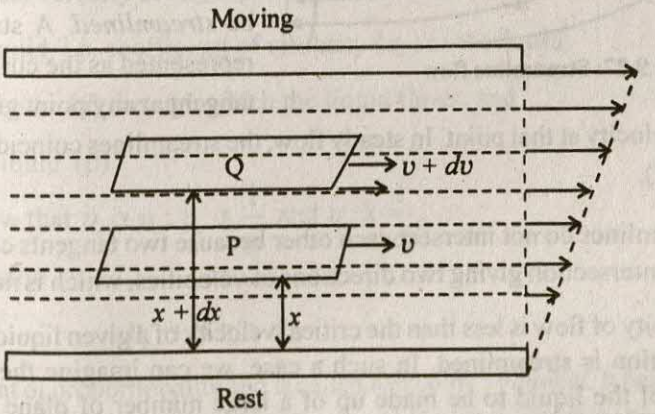


Fig. 9.26 : Flow of a liquid in a tube: Different layers move with different velocities

Thus, the velocity changes by dv in going through a distance dx perpendicular to it. The quantity dv/dx is called the **velocity gradient**.

The viscous force F between two layers of the fluid is proportional to

- area (A) of the layer in contact : $F \propto A$

- velocity gradient (dv/dx) in a direction perpendicular to the flow of liquid : $F \alpha\, dv/dx$

On combining these, we can write

$$F \alpha\, A\, dv/dx$$

or

$$F = -\eta A\,(dv/dx) \qquad (9.15)$$

where η is constant of proportionality and is called **coefficient of viscosity**. The negative sign indicates that force is frictional in nature and opposes motion.

The SI unit of coefficient of viscosity is Nsm^{-2}. In cgs system, the unit of viscosity is poise.

$$1 \text{ poise} = 0.1 \text{ Nsm}^{-2}$$

Dimensions of coefficient of viscosity are $[ML^{-1} T^{-1}]$

Table 10.1 : Viscosity of a few typical fluids

Name of fluid	T [°C]	Viscosity η (PR)
Water	20	1.0×10^{-3}
Water	100	0.3×10^{-3}
blood	37	2.7×10^{-3}
Air	40	1.9×10^{-5}

9.8 Types of Liquid Flow

Have you ever seen a river in floods? Is it similar to the flow of water in a city water supply system? If not, how are the two different? To discover answer to such questions, let as study the flow of liquids.

9.8.1 Streamline Motion

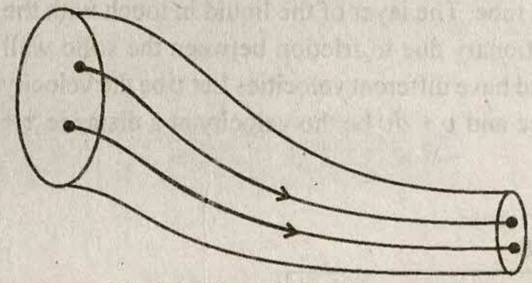


Fig. 9.27: Streamline flow

The path followed by fluid particles is called line of flow. If every particle passing through a given point of the path follows the same line of flow as that of preceding particles, the flow is said to be *streamlined.* A streamline can be represented as the curve or path whose tangent at any point gives the direction of the liquid velocity at that point. In steady flow, the streamlines coincide with the line of flow (Fig. 9.27).

Note that streamlines do not intersect each other because two tangents can then be drawn at the point of intersection giving two directions of velocities, which is not possible.

When the velocity of flow is less than the critical velocity of a given liquid flowing through a tube, the motion is streamlined. In such a case, we can imagine the entire thickness of the stream of the liquid to be made up of a large number of plane layers (laminae) one sliding past the other, i.e. one flowing over the other. Such a flow is called *laminar flow.*

If the velocity of flow exceeds the critical velocity v_c, the mixing of streamlines takes place and the flow path becomes zig-zag. Such a motion is said to be *turbulent.*

9.8.2 Equation of Continuity

If an incompressible, non-viscous fluid flows through a tube of non-uniform cross section, the product of the area of cross section and the fluid speed at any point in the tube is constant for a streamline flow. Let A_1 and A_2 denote the areas of cross section of the tube where the fluid is entering and leaving, as shown in Fig. 9.28. If υ_1 and υ_2 are the speeds of the fluid at the ends A and B respectively, and ρ is the density of the fluid, then the liquid entering the tube at A covers a distance υ_1 in one second. So volume of the liquid entering per second= $A_1 \times \upsilon_1$. Therefore

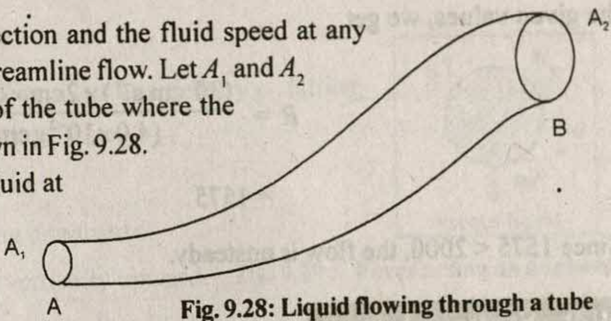


Fig. 9.28: Liquid flowing through a tube

Mass of the liquid entering per second at point A = $A_1 \upsilon_1 \rho$

Similarly, mass of the liquid leaving per second at point B = $A_2 \upsilon_2 \rho$

Since there is no accumulation of fluid inside the tube, the mass of the liquid crossing any section of the tube must be same. Therefore, we get

$$A_1 \upsilon_1 \rho = A_2 \upsilon_2 \rho$$

or

$$A_1 \upsilon_1 = A_2 \upsilon_2$$

This expression is called **equation of continuity**.

9.8.3 Critical Velocity and Reynolds's Number

We now know that when the velocity of flow is less than a certain value, valled *critical velocity*, the flow remains streamlined. But when the velocity of flow exceeds the critical velocity, the flow becomes turbulent.

The value of critical velocity of any liquid depends on the

- nature of the liquid, i.e. coefficient of viscosity (η) of the liquid;
- diameter of the tube (d) through which the liquid flows; and
- density of the liquid (ρ).

Experiments show that $\upsilon_c \; \alpha \; \eta$; $\upsilon_c \; \alpha \; \frac{1}{\rho}$ and $\upsilon_c \; \alpha \; \frac{1}{d}$.

Hence, we can write

$$\upsilon_c = R.\eta/\rho\, d \qquad (9.16)$$

where R is constant of proportionality and is called Reynolds's Number. It has no dimensions. Experiments show that if R is below 1000, the flow is laminar. The flow becomes unsteady when R is between 1000 and 2000 and the flow becomes turbulent for R greater than 2000.

Example 9.1: The average speed of blood in the artery (d =2.0 cm) during the resting part of heart's cycle is about 30 cm s^{-1}. Is the flow laminar or turbulent? Density of blood 1.05 g cm^{-3}; and $\eta = 4.0 \times 10^{-2}$ poise.

Solution: From Eqn. (9.16) we recall that Reynold's number $R = \upsilon_c \rho d/\eta$. On substituting the given values, we get

$$R = \frac{(30\ \text{cm s}^{-1}) \times 2\,\text{cm} \times (1.05\,\text{g cm}^{-3})}{(4.0 \times 10^{-2}\,\text{g cm}^{-1}\text{s}^{-1})}$$

$$= 1575$$

Since $1575 < 2000$, the flow is unsteady.

9.9 Stokes' Law

George Stokes gave an empirical law for the magnitude of the tangential backward viscous force F acting on a freely falling smooth spherical body of radius r in a highly viscous liquid of coefficient of viscosity η moving with velocity υ. This is known as Stokes' law.

According to Stokes' law

$$F \propto \eta\, r\, \upsilon$$

or $$F = K \eta\, r\, \upsilon$$

where K is constant of proportionality. It has been found experimentally that $K = 6\pi$.

Hence Stokes' law can be written as

$$F = 6\pi\, \eta\, r\, \upsilon \qquad (9.17)$$

Stokes' Law can also be derived using the method of dimensions as follows:

According to Stokes, the viscous force depends on:

- coefficient of viscosity (η) of the medium
- radius of the spherical body (r)
- velocity of the body (υ)

Then $$F \propto \eta^a\, r^b\, \upsilon^c$$

or $$F = K \eta^a\, r^b\, \upsilon^c$$

where K is constant of proportionality

Taking dimensions on both the sides, we get

$$[\text{MLT}^{-2}] = [\text{ML}^{-1}\text{T}^{-1}]^a\, [\text{L}]^b\, [\text{LT}^{-1}]^c$$

or $$[\text{MLT}^{-2}] = [\text{M}^a\, \text{L}^{-a+b+c}\, \text{T}^{-a-c}]$$

Comparing the exponents on both the sides and solving the equations we get $a = b = c = 1$.

Hence $$F = K \eta\, r\, \upsilon$$

9.9.1 Terminal Velocity

Let us consider a spherical body of radius r and density ρ falling through a liquid of density σ.

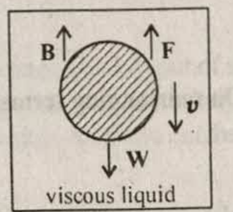


Fig. 9.29 : Force acting on a sphere falling in viscous fluid

The forces acting on the body will be

(i) Weight of the body **W** acting downward.

(ii) The viscous force **F** acting vertically upward.

(iii) The buoyant force **B** acting upward.

Under the action of these forces, at some instant the net force on the body becomes zero, (since the viscous force increases with the increase of velocity). Then, the body falls with a constant velocity known as **terminal velocity**. We know that magnitude of these forces are

$$F = 6\pi \eta r \upsilon_0$$

where υ_0 is the terminal velocity.

$$W = (4/3)\pi r^3 \rho g$$

and

$$B = (4/3)\pi r^3 \sigma g$$

The net force is zero when object attains terminal velocity. Hence

$$6\pi \eta r \upsilon_0 = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

Hence

$$\upsilon_0 = \frac{2r^2(\rho - \sigma)g}{9\eta} \qquad (9.18)$$

9.9.2 Applications of Stokes' Law

A. Parachute

When a soldier jumps from a flying aeroplane, he falls with acceleration due to gravity g but due to viscous drag in air, the acceleration goes on decreasing till he acquires terminal velocity. The soldier then descends with constant velocity and opens his parachute close to the ground at a pre-calculated moment, so that he may land safely near his destination.

B. Velocity of rain drops

When raindrops fall under gravity, their motion is opposed by the viscous drag in air. When viscous force becomes equal to the force of gravity, the drop attains a terminal velocity. That is why rain drops reaching the earth do not have very high kinetic energy.

Example 9.2: Determine the radius of a drop of rain falling through air with terminal velocity 0.12 ms^{-1}. Given $\eta = 1.8 \times 10^{-5}$ kg m^{-1} s^{-1}, $\rho = 1.21$ kg m^{-3}, $\sigma = 1.0 \times 10^{3}$ kg m^{-3} and $g = 9.8$ m s^{-2}.

Solution: We know that terminal velocity is given by

$$\upsilon_0 = \frac{2r^2(\rho-\sigma)g}{9\eta}$$

On rearranging terms, we can write

$$r = \sqrt{\frac{9\eta\,\upsilon_0}{2(\rho-\sigma)g}}$$

$$= \sqrt{\frac{9\times1.8\times10^{-5}\times0.12}{2\,(1000-1.21)\,9.8}}\ \text{m}$$

$$= 10^{-5}\ \text{m}$$

Intext Questions 9.4

1. Differentiate between streamline flow and turbulent flow?

....

2. Can two streamlines cross each other in a flowing liquid?

....

3. Name the physical quantities on which critical velocity of a viscous liquid depends.

....

4. Calculate the terminal velocity of a rain drop of radius 0.01m if the coeflicient of viscosity of air is 1.8×10^{-5} Ns m^{-2} and its density is 1.2 kg m^{-3}. Density of water = 1000 kg m^{-3}. Take $g = 10$ m s^{-2}.

....

5. When a liquid contained in a tumbler is stirred and placed for some time, it comes to rest, Why?

....

Daniel Bernoulli (1700-1782)

Daniel Bernoulli, a Swiss Physicist and mathematician was born in a family of mathematicians on February 8, 1700. He made important contributions in hydrodynamics. His famous work, *Hydrodyanamica* was published in 1738. He also explained the behavior of gases with changing pressure and temperature, which led to the development of kinetic theory of gases.

He is known as the founder of mathematical physics. Bernoulli's principle is used to produce vacuum in chemical laboratories by connecting a vessel to a tube through which water is running rapidly.

9.10 Bernoulli's Principle

Have you ever thought how air circulates in a dog's burrow, smoke comes quickly out of a chimney or why car's convertible top bulges upward at high speed? You must have definitely experienced the bulging upwards of your umbrella on a stormy- rainy day. All these can be understood on the basis of Bernoulli's principle.

Bernoulli's Principle states that **where the velocity of a fluid is high, the pressure is low and where the velocity of the fluid is low, pressure is high.**

9.10.1 Energy of a Flowing Fluid

Flowing fluids possess three types of energy. We are familiar with the kinetic and potential energies. The third type of energy possessed by the fluid is pressure energy. It is due to the pressure of the fluid. The pressure energy can be taken as the product of pressure difference and its volume. If an element of liquid of mass m, and density d is moving under a pressure difference p, then

$$\text{Pressure energy} = p \times (m/d) \text{ joule}$$

Pressure energy per unit mass = (p/d) J kg^{-1}

9.10.2 Bernoulli's Equation

Bernoulli developed an equation that expresses this principle quantitatively. Three important assumptions were made to develop this equation:

1.The fluid is incompressible, i.e. its density does not change when it passes from a wide bore tube to a narrow bore tube.

2.The fluid is non-viscous or the effect of viscosity is not to be taken into account.

3.The motion of the fluid is streamlined.

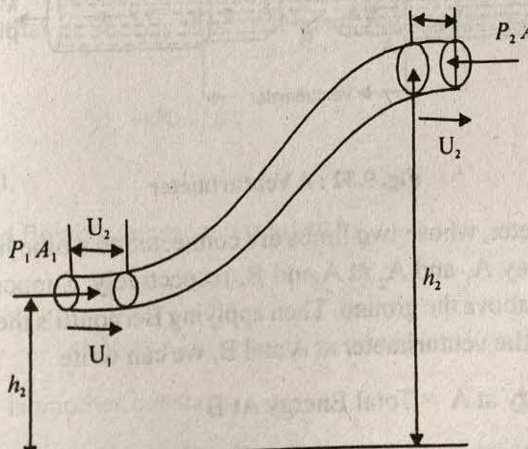


Fig. 9.30

We consider a tube of varying cross section shown in the Fig. 9.30. Suppose at point A the pressure is P_1, area of cross section A_1, velocity of flow υ_1, height above the ground h_1 and at B, the pressure is P_2 ,area of cross-section A_2 velocity of flow = υ_2, and height above the ground h_2.

Since points A and B can be any two points along a tube of flow, we write Bernoulli's equation

$P + 1/2\, dv^2 + h\, dg = \text{Constant.}$

That is, the sum of pressure energy, kinetic energy and potential energy of a fluid remains constant in streamline motion.

Activity 9.4

1. Take a sheet of paper in your hand.
2. Press down lightly on horizontal part of the paper as shown in Fig. 9.31 so that the paper curves down.
3. Blow on the paper along the horizontal line.

Watch the paper. It lifts up because speed increases and pressure on the upper side of the paper decreases.

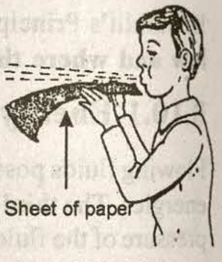


Fig. 9.31

9.10.3 Applications of Bernoulli's Theorem

Bernoulli's theorem finds many applications in our lives. Some commonly observed phenomena can also be explained on the basis of Bernoulli's theorem.

A. Flow meter or Venturimeter

It is a device used to measure the rate of flow of liquids through pipes. The device is inserted in the flow pipe, as shown in the Fig. 9.32

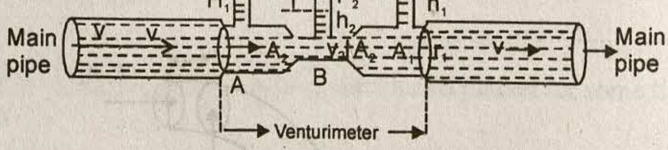


Fig. 9.32 : A Venturimeter

It consists of a manometer, whose two limbs are connected to a tube having two different cross-sectional areas say A_1 and A_2 at A and B, respectively. Suppose the main pipe is horizontal at a height h above the ground. Then applying Bernoulli's theorem for the steady flow of liquid through the venturimeter at A and B, we can write

$$\text{Total Energy at A} = \text{Total Energy At B}$$

$$\frac{1}{2} m v_1^2 + mgh + \frac{mp_1}{d} = \frac{1}{2} m v_2^2 + mgh + \frac{mp_2}{d}$$

On rearranging terms we can write,

$$(p_1 - p_2) = \frac{d}{2}(v_2^2 - v_1^2) = \frac{v_1^2 d}{2}\left[\left(\frac{v_2}{v_1}\right)^2 - 1\right] \qquad (9.19)$$

It shows that points of higher velocities are the points of lower pressure (because of the sum of pressure energy and K.E. remain constant). This is called *Venturi's Principle*.

For steady flow through the ventrurimeter, volume of liquid entering per second at A = liquid volume leaving per second at B. Therefore

$$A_1 \upsilon_1 = A_2 \upsilon_2 \qquad (9.20)$$

(The liquid is assumed incompressible i.e., velocity is more at narrow ends and vice versa.

Using this result in Eqn. (9.19), we conclude that pressure is lesser at the narrow ends;

$$p_1 - p_2 = \frac{\upsilon_1^2 d}{2}\left[\frac{A_1^2}{A_2^2} - 1\right]$$

$$= \frac{1}{2} d\upsilon_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$\upsilon_1 = \sqrt{\frac{2(p_1 - p_2)}{d\left(\frac{A_1^2}{A_2^2}\right) - 1}} \qquad (9.21)$$

If *h* denotes level difference between the two limbs of the venturimeter, then

$$p_1 - p_2 = h\,d\,g$$

and $\upsilon_1 = \sqrt{2hg/[(A_1^2/A_2^2) - 1]}$

From this we note that $\upsilon_1 \propto \sqrt{h}$ since all other parameters are constant for a given venturimeter. Thus

$$\upsilon_1 = K\sqrt{h};$$

where K is constant.

The volume of liquid flowing per second is given by

$$V = A_1\,\upsilon_1 = A_1 \times K\sqrt{h}$$

or $V = K'h$

where $K' = K A_1$ is another constant.

Bernaulli's principle has many applications in the design of many useful appliances like atomizer, spray gun, Bunsen burner, carburetor, Aerofoil, etc.

(i) Atomizer : An atomizer is shown in Fig. 9.33. When the rubber bulb A is squeezed, air blows through the tube B and comes out of the narrow orifice with larger velocity creating a region of low pressure in its neighborhood. The liquid (scent or paint) from the vessel is, therefore, sucked into the tube to come out to the nozzles N. As the liquid reaches the nozzle, the air stream from the tube B blows it into a fine spray.

(ii) Spray gun : When the piston is moved in, it blows the air out of the narrow hole 'O' with large velocity creating a region of low pressure in its neighborhood. The liquid (e.g. insecticide) is sucked through the narrow tube attached to the vessel end having its opening just below 'O'. The liquid on reaching the end gets sprayed by out blown air from the piston (Fig. 9.34).

(iii) Bunsen Burner : When the gas emerges out of the nozzle N, its velocity being high the pressure becomes low in its vicinity. The air, therefore, rushed in through the side hole A and gets mixed with the gas. The mixture then burns at the mouth when ignited, to give a hot blue flame (Fig.9.35).

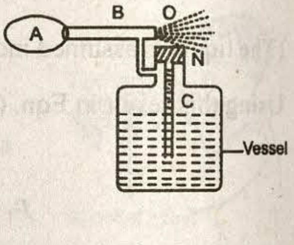


Fig. 9.33 : Atomizer

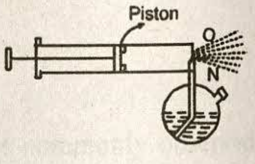


Fig. 9.34 : Spray gun

(iv) Carburetor : The carburetor shown in Fig. 9.36. is a device used in motor cars for supplying a proper mixture of air and petrol vapours to the cylinder of the engine. The energy is supplied by the explosion of this mixture inside the cylinders of the engine. Petrol is contained in the float chamber. There is a decrease in the pressure on the side A due to motion of the piston.This causes the air from outside to be sucked in with large velocity. This causes a low pressure near the nozzle B (due to constriction, velocity of air sucked is more near B) and, therefore, petrol comes out of the nozzle B which gets mixed with the incoming. Air. The mixture of vaporized petrol and air forming the fuel then enters the cylinder through the tube A.

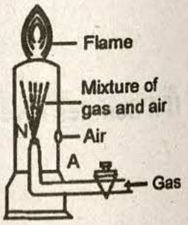


Fig. 9.35 : Bunsen Burner

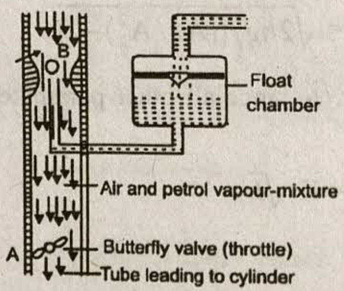


Fig. 9.36 : Carburettor

(Sometimes when the nozzle B gets choked due to deposition of carbon or some impurities, it checks the flow of petrol and the engine not getting fuel stops working. The nozzle has therefore, to be opened and cleaned.

(v) Aerofoil : When a solid moves in air , streamlines are formed . The shape of the body of the aeroplane is designed specially as shown in the Fig. 9.37. When the aeroplane runs on its runway, high velocity streamlines of air are formed. Due to crowding of more streamlines on the upper side, it becomes a region of more velocity and hence of comparatively low pressure region than below it. This pressure difference gives the lift to the aeroplane.

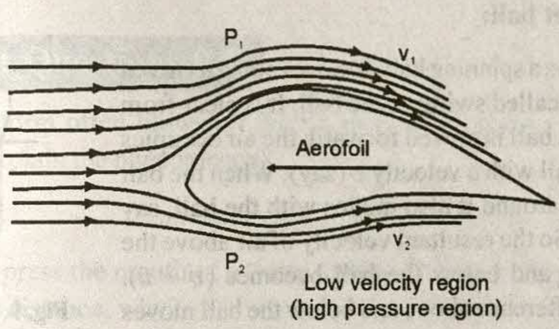


Fig. 9.37 : Crowding of streamlines on the upper side.

Based on this very principle i.e., the regions of high velocities due to crowding of steam lines are the regions of low pressure, following are interesting demonstrations.

(a) Attracted disc paradox : When air is blown through a narrow tube handle into the space between two cardboard sheets [Fig. 9.38] placed one above the other and the upper disc is lifted with the handle, the lower disc is attracted to stick to the upper disc and is lifted with it. This is called attracted disc paradox,

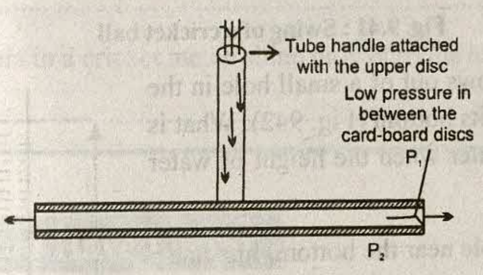


Fig. 9.38 : Attracted disc paradox

(b) Dancing of a ping pong ball on a jet of water:

If a light hollow spherical ball (ping-pong ball or table tennis ball) is gently put on a vertical stream of water coming out of a vertically upward directed jet end of a tube, it keeps on dancing this way and that way without falling to the ground (Fig.9.39). When the ball shifts to the lefts , then most of the jet streams pass by its right side thereby creating a region of high velocity and hence low pressure on its right side in comparison to that on the left side and the ball is again pushed back to the center of the jet stream .

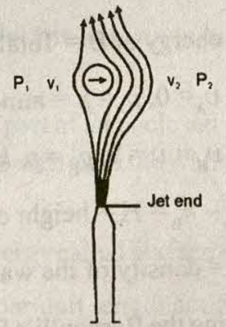


Fig. 9.39 : Dancing Pring Pongball

(c) Water vacuum pump or aspirator or filter pump : Fig. 9.40 shows a filter pump used for producing moderately low pressures. Water from the tap is allowed to come out of the narrow jet end of the tube A . Due to small aperture of the nozzle, the velocity becomes high and hence a low-pressure region is created around the nozzle N. Air is, therefore, sucked from the vessel to be evacuated through the tube B; gets mixed with the steam of water and goes out through the outlet. After a few minutes., the pressure of air in the vessel is decreased to about 1 cm of mercury by such a pump

(d) Swing of a cricket ball:

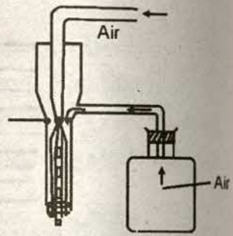


Fig. 9.40 : Filter Pump

When a cricketer throws a spinning ball, it moves along a curved path in the air. This is called swing of the ball. It is clear from Fig. 9.41. That when a ball is moved forward, the air occupies the space left by the ball with a velocity υ (say). When the ball spins, the layer of air around it also moves with the ball, say with the velocity 'u'. So the resultant velocity of air above the ball becomes $(\upsilon - u)$ and below the ball becomes $(\upsilon + u)$. Hence, the pressure difference above and below the ball moves the ball in a curved path.

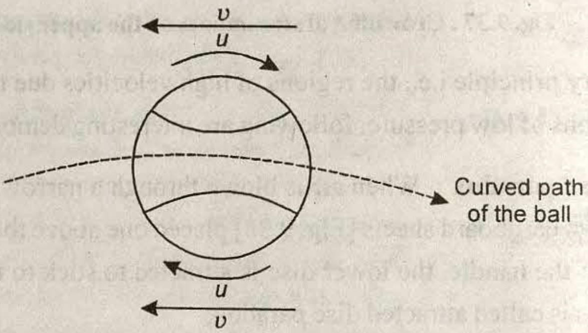


Fig. 9.41 : Swing of a cricket ball

Example 9.3: Water flows out of a small hole in the wall of a large tank near its bottom (Fig. 942). What is the speed of efflux of water when the height of water level in the tank is 2.5m?

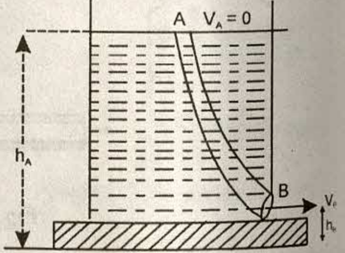


Fig. 9.42

Solution: Let B be the hole near the bottom. Imagine a tube of flow A to B for the water to flow from the surface point A to the hole B. We can apply the Bernoulli's theorem to the points A and B for the streamline flow of small mass m .

Total energy at B = Total energy at A

At A, $\upsilon_A = 0$, $p_A = p$ = atmospheric pressure, h = height above the ground.

At B, $\upsilon_B = \upsilon = ?$, $p_B = p$, h_B = height of the hole above the ground.

Let $h_A - h_B = H$ = height of the water level in the vessel = 2.5m

and d = density of the water.

Applying the Bernoulli's Principle and substituting the values we get,

$$\tfrac{1}{2} m\, \upsilon_B^2 = mg\,(h_A - h_B)$$

or

$$\upsilon_B = \sqrt{2g(h_A - h_B)}$$

$$= \sqrt{2 \times 9.8 \times 2.5}$$

$$= 7 \text{ m s}^{-1}$$

Intext Questions 9.5

1. The windstorm often blows off the tin roof of the houses, How does Bernoulli's equation explain the phenomenon?

....

2. When you press the mouth of a water pipe used for watering the plants, water goes to a longer distance, why?

....

3 What are the conditions necessary for the application of Bernoulli's theorem to solve the problems of flowing liquid?

....

4. Water flows along a horizontal pipe having non-uniform cross section. The pressure is 20 mm of mercury where the velocity is 0.20m/s. find the pressure at a point where the velocity is 1.50 m/s?

....

5. Why do bowlers in a cricket match shine only one side of the ball?

....

What You Have Learnt

- Hydrostatic pressure P at a depth h below the free surface of a liquid of density is given by

$$P = hdg$$

- The upward force acting on an object submerged in a fluid is known as buoyant force.
- According to Pascal's law, when pressure is applied to any part of an enclosed liquid, it is transmitted undiminished to every point of the liquid as well as to the walls of the container.
- The liquid molecules in the liquid surface have potential energy called surface energy.
- The surface tension of a liquid may be defined as force per unit length acting on a imaginary line drawn in the surface. It is measured in Nm^{-1}.
- Surface tension of any liquid is the property by virtue of which a liquid surface acts like a stretched membrane.
- Angle of contact is defined as the angle between the tangent to the liquid surface and the wall of the container at the point of contact as measured from within the liquid.
- The liquid surface in a capillary tube is either concave or convex. This curvature is

due to surface tension. The rise in capillary is given by

$$h = \frac{2T \cos \theta}{r d g}$$

- The excess pressure P on the concave side of the liquid surface is given by

  $P = \frac{2T}{R}$, where T is surface tension of the liquid

  $P = \frac{2T}{R}$, for air bubble in the liquid and

  $P = \frac{4T'}{r}$, where T' is surface tension of soap solution, for soap bubble in air

- Detergents are considered better cleaner of clothes because they reduce the surface tension of water-oil.
- The property of a fluid by virtue of which it opposes the relative motion between its adjacent layers is known as viscosity.
- The flow of liquid becomes turbulent when the velocity is greater than a certain value called critical velocity (υ_c) which depends upon the nature of the liquid and the diameter of the tube i.e. (η.P and d).
- Coefficient of viscosity of any liquid may be defined as the magnitude of tangential backward viscus force acting between two successive layers of unit area in contact with each other moving in a region of unit velocity gradient.
- Stokes' law states that tangential backward viscous force acting on a spherical mass of radius r falling with velocity 'υ' in a liquid of coefficient of viscosity η is given by

  $$F = 6\pi \eta r \upsilon.$$

- Bernoulli's theorem states that the total energy of an element of mass (m) of an incompressible liquid moving steadily remains constant throughout the motion. Mathematically, Bernoullis's equation as applied to any two points A and B of tube of flow

$$\frac{1}{2} m v_A^2 + m g h_A + \frac{mP_A}{d} = \frac{1}{2} m v_B^2 + m g h_B + \frac{mP_B}{d}$$

Terminal Exercises

1. Derive an expression for hyhostatic pressure due to a liquid column.
2. State pascal's law. Explain the working of hydraulic press.

3. Define surface tension. Find its dimensional formula.
4. Describe an experiment to show that liquid surfaces behave like a stretched membrane.
5. The hydrostatic pressure due to a liquid filled in a vessel at a depth 0.9 m is $3.0\ \text{N m}^2$. What will be the hydrostatic pressure at a hole in the side wall of the same vessel at a depth of 0.8 m.
6. In a hydraulic lift, how much weight is needed to lift a heavy stone of mass 1000 kg? Given the ratio of the areas of cross section of the two pistons is 5. Is the work output greater than the work input? Explain.
7. A liquid filled in a capillary tube has convex meniscus. If F_a is force of adhesion, F_c is force of cohesion and θ = angle of contact, which of the following relations should hold good?
   (a) $F_a > F_c \sin\theta$; (b) $F_a < F_c \sin\theta$; (c) $F_a \cos\theta = F_c$; (d) $F_a \sin\theta > F_c$
8. 1000 drops of water of same radius coalesce to form a larger drop. What happens to the temperature of the water drop? Why?
9. What is capillary action? What are the factors on which the rise or fall of a liquid in a capillary tube depends?
10. Calculate the approximate rise of a liquid of density 10^3 kg m^{-3} in a capillary tube of length 0.05 m and radius 0.2×10^{-3} m. Given surface tension of the liquid for the material of that capillary is 7.27×10^{-2} N m^{-1}.
11. Why is it difficult to blow water bubbles in air while it is easier to blow soap bubble in air?
12. Why the detergents have replaced soaps to clean oily clothes.
13. Two identical spherical balloons have been inflated with air to different sizes and connected with the help of a thin pipe. What do you expect out of the following observations?

    (i) The air from smaller balloon will rush into the bigger balloon till whole of its air flows into the later.

    (ii) The air from the bigger balloon will rush into the smaller balloon till the sizes of the two become equal.

    What will be your answer if the balloons are replaced by two soap bubbles of different sizes.
14. Which process involves more pressure to blow a air bubble of radius 3 cm inside a soap solution or a soap bubble in air? Why?
15. Differentiate between laminar flow and turbulent flow and hence define critical velocity.
16. Define viscosity and coefficient of viscosity. Derive the units and dimensional formula of coefficient of viscosity. Which is more viscous : water or glycerine? Why?
17. What is Reynold's number? What is its significance? Define critical velocity on the basis of Reynold's number.
18. State Bernoulli's principle. Explain its application in the design of the body of an aeroplane.

19. Explain Why :

    (i) A spinning tennis ball curves during the flight?

    (ii) A ping pong ball keeps on dancing on a jet of water without falling on to either side?

    (iii) The velocity of flow increases when the aperture of water pipe is decreased by squeezing its opening.

    (iv) A small spherical ball falling in a viscous fluid attains a constant velocity after some time.

    (v) If mercury is poured on a flat glass plate; it breaks up into small spherical droplets.

20. Calculate the terminal velocity of an air bubble with 0.8 mm in diameter which rises in a liquid of viscosity of 0.15 kg m^{-1} s^{-1} and density 0.9 g m^{-3}. What will be the terminal velocity of the same bubble while rising in water? For water $\eta = 10^{-2}$ kg m^{-1} s^{-1}.

21. A pipe line 0.2 m in diameter, flowing full of water has a constriction of diameter 0.1 m. If the velocity in the 0.2 m pipe-line is 2 m s^{-1}. Calculate

    (i) the velocity in the constriction, and

    (ii) the discharge rate in cubic meters per second.

22. (i) With what velocity in a steel ball 1 mm is radius falling in a tank of glycerine at an instant when its acceleration is one-half that of a freely falling body?

    (ii) What is the terminal velocity of the ball? The density of steel and of glycerine are 8.5 gm cm^{-3} and 1.32 g cm^{-3} respectively; viscosity of glycerine is 8.3 Poise.

23. Water at 20°C flows with a speed of 50 cm s^{-1} through a pipe of diameter of 3 mm.

    (i) What is Reynold's number?

    (ii) What is the nature of flow?

    Given, viscosity of water at 20°C as = 1.005×10^{-2} Poise; and
    Density of water at 20°C as = 1 g cm^{-3}.

24. Modern aeroplane design calls for a lift of about 1000 N m^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the velocity of flow past the lower wing surface is 100 m s^{-1}, what is the required velocity over the upper surface to give a desired lift of 1000 N m^{-2}? The density of air is 1.3 kg m^{-3}.

25. Water flows horizontally through a pipe of varying cross-section. If the pressure of water equals 5 cm of mercury at a point where the velocity of flow is 28 cm s^{-1}, then what is the pressure at another point, where the velocity of flow is 70 cm s^{-1}? [Tube density of water 1 g cm^{-3}].

Answers to Intext Questions

9.1

1. Because then the weight of the person applies on a larger area hence pressure on snow decreases.

2. $P = P_a + \rho gh$

   $P = 1.5 \times 10^7\ Pa$

3. Pressure applied by the weight of the boy = $\frac{2.5}{0.05}$ = 500 N m^{-2}.

   Pressure due to the weight of the elephant = $\frac{5000}{10}$ = 500 N m^{-2}.

   ∴ The boy can balance the elephant.

4. Because of the larger area of the rod, pressure on the skin is small.

5. $\frac{50}{0.1} = \frac{w}{10}$, w = 5000 kg wt.

9.2

1. Force of attraction between molecules of same substance is called force of cohesion and the force of attractive between molecules of different substance is called force of adhesion.

2. Surface tension leads to the minimum surface area and for a given volume, sphere has minimum surface area.

3. No, they have tightly bound molecules.

4. Due to surface tension forces.

5. For air bubble in water

$$P = \frac{2T}{r} = \frac{2\times 727\times10^{-3}}{2\times10^{-2}} = 72.7 \text{ N m}^{-2}.$$

   For soap bubble in air

$$P' = \frac{4T'}{r'} = \frac{4\times 25\times10^{-3}}{4\times10^{-2}} = 2.5 \text{ N m}^{-2}.$$

9.3

1. No.

2. Yes, the liquid will rise.

3. Mercury has a convex meniscus and the angle of contact is obtuse. The fall in the level of mercury in capillary makes it difficult to enter.

4. $r = \frac{2T}{h\rho g} = \frac{2\times7.2\times10^{-2}}{3\times1000\times10}$

   $= 4.8 \times 10^{-6}$m.

5. Due to capillary action.

9.4

1. If every particle passing through a given point of path follows the same line of flow as that of preceding particle the flow is stream lined, if its zig-zag, the flow is turbulent.
2. No, otherwise the same flow will have two directions.
3. Critical velocity depends upon the viscous nature of the liquid, the diameter of the tube and density of the liquid.
4. .012 ms^{-1}
5. Due to viscous force.

9.5

1. High velocity of air creates low pressure on the upper part.
2. Decreasing in the area creates large pressure.
3. The fluid should be incompressible and non-viscous on (very less). The motion should be steamlined.
4. $(P_1 - P_2) = \frac{1}{2} d (v_2^2 - v_1^2)$
5. So that the stream lines with the two surfaces are different. More swing in the ball will be obtained.

Answers to the Terminal Exercises

5. 2.67 N m^{-2}.

6. 200 N, No.

20. 2.1 mm s^{-1}, 35 cm s^{-1}.

21. 8 m s^{-1}, 6.3×10^{-2} m^3 s^{-1}.

22. 7.8 mm s^{-1}, 0.19 m s^{-1}.

23. 1500, Unsteady.

24. 2 cm of mercury.

SENIOR SECONDARY COURSE

PHYSICS

STUDENT'S ASSIGNMENT – 2

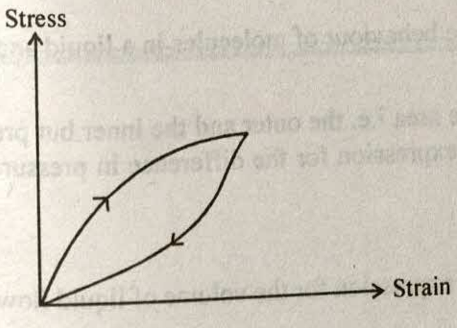
Maximum Marks: 50 **Time : $1\frac{1}{2}$ Hours**

INSTRUCTIONS

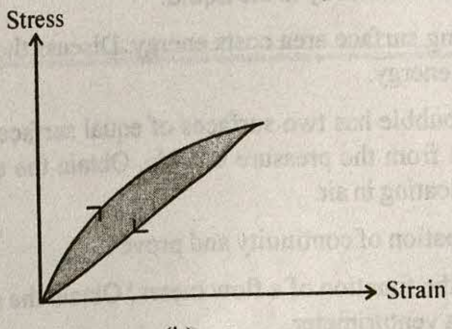
- Answer All the questions on a seperate sheet of paper
- Give the following information on your answer sheet:
  - Name
  - Enrolment Number
  - Subject
  - Assignment Number
  - Address
- Get your assignment checked by the subject teacher at your study centre so that you get positive feedback about your performance.

Do not send your assignment to NIOS

1. Stress-straits graph for two samples of rubber are shown in the figures given below. Which of the two will serve as better shock absorber? (1)



(a)



(b)

2. Two wires A and B having equal lengths and made of the same metal are subjected to equal loads. If extension in A is twice the extension in B what is the ratio of the radii of A and B. (1)
3. Why are the walls of a dam made thicker at the base? (1)
4. A balloon filled with helium gas does not rise in air indefinitely but halts after a certain height. Why? (1)
5. How does the viscosity of a gas change with increase in temperature of the gas? (1)
6. Which is more elastic iron or rubber? (1)

7. Is Surface tension dependent on the area of the surface? (1)

8. For what values of Raynold number is the flow of a fluid stream-lined. (1)

9. When solid rubber ball is taken from the surface to bottom of a lake the reduction in its volume is 0.0012%. The depth of the lake is 0.360 *km*, density of water is $1 g\ cm^{-3}$ and acceleration due to gravity is $10\ N\ kg^{-1}$. Calculate bulk modulus of rubber. [Ans : $3 \times 10^{11} N\ m^{-2}$] (2)

10. Show the variation of stress with strain when a metallic wire of uniform cross.section is subjected to an increasing load. (2)

11. Explain why the detergents should have small angle of contact. (2)

12. A 40 kg girl, wearing high heel shoes, balances on a single heel which is circular and has a diameter 10 mm. What is the pressure exerted by the heel on the floor? (2)

13. (*i*) Why does a spinning cricket ball in air not follow a parabolic trajectory?

    (*ii*) Discuss the magnus effect. (2 + 2)

14. State Bernoulli's principle.

    A fully loaded aircraft has a mass 330 tonnes and total wing area 500 m^2. It is in level flight with a speed of 960 km h^{-1}. Estimate the pressure difference between the lower and upper surfaces of the wings. Also estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. The density of air is 1.2 kg m^{-3}.

    Hint: $$\left\{\begin{matrix} \Delta P' = \dfrac{F}{A} = 6.5 \times 10^{3}\ Nm^{-2}, \dfrac{V_2 - V_1}{V_{av}} = \dfrac{\Delta P}{P\bar{v}^2} = 0.08 \\ V_2 - V_1 = \dfrac{2\Delta P}{P(V_2 + V_1)} \end{matrix}\right\}$$ (4)

15. A smooth spherical body of density(ρ) and radius(*r*), falling freely in a highly viscous liquid of density σ and coefficient of viscosity(η) with a velocity (*υ*), state the law for the magnitude of the tangential backward viscous force (F) acting on the body. Obtain the expression for the constant velocity acquired by the spherical body in the liquid. (4)

16. Increasing surface area costs energy. Discuss the behaviour of molecules in a liquid and hence explain surface energy. (4)

17. A soap bubble has two surfaces of equal surface area i.e. the outer and the inner but pressure inside is different from the pressure outside. Obtain the expression for the difference in pressure inside a soap bubble floating in air. (4)

18. State equation of continuity and prove it. (4)

19. What is the function of a flow meter? Obtain the expression for the volume of liquid flowing per second through a venturimeter. (5)

20. State three assumptions required to develop Bernoulli's equation. Show that pressure energy, kinetic energy, and potential energy per unit Volume of a fluid remains constant in a stream-line motion. (5)

    or

    If a capillary tube is dipped in water what do you observe? What do you call this phenomenon? Obtain the expression for this phenomenon relating the symbols T, *r*, *h*, θ, *f* and *g* where symbols have their usual meaning. Also discuss what would happen if the thin tube of uniform bore immersed in water is of insufficient length.

MODULE - III
THERMAL PHYSICS

10. Kinetic Theory of Gases
11. Thermodynamics
12. Heat Transfer and Solar Energy

10

KINETIC THEORY OF GASES

As you have studied in the previous lessons, at standard temperature and pressure, matter exists in three states – solid, liquid and gas. These are composed of atoms/molecules which are held together by intermolecular forces. At room temperature, these atoms/ molecules have finite thermal energy. If thermal energy increases, molecules begin to move more freely. This state of matter is said to be the gaseous state. In this state, intermolecular forces are very weak and very small compared to their kinetic energy.

Under different conditions of temperature, pressure and volume, gases exhibit different properties. For example, when the temperature of a gas is increased at constant volume, its pressure increases. In this lesson you will learn the kinetic theory of gases which is based on certain simplifying assumptions. You will also learn the kinetic interpretation of temperature and its relationship with the kinetic energy of the molecules. Why the gases have two types of heat capacities will also be explained in this lesson.

Objectives

After studying this lesson, you should be able to :

- *state the assumptions of kinetic theory of gases;*
- *derive the expression for pressure* $P = \frac{1}{3}\rho\overline{c^2}$ *;*
- *explain how rms velocity and average velocity are related to temperature;*
- *derive gas laws on the basis of kinetic theory of gases;*
- *give kinetic interpretation of temperature and compute the mean kinetic energy of a gas;*
- *explain the law of equipartition of energy;*
- *explain why a gas has two heat capacities; and*
- *derive the relation* $c_p - c_V = R/J$.

10.1 Kinetic Theory of Gases

You now know that matter is composed of very large number of atoms and molecules. Each of these molecules shows the characteristic properties of the substance of which it is a part. Kinetic theory of gases attempts to relate the macroscopic or bulk properties such as pressure, volume and temperature of an ideal gas with its microscopic properties such as speed and mass of its individual molecules. The kinetic theory is based on certain assumptions. (A gas whose molecules can be treated as point masses and there is no intermolecular force between them is said to be ideal.) A gas at room temperature and atmospheric pressure (low pressure) behaves like an ideal gas.

10.1.1 Assumptions of Kinetic Theory of Gases

Clark Maxwell in 1860 showed that the observed properties of a gas can be explained on the basis of certain assumptions about the nature of molecules, their motion and interaction between them. These resulted in considerable simplification. We now state these.

(i) A gas consists of a very large number of identical rigid molecules, which move with all possible velocities randomly. The intermolecular forces between them are negligible.

(ii) Gas molecules collide with each other and with the walls of the container. These collisions are perfectly elastic.

(iii) Size of the molecules is negligible compared to the separation between them.

(iv) Between collisions, molecules move in straight lines with uniform velocities.

(v) Time taken in a collision is negligible as compared to the time taken by a molecule between two successive collisions.

(vi) Distribution of molecules is uniform throughout the container.

To derive an expression for the pressure exerted by a gas on the walls of the container, we consider the motion of only one molecule because all molecules are identical (Assumption i). Moreover, since a molecule moving in space will have velocity components along *x*, *y* and *z*–directions, in view of assumption (vi)it is enough for us to consider the motion only along one dimension, say *x*-axis.(Fig. 10.1). Note that if there were $N(= 6 \times 10^{26}$ molecules $\text{m}^{-3})$, instead of considering $3N$ paths, the assumptions have reduced the roblem to only one molecule in one dimension. Let us consider a molecule having velocity C in the face LMNO. Its *x*, *y* and z components are u, v and w, respectively. If the mass of the molecule is m and it is moving with a speed *u along x–axis*, its momentum will be mu towards the wall and normal to it. On striking the wall, this molecule will rebound in the opposite direction with the same speed u, since the collision has been assumed to be perfectly elastic (Assumption ii). The momentum of the molecule after it rebounds is $(-mu)$. Hence, the change in momentum of a molecule is

$$mu - (-mu) = 2mu$$

If the molecule travels from face LMNO to the face ABCD with speed u along *x–axis* and rebounds back without striking any other molecule on the way, it covers a distance $2l$ in time $2l/u$. That is, the time interval between two successive collisions of the molecules with the wall is $2l/u$.

According to Newton's second law of motion, the rate of change of momentum is equal to the impressed force. Therefore

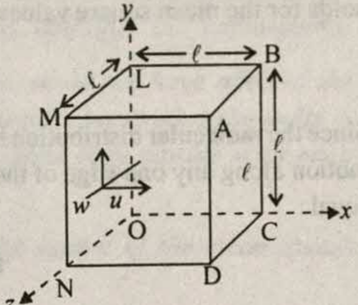


Fig. 10.1 : **Motion of a molecule in a container**

$$\text{Rate of change of momentum at ABCD} = \frac{\text{Change in momentum}}{\text{Time}}$$

$$= \frac{2mu}{2l/u} = \frac{mu^2}{l}$$

This is the rate of change of momentum of one molecule. Since there are N molecules of the gas, the total rate of change of momentum or the total force exerted on the wall ABCD due to the impact of all the N molecules moving along x-axis with speeds, $u_1, u_2, ..., u_N$ is given by

$$\text{Force on ABCD} = \frac{m}{l}\left(u_1^2 + u_2^2 + u_3^2 + ... + u_N^2\right)$$

We know that pressure is force per unit area. Therefore, the pressure P exerted on the wall ABCD of areas l^2 by the molecules moving along x-axis is given by

$$\text{P} = \frac{\frac{m}{l}\left(u_1^2 + u_2^2 + ... + u_N^2\right)}{l^2}$$

$$= \frac{m}{l^3}\left(u_1^2 + u_2^2 + ... + u_N^2\right) \tag{10.1}$$

If $\overline{u^2}$ represents the mean value of the squares of all the speed components along x-axis, we can write

$$\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + ... + u_N^2}{N}$$

or

$$N\overline{u^2} = u_1^2 + u_2^2 + u_3^2 + ... + u_N^2$$

Substituting this result in Eqn. (10.1), we get

$$\text{P} = \frac{Nm\overline{u^2}}{\ell^3} \tag{10.2}$$

It can be shown by geometry that

$$c^2 = u^2 + v^2 + w^2$$

since u, v and w are components of c along the three orthogonal axes. This relation also holds for the mean square values, i.e.

$$\overline{c^2} = \overline{u^2} + \overline{\upsilon^2} + \overline{w^2}$$

Since the molecular distribution has been assumed to be isotropic, there is no preferential motion along any one edge of the cube. This means that the mean value of u^2, υ^2, w^2 are equal :

$$\overline{u^2} = \overline{\upsilon^2} = \overline{w^2}$$

so that $$\overline{u^2} = \frac{\overline{c^2}}{3}$$

Substituting this result in Eqn. (10.2), we get

$$P = \frac{1}{3}\frac{Nm}{\ell^3}\overline{c^2}$$

But l^3 defines the volume V of the container or the volume of the gas. Hence, we get

$$PV = \frac{1}{3}Nm\overline{c^2} = \frac{1}{3}M\overline{c^2} \qquad (10.3)$$

Note that the left hand side has macroscopic properties i.e. pressure and volume and the right hand side has only microscopic properties i.e. mass and mean square speed of the molecules.

Eqn (10.3) can be re-written as

$$P = \frac{1}{3}\frac{Nm}{V}\overline{c^2}$$

If $\rho = \frac{mN}{V}$ is the density of the gas, we can write

$$P = \frac{1}{3}\rho\overline{c^2}$$

or $$\overline{c^2} = \frac{3P}{\rho} \qquad (10.4)$$

If we denote the ratio N/V by number density n, Eqn. (10.3) can also be expressed as

$$P = \frac{1}{3}mn\overline{c^2} \qquad (10.3a)$$

The following points about the above derivation should be noted:

(i) *From Eqn. (10.4) it is clear that the shape of the container does not play any* *role in kinetic theory; only volume is of significance. Instead of a cube we could have taken any other container. A cube only simplifies our calculations.*

(ii) *We ignored the intermolecular collisions but these would not have affected the result, because, the average momentum of the molecules on striking the walls is unchanged by their collision; same is the cose when they collide with each other.*

(iii) *The mean square speed* $\overline{c^2}$ *is not the same as the square of the mean speed.* This is illustrated by the following example.

Suppose we have five molecules and their speeds are 1, 2, 3, 4, 5 units, respectively. Then their mean speed is

$$\frac{1+2+3+4+5}{5} = 3 \text{ units}$$

Its square is 9 (nine).

On the other hand, the mean square speed is

$$\frac{1^2+2^2+3^2+4^2+5^2}{5} = \frac{55}{5} = 11$$

Thus we see that mean square speed is not the same as square of mean speed.

Example 10.1 : Calculate the pressure exerted by 10^{22} molecules of oxygen, each of mass 5×10^{-26} kg, in a hollow cube of side 10 cm where the average translational speed of molecule is 500 m s^{-1}.

Solution : Change in momentum $2mu = 2 \times (5 \times 10^{-26} \text{ kg}) \times (500 \text{ ms}^{-1})$

$$= 5 \times 10^{-23} \text{ kg ms}^{-1}.$$

Time taken to make successive impacts on the same face is equal to the time spent in travelling a distance of 2×10 cm or 2×10^{-1} m. Hence

$$\text{Time} = \frac{2 \times 10^{-2} \text{ m}}{500 \text{ ms}^{-1}} = 4 \times 10^{-4} \text{ s}$$

$$\therefore \quad \text{Rate of change of momentum} = \frac{5 \times 10^{-23} \text{ kg ms}^{-1}}{4 \times 10^{-4} \text{ s}} = 1.25 \times 10^{-19} \text{ N}$$

The force on the side due to one third molecules

and
$$f = \frac{1}{3} \times 1.25 \times 10^{-19} \times 10^{22} \not\simeq 416.7 \text{ N}$$

$$\text{pressure} = \frac{\text{Force}}{\text{Area}} = \frac{417 \text{ N}}{100 \times 10^{-4} \text{ m}^2}$$

$$= 4.2 \times 10^{-4} \text{ N m}^{-2}$$

Notes

Intext Questions 10.1

1. (i) A gas fills a container of any size but a liquid does not. Why?

   (ii) Solids have more ordered structure than gases. Why?

   ....

2. What is an ideal gas?

   ....

3. How is pressure related to density of molecules?

   ....

10.2 Kinetic Interpretation of Temperature

From Eqn. (10.3) we recall that

$$P\,V = \frac{1}{3} m N \overline{c^2}$$

Also, for n moles of a gas, the equation of state is $PV = n\,RT$, where gas constant R is equal to 8.3 J mol^{-1} K^{-1}. On combining this result with the expression for pressure, we get

$$n\,R\,T = \frac{1}{3} m N \overline{c^2}$$

Multiplying both sides by $\frac{3}{2n}$ we have

$$\frac{3}{2} R\,T = \frac{1}{2}\frac{Nm\overline{c^2}}{n} = \frac{1}{2} m N_A \overline{c^2}$$

where $\frac{N}{n} = N_A$ is Avogadro's number. It denotes the number of atoms or molecules in one mole of a substance. Its value is 6.023×10^{23} per gram mole. In terms of N_A, we can write

$$\frac{3}{2}\left(\frac{R}{N_A}\right) T = \frac{1}{2} m \overline{c^2}$$

But $\frac{1}{2} m \overline{c^2}$ is the mean kinetic energy of a molecule. Therefore, we can write

$$\frac{1}{2} m \overline{c^2} = \frac{3}{2}\left(\frac{R}{N_A}\right) T = \frac{3}{2} k\,T \qquad (10.5)$$

where

$$k = \frac{R}{N_A} \qquad (10.6)$$

is ***Boltzmamn constant.*** The value of k is 1.38×10^{-23} J K^{-1}.

In terms of k, the mean kinetic energy of a molecule of the gas is given as

$$\bar{\varepsilon} = \frac{1}{2} m \overline{c^2} = \frac{3}{2} k T \qquad (10.7)$$

Hence, kinetic energy of a gram mole of a gas is $\frac{3}{2}RT$

This relationship tells us that the kinetic energy of a molecule depends only on the absolute temperature T of the gas and it is quite independent of its mass. This fact is known as the ***kinetic interpretation of temperature.***

Clearly, at T = 0, the gas has no kinetic energy. In other words, all molecular motion ceases to exist at absolute zero and the molecules behave as if they are frozen in space. According to modern concepts, the energy of the system of electrons is not zero even at the absolute zero. The energy at absolute zero is known as *zero point energy*.

From Eqn.(10.5), we can write the expression for the square root of $\overline{c^2}$, called **root mean square speed :**

$$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$$

This expression shows that at any temperature T, the c_{rms} is inversely proportional to the square root of molar mass. It means that lighter molecule, on an average, move faster than heavier molecules. For example, the molar mass of oxygen is 16 times the molar mass of hydrogen. So according to kinetic theory, the hydrogen molecules should move 4 times faster then oxygen molecules. It is for this reason that lighter gases are in the above part of our atmosphere. This observed fact provided an early important evidence for the validity of kinetic theory.

10.3 Deduction of Gas Laws from Kinetic Theory

(i) Boyle's Law

We know that the pressure P exerted by a gas is given by Eqn. (11.3) :

$$PV = \frac{1}{3} M \overline{c^2}$$

When the temperature of a given mass of the gas is constant, the mean square speed is constant. Thus, both M and $\overline{c^2}$ on the right hand side of Eqn. (10.3) are constant. Thus, we can write

$$PV = \text{Constant} \qquad (10.9)$$

This is Boyle's law, which states that ***at constant temperature, the pressure of a given mass of a gas is inversely proportional to the volume of the gas.***

(ii) Charle's Law

From Eqn. (10.3) we know that

$$PV = \frac{1}{3} M \overline{c^2}$$

or

$$V = \frac{1}{3} \frac{M}{P} \overline{c^2}$$

i.e, $V \propto \overline{c^2}$, if M and P do not vary or M and P are constant. But $\overline{c^2} \propto T$

$$\therefore \qquad V \propto T \qquad (10.9)$$

This is Charle's law : The volume of a given mass of a gas at constant pressure is directly proportional to temperature.

Robert Boyle
(1627 – 1691)

British experimentalist Robert Boyle is famous for his law relating the pressure and volume of a gas (PV = constant). Using a vacuum pump designed by Robert Hook, he demonstrated that sound does not travel in vacuum. He proved that air was required for burning and studied the elastic properties of air.

A founding fellow of Royal Society of London, Robert Boyle remained a bachalor throughout his life to pursue his scientific interests. Crater Boyle on the moon is named in his honour.

(iii) Gay Lussac's Law – According to kinetic theory of gases, for an ideal gas

$$P = \frac{1}{3} \frac{M}{V} \overline{c^2}$$

For a given mass (M constant) and at constant volume (V constant),

$$P \propto \overline{c^2}$$

But

$$\overline{c^2} \propto T$$

$$\therefore \qquad P \propto T \qquad (10:11)$$

which is Gay Lussac's law. It states that **the pressure of a given mass of a gas is directly proportional to its absolute temperature T, if its volume remains constant.**

(iv) Avogadro's Law

Let us consider two different gases 1 and 2. Then from Eqn. (10.3), we recall that

$$P_1 V_1 = \frac{1}{3} m_1 N_1 \overline{c_1^2}$$

and

$$P_2 V_2 = \frac{1}{3} m_2 N_2 \overline{c_2^2}$$

If their pressure and volume are the same, we can write

$$P_1 V_2 = P_2 V_2$$

Hence

$$\frac{1}{3} m_1 N_1 \overline{c_1^2} = \frac{1}{3} m_2 N_2 \overline{c_2^2}$$

Since the temperature is constant, their kinetic energies will be the same, i.e.

$$\frac{1}{2} m_1 \overline{c_1^2} = \frac{1}{2} m_2 \overline{c_2^2}$$

Using this result in the above expression, we get $N_1 = N_2$ (10.12)

That is, ***equal volume of ideal gases under the same conditions of temperature and pressure contain equal number of molecules. This statement is Avogadro's Law.***

(v) Dalton's Law of Partial Pressure

Suppose we have a number of gases or vapours, which do not react chemically. Let their densities be ρ_1, ρ_2, ρ_3 ... and mean square speeds $\overline{c_1^2}, \overline{c_2^2}, \overline{c_3^2}$ ... respectively. We put these gases in the same enclosure. They all will have the same volume. Then the resultant pressure P will be given by

$$P = \frac{1}{3}\rho_1 \overline{c_1^2} + \frac{1}{3}\rho_2 \overline{c_2^2} + \frac{1}{3}\rho_3 \overline{c_3^2} + \ldots$$

Here $\frac{1}{3}\rho_1 \overline{c_1^2}, \frac{1}{3}\rho_2 \overline{c_2^2}, \frac{1}{3}\rho_3 \overline{c_3^2}$ ... signify individual (or partial) pressures of different gases or vapours. If we denote these by P_1, P_2, P_3, respectively we get

$$P = P_1 + P_2 + P_3 + \ldots. \qquad (10.13)$$

In other words, ***the total pressure exerted by a gaseous mixture is the sum of the partial pressures that would be exerted, if individual gases occupied the space in turn. This is Dalton's law of partial pressures.***

(vi) Graham's law of diffusion of gases

Graham investigated the diffusion of gases through porous substances and found that ***the rate of diffusion of a gas through a porous partition is inversely proportional to the square root of its density. This is known as Graham's law of diffusion.***

On the basis of kinetic theory of gases, the rate of diffusion through a fine hole will be proportional to the average or root mean square velocity c_{rms}. From Eqn. (10.4) we recall that

$$\overline{c^2} = \frac{3P}{\rho}$$

or

$$\sqrt{\overline{c^2}} = c_{rms} = \sqrt{\frac{3P}{\rho}}$$

That is, the root mean square velocities of the molecules of two gases of densities ρ_1 and ρ_2 respectively at a pressure P are given by

$$(c_{rms})_1 = \sqrt{\frac{3P}{\rho_1}} \quad \text{and} \quad (c_{rms})_2 = \sqrt{\frac{3P}{\rho_2}}$$

Thus,

$$\frac{\text{Rate of diffusion of one gas}}{\text{Rate of diffusion of other gas}} = \frac{(c_{rms})_1}{(c_{rms})_2} = \sqrt{\frac{\rho_2}{\rho_1}} \qquad (10.14)$$

Thus, rate of diffusion of gases is inversely proportional to the square root of their densities at the same pressure, which is Graham's law of diffusion.

Example 10.2 : Calculate is the root mean square speed of hydrogen molecules at 300 k. Take m(H_2) as 3.347×10^{-27} kg and $k = 1.38 \times 10^{-23}$ J mol^{-1} K^{-1}

Solution : We know that

$$c_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\,'\,(1.38\,'\,10^{-27}\,\mathrm{J\,K^{-1}})(300\,\mathrm{K})}{3.347\,'\,10^{-27}\,\mathrm{kg}}}$$

$$= 1927 \text{ m s}^{-1}$$

Intext Questions 10.2

1. Five gas molecules chosen at random are found to have speeds 500 ms^{-1}, 600 ms^{-1}, 700 ms^{-1}, 800 ms^{-1}, and 900 ms^{-1}. Calculate their RMS speed.

....

2. If equal volumes of two non–reactive gases are mixed, what would be the resultant pressure of the mixture?

....

3. When we blow air in a balloon, its volume increases and the pressure inside is also more than when air was not blown in. Does this situation contradict Boyle's law?

....

Example 10.3 : At what temperature will the root mean square velocity of hydrogen be double of its value at S.T.P., pressure being constant (STP = Standard temperature and pressure).

Solution : From Eqn. (10.8), we recall that

$$c_{rms} \propto \sqrt{T}$$

Let the rms velocity at S.T.P. be c_0.

If T K is the required temperature, the velocity $c = 2\,c_0$ as given in the problem

$$\therefore \qquad \frac{c}{c_0} = \frac{2c_0}{c_0} = \sqrt{\frac{T}{T_0}}$$

Squaring both sides, we get

$$4 = \frac{T}{T_0}$$

or

$$T = 4T_0$$

Since $T_0 = 273$K, we get

$$T = 4 \times 273\text{K} = 1092\text{K} = 819^{\circ}\text{C}$$

Example 10.4 : Calculate the average kinetic energy of a gas at 300 K. Given $k = 1.38 \times 10^{-23}$ JK^{-1}.

Solution : We know that

$$\frac{1}{2} M \bar{c^2} = \frac{3}{2} \text{k T}$$

Since $k = 1.38 \times 10^{-23}$ J K^{-1} and T= 300 K, we get

$$\therefore \qquad \bar{E} = \frac{3}{2}\,(1.38 \times 10^{-23}\,\text{J K}^{-1})\,(300\text{ K})$$

$$= 6.21 \times 10^{-21}\text{ J}$$

10.4.1 The Law of Equipartition of Energy

We now know that kinetic energy of a molecule of a gas is given by $\frac{1}{2} m\bar{c^2} = \frac{3}{2}\text{kT}$.

Since the motion of a molecule can be along *x*, *y*, and *z* directions equally probably, the average value of the components of velocity *c* (i.e., *u*, *v* and *w*) along the three directions should be equal. That is to say, for a molecule all the three directions are equivalent :

$$\bar{u} = \bar{v} = \bar{w}$$

and

$$\bar{u^2} = \bar{v^2} = \bar{w^2} = \frac{1}{3}\,\bar{c^2}$$

Since

$$c^2 = u^2 + \bar{v}^2 + w^2$$

Notes

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

Multiplying throughout by $\frac{1}{2}m$, where m is the mass of a molecule, we have

$$\frac{1}{2}m\,\overline{u^2} = \frac{1}{2}m\,\overline{v^2} = \frac{1}{2}m\,\overline{w^2}$$

But $\frac{1}{2}m\,\overline{u^2}$ = E = total mean kinetic energy of a molecule along x–axis. Therefore,

$E_x = E_y = E_z$. But the total mean kinetic energy of a molecule is $\frac{3}{2}$ k T. Hence, we get an important result :

$$E_x = E_y = E_z = \frac{1}{2}\text{k T}$$

Since three velocity components u, v and w correspond to the three degree of freedom of the molecule, we can conclude that total kinetic energy of a dynamical system is equally divided among all its degrees of freedom and it is equal to $\frac{1}{2}$ k T for each degree of freedom. This is the law of equipartition of energy and was deduced by Ludwing Boltzmann. Let us apply this law for different types of gases.

So far we have been considering only translational motion. For a ***monoatomic molecule***, we have only translational motion because they are not capable of rotation (although they can spin about any one of the three mutually perpendicular axes if it is like a finite sphere). Hence, for one molecule of a ***monoatomic gas***, total energy

$$E = \frac{3}{2}\text{k T} \qquad (10.15)$$

A ***diatomic molecule*** can be visualised as if two spheres are joined by a rigid rod. Such a molecule can rotate about any one of the three mutually perpendicular axes. However, the rotational inertia about an axis along the rigid rod is negligible compared to that about an axis perpendicular to the rod. It means that rotational energy consists of two terms such as $\frac{1}{2}\text{I}\,\omega_y^2$ and $\frac{1}{2}\text{I}\,\omega_z^2$.

Now the special description of the centre of mass of a diatomic gas molecules will require three coordinates. Thus, for a diatomic gas molecule, both rotational and translational motion are present but it has 5 degrees of freedom. Hence

$$E = 3\left(\frac{1}{2}\text{kT}\right) + 2\left(\frac{1}{2}\text{kT}\right)$$

$$= \frac{5}{2}\text{k T} \qquad (10.16)$$

Ludwing Boltzmann
(1844 – 1906)

Born and brought up in Vienna (Austria), Boltzmann completed his doctorate under the supervision of Josef Stefan in 1866. He also worked with Bunsen, Kirchhoff and Helmholtz. A very emotional person, he tried to commit suicide twice in his life and succeeded in his second attempt. The cause behind these attempts, people say, were his differences with Mach and Ostwald.

He is famous for his contributions to kinetic theory of gases, statistical mechanics and thermodynamics. Crater Bolzmann on moon is named in his memory and honour.

10.5 Heat Capacities of Gases

We know that the temperature of a gas can be raised under different conditions of volume and pressure. For example, the volume or the pressure may be kept constant or both may be allowed to vary in some arbitrary manner. In each of these cases, the amount of thermal energy required to increase unit rise of temperature in unit mass is different. Hence, we say that a gas has two different heat capacities.

If we supply an amount of heat ΔQ to a gas to raise its temperature through ΔT, the heat capacity is defined as

$$\text{Heat capacity} = \frac{\Delta Q}{\Delta T}$$

The heat capacity of a body per unit mass of the body is termed as ***specific heat capacity*** of the substance and is usually denoted by c. Thus

$$\text{Specific heat capacity, c} = \frac{\text{heat capacity}}{m} \qquad (10.17)$$

Eqns. (10.16) and (10.17) may be combined to get

$$c = \frac{\Delta Q}{m\,\Delta T} \qquad (10.18)$$

Thus, specific heat capacity of a material is the heat required to raise the temperature of its unit mass by 1 °C (or 1 K).

The SI unit of specific heat capacity is kilo calories per kilogram per kelvin (kcal $kg^{-1}K^{-1}$). It may also the expressed in joules per kg per K. For example the specific heat capacity of water is

$$1 \text{ kilo cal } kg^{-1}\,K^{-1} = 4.2 \times 10^3\ J\ kg^{-1}\,K^{-1}.$$

The above definition of specific heat capacity holds good for solids and liquids but not for gases, because it can vary with external conditions. In order to study the heat capacity of a gas, we keep the pressure or the volume of a gas constant. Consequently, we define two specific heat capacities :

(i) Specific heat at constant volume, denoted as c_v.

(ii) Specific heat at constant pressure, denoted as c_p.

(a) ***The specific heat capacity of a gas at constant volume*** (c_v) is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1K, when its volume is kept constant :

$$c_v = \left(\frac{\Delta Q}{\Delta T}\right)_V \tag{10.19}$$

(b) ***The specific heat capacity of a gas at constant pressure*** (c_p) is defined as the amount of heat required to raise the temperature of unit mass of a gas through 1K when its pressure is kept constant.

$$c_p = \left(\frac{\Delta Q}{\Delta T}\right)_P \tag{10.20}$$

When 1 mole of a gas is considered, we define ***molar heat capacity.***

We know that when pressure is kept constant, the volume of the gas increases. Hence in the second case note that the heat required to raise the temperature of unit mass through 1 degree at constant pressure is used up in two parts :

(i) heat required to do external work to produce a change in volume of the gas, and

(ii) heat required to raise the temperature of the gas through one degree (c_v).

This means the specific heat capacity of a gas at constant pressure is greater than its specific heat capacity at constant volume by an amount which is thermal equivalent of the work done in expending the gas against external pressure. That is

$$\boxed{c_p = W + c_v} \tag{10.21}$$

10.6 Relation between c_p and c_v

Let us consider one mole of an ideal gas enclosed in a cylinder fitted with a frictionless movable piston (Fig. 10.2). Since the gas has been assumed to be ideal (perfect), there is no intermolecular force between its molecules. When such a gas expands, some work is done in overcoming internal pressure.

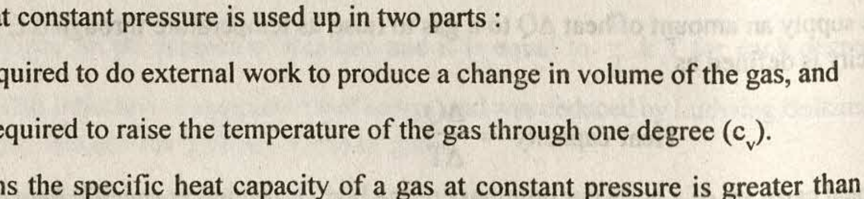


Fig.10.2 : Gas heated at constant pressure

Let P be the external pressure and *A* be the cross sectional area of the piston. The force acting on the piston = P × *A*. Now suppose that the gas is heated at constant pressure by 1K and as a result, the piston moves outward through a distance *x*, as shown in Fig. 10.2.

Let V_1 be the initial volume of the gas and V_2 be the volume after heating. Therefore, the work W done by the gas in pushing the piston through a distance x, against external pressure P is given by

$$W = P \times A \times x$$

$$= P \times (\text{Increase in volume})$$

$$= P\,(V_2 - V_1)$$

We know from Eqn. (10.22) that $c_p - c_v$ = Work done (W) against the external pressure in raising the temperature of 1 mol of a gas through 1 K, i.e.

$$c_p - c_v = P\,(V_2 - V_1) \qquad (10.22)$$

Now applying perfect gas equation to these two stages of the gas i.e. before and after heating, we have

$$PV_1 = RT \qquad (10.23)$$

$$PV_2 = R\,(T + 1) \qquad (10.24)$$

Substracting Eqn. (10.23) from Eqn.(10.24), we get

$$P\,(V_2 - V_1) = R \qquad (10.25)$$

Hence from Eqns. (10.19) and (10.22) we get

$$c_p - c_v = R \qquad (10.26)$$

where R is in J mol^{-1} K^{-1}

Converting joules into calories, we can write

$$c_p - c_v = \frac{R}{J} \qquad (10.27)$$

where J = 4.18 cal is the mechanical equivalent of heat.

Example 10.5 : Calculate the value of c_p and c_v for a monoatomic, diatomic and triatomic gas molecules.

Solution : We know that the average KE for 1 mol of a gas is given as

$$E = \frac{3}{2} R\,T$$

Now c_v is defined as the heat required to raise the temperature of 1 mole of a gas at constant volume by one degree i.e. if E_T denotes total energy of gas at T K and E_{T+1} signifies total energy of gas at (T + 1) K, then $c_v = E_{T+1} - E_T$.

(i) We know that for monoatomic gas, total energy $= \frac{3}{2}\,R\,T$

Notes

$$\therefore \quad \text{monoatomic gas } c_V = \frac{3}{2}R(T+1) - \frac{3}{2}RT = \frac{3}{2}R.$$

Hence $$c_p = c_V + R = \frac{3}{2}R + R = \frac{5}{2}R.$$

(ii) For diatomic gases, total energy $= \frac{5}{2}RT$

$$\therefore \quad c_V = \frac{5}{2}R(T+1) - R\frac{5}{2}RT = \frac{5}{2}R$$

$$c_p = c_V + R = \frac{5}{2}R + R = \frac{7}{2}R.$$

(iii) You should now find out c_v and c_p for triatomic gas.

Intext Questions 10.3

1. What is the total energy of a nitrogen molecule?

....

2. Calculate the value of c_p and c_V for nitrogen (given, $R = 8.3 \text{J mol}^{-1} \text{K}^{-1}$).

....

3. Why do gases have two types of specific heat capacities?

....

Brownian Motion and Mean Free Path

Scottish botanist Robert Brown, while observing the pollen grains of a flower suspended in water, under his microscope, found that the pollen grains were tumbling and tossing and moving about in a zigzag random fashion. The random motion of pollen grains, was initially attributed to live objects. But when motion of pollens of dead plants and particles of mica and stone were seen to exhibit the same behaviour, it became clear that the motion of the particles, now called **Brownian motion,** was caused by unbalanced forces due to impacts of water molecules. Brownian motion provided a direct evidence in favour of kinetic theory of matter. The Brownian displacement was found to depend on.

(i) Size of the particles of the suspension – smaller the particles, more the chances of inbalanced impacts and more pronounced the Brownian motion.

(ii) The Brownian motion also increases with the increase in the temperature and decreases with the viscosity of the medium.

Due to mutual collisions, the molecules of a fluid also move on zig-zag paths. The average distance between two successive collisions of the molecules is called mean free path. The mean free path of a molecule is given by

$$\sigma = \frac{1}{\sqrt{2}\, n \pi d^2}$$

where n is the number density and d the diameter of the molecules.

What You Have Learnt

- Kinetic theory assumes the existence of atoms and molecules of a gas and applies the law of mechanics to large number of them using averaging technique.
- Kinetic theory relates macroscopic properties to microscopic properties of individual molecules.
- The pressure of a gas is the average impact of its molecules on the unit area of the walls of the container.
- Kinetic energy of a molecule depends on the absolute temperature T and is independent of its mass.
- At absolute zero of temperature, the kinetic energy of a gas is zero and molecular motion ceases to exist.
- Gas law can be derived on the basis of kinetic theory. This provided an early evidence in favour of kinetic theory.
- Depending on whether the volume or the pressure is kept constant, the amount of heat required to raise the temperature of unit mass of a gas by 1°C is different. Hence there are two specific heats of gas :

  i) Specific heat capacity at constant volume (c_V)

  ii) Specific heat capacity at constant pressure (c_p)

  These are related as $c_p = W + c_V$

  $$c_p - c_V = \frac{R}{J}$$

- The law of equipartition of the energy states that the total kinetic energy of a dynamical system is distributed equally among all its degrees of freedom and it is equal to $\frac{1}{2}$ k T per degree of freedom.
- Total energy for a molecule of (i) a monatomic gas is $\frac{3}{2}$ k T, (ii) a diatomic gas is $\frac{5}{2}$, and (iii) a triatomic gas is 3 k T.

Notes

Terminal Exercise

1. Can we use Boyle's law to compare two different ideal gases?

2. What will be the velocity and kinetic energy of the molecules of a substance at absolute zero temperature?

3. If the absolute temperature of a gas is raised four times, what will happen to its kinetic energy, root-mean square velocity and pressure?

4. What should be the ratio of the average velocities of hydrogen molecules (molecular mass = 2) and that of oxygen molecules (molecular mass = 32) in a mixture of two gases to have the same kinetic energy per molecule?

5. If three molecules have velocities 0.5, 1 and 2 km s^{-1} respectively, calculate the ratio between their root mean square and average speeds.

6. Explain what is meant by the root-mean square velocity of the molecules of a gas. Use the concepts of kinetic theory of gases to derives an expression for the root-mean square velocity of the molecules in term of pressure and density of the gas.

7. i) Calculate the average translational kinetic energy of a neon atom at 25 °C.

   ii) At what temperature does the average energy have half this value?

8. A container of volume of 50 cm^3 contains hydrogen at a pressure of 1.0 Pa and at a temperature of 27 °C. Calculate (a)the number of molecules of the gas in the container, and (b)their root-mean square speed.

   ($R= 8.3$ J mol^{-1} K^{-1} , $N = 6 \times 10^{23}$ mol^{-1}. Mass of 1 mole of hydrogen molecule = 20×10^{-3} kg mol^{-1}).

9. A closed container contains hydrogen which exerts pressure of 20.0 mm Hg at a temperature of 50 K.

   (a)At what temperature will it exert pressure of 180 mm Hg?

   b) If the root-mean square velocity of the hydrogen molecules at 10.0 K is 800 m s^{-1}, what will be their root-mean square velocity at this new temperature?

10. State the assumptions of kinetic theory of gases.

11. Find an expression for the pressure of a gas.

12. Deduce Boyle's law and Charle's law from kinetic the theory of gases.

13. What is the interpretation of temperature on the basis of kinetic theory of gases?.

14. What is Avagardo's law? How can it be deduced from kinetic theory of gases

15. Calculate the root-mean square of the molecules of hydrogen at 0 °C and at 100 °C

    (Density of hydrogen at 0°C and 760 mm of mercury pressure = 0.09 kg m^{-3}).

16. Calculate the pressure in mm of mercury exerted by hydrogen gas if the number of molecules per m^3 is 6.8×10^{24} and the root-mean square speed of the molecules is

1.90×10 m s^{-1}. Avogadro's number 6.02×10^{23} and molecular weight of hydrogen = 2.02).

17. Define specific heat of a gas at constant pressure. Derive the relationship between c_p and c_V.

18. Define specific heat of gases at constant volume. Prove that for a triatomic gas $c_V = 3R$

19. Calculate c_p and c_V for argon. Given R = 8.3 J mol^{-1} K^{-1}.

Answers to Intext Questions

10.1

1. (i) Because in a gas the cohesive force between the molecules are extremely small as compared to the molecules in a liquid.

   (ii) Because the molecules in a solid are closely packed. The bonds between the molecules are stronger giving a ordered structure.

2. The gas which follows the kinetic theory of molecules is called as an ideal gas.

3. $P = \frac{1}{3}\rho\overline{c^2}$

10.2

1. Average speed $\overline{c}$

$$= \frac{500+600+700+800+900}{5}$$

$= 700$ m s^{-1}

Average value of $\overline{c^2}$

$$= \frac{500^2+600^2+700^2+800^2+900^2}{5}$$

$= 510{,}000$ m^2 s^{-2}

$c_{rms} = \sqrt{\overline{c^2}} = \sqrt{510{,}000} = 714$ m s^{-1}

c_{rms} and $\overline{c}$ are not same

2. The resultant pressure of the mixture will be the sum of the pressure of gases 1 and 2 respectively i.e. $P = P_1 + P_2$.

3. Boyle's law is not applicable.

10.3

1. For each degree of freedom, energy = $\frac{1}{2}$k T

   ∴ for 5 degrees of freedom for a molecule of nitrogen, total energy = $\frac{5}{2}$k T.

2. c_V for a diatomic molecule = $\frac{5}{2}$ R

   $c_V = \frac{5}{2} \times 8.3 \text{ J mol}^{-1} \text{ K}^{-1} = 20.75 \text{ J mol}^{-1} \text{ K}^{-1}$.

   $c_p = c_V + R = 29.05 \text{ J mol}^{-1} \text{ C}^{-1}$.

Answers to Terminal Problem

2. zero
3. becomes 4 times, doubles, becomes 4 time.
4. 4 : 1
5. 2
7. 6.18×10^{-21} ms^{-1}, – 124 °C
8. 12×10^{20}, 7.9×10^{11} m s^{-1}
9. 2634°C, 2560 ms^{-1}
15. 1800 ms^{-1}, 2088 ms^{-1}
16. 3.97×10^{3} Nm^{-2}
17. 12.45 J mol^{-1} K^{-1}, 20.75 J mol^{-1} K^{-1}.

11

THERMODYNAMICS

You are familiar with the sensation of hotness and coldness. When you rub your hands together, you get the feeling of warmth. You will agree that the cause of heating in this case is mechanical work.This suggests that there is a relationship between mechanical work and thermal effect. A study of phenomena involving thermal energy transfer between bodies at different temperatures forms the subject matter of thermodynamics, which is a phenomenological science based on experience. A quantitative description of thermal phenomena requires a definition of temperature, thermal energy and internal energy. And the laws of thermodynamics provide relationship between the direction of flow of heat, work done on/by a system and the internal energy of a system.

In this lesson you will learn three laws of thermodynamics : the zeroth law, the first law and the second law of thermodynamics. These laws are based on experience and need no proof. As such, the zeroth, first and second law introduce the concept of temperature, internal energy and entropy, respectively. While the first law is essentially the law of conservation of energy for a thermodynamic system, the second law deals with conversion of heat into work and vice versa.You will also learn that Carnot's engine has maximum efficiency for conversion of heat into work.

Objectives

After studying this lesson, you should be able to :

- *draw indicator diagrams for different thermodynamic processes and show that the area under the indicator diagram represents the work done in the process;*
- *explain thermodynamic equilibrium and state the Zeroth law of thermodynamics;*
- *explain the concept of internal energy of a system and state first law of thermodynamics;*
- *apply first law of thermodynamics to simple systems and state its limitations;*
- *define triple point;*
- *state the second law of thermodynamics in different forms; and*
- *describe Carnot cycle and calculate its efficiency.*

Notes

11.1 Concept of Heat and Temperature

11.1.1 Heat

Energy has pervaded all facets of human activity ever since man lived in caves. In its manifestation as heat, energy is intimate to our existence. The energy that cooks our food, lights our houses, runs trains and aeroplanes originates in heat released in burning of wood, coal, gas or oil. You may like to ask : What is heat? To discover answer to this question, let us consider as to what happens when we inflate the tyre of a bicycle using a pump. If you touch the nozzle, you will observe that pump gets hot. Similarly, when you rub you hands together, you get the feeling of warmth. You will agree that in these processes heating is not caused by putting a flame or something hot underneath the pump or the hand. Instead, heat is arising as a result of mechanical work that is done in compressing the gas in the pump and forcing the hand to move against friction. These examples, in fact, indicate a relation between mechanical work and thermal effect.

We know from experience that a glass of ice cold water left to itself on a hot summer day eventually warms up. But a cup of hot coffee placed on the table cools down. It means that energy has been exchanged between the sysem – water or coffee – and its surrounding medium. This energy transfer continues till thermal equilibrium is reached. That is until both – the system and the suroundings – are as the same temperature. It also shows that the direction of energy transfer is always from the body at high temprature to a body at lower temperature. You may now ask : In what form is energy being transferred? In the above examples, energy is said to be transfered in the form of heat. So we can say that heat is the *form of energy transferred between two (or more) systems or a system and its surroundings because of temperature difference.*

You may now ask. What is the nature of this form of energy? The answer to this question was provided by Joule through his work on the equivalence of heat and mechanical work : Mechanical motion of molecules making up the system is associated with heat.

The unit of heat is calorie. One calorie is defined as the quantity of heat energy required to raise the temperature of 1 gram of water from 14.5°C to 15.5°C. It is denoted as cal.

Kilocalorie (k cal) is the larger unit of heat energy :

$$1 \text{ kcal} = 10^3 \text{ cal.}$$

Also $$1 \text{ cal} = 4.18 \text{ J}$$

11.1.2 Concept of Temperature

While studying the nature of heat, you learnt that energy exchange between a glass of cold water and its surroundings continues until thermal equilibrium was reached. All bodies in thermal equilibrium have a common property, called temperature, whose value is same for all of them. Thus, we can say that temperature of a body is the property which determines whether or not it is in thermal equilibrium with other bodies.

11.1.3. Thermodynamic Terms

(i) **Thermodynamic system :** A thermodynamic system refers to a definite quantity of matter which is considered unique and separated from everything else, which can influence it. Every system is enclosed by an arbitrary surface, which is called its boundary. The boundary may enclose a solid, a liquid or a gas. It may be real or imaginary, either at rest or in motion and may change its size and shape. The region of space outside the boundary of a system constitutes its surroundings.

Notes

(a) **Open System :** It is a system which can exchange mass and energy with the surroundings. A water heater is an open system.

(b) **Closed system :** It is a system which can exchange energy but not mass with the surroundings. A gas enclosed in a cylinder fitted with a piston is a closed system.

(c) **Isolated system :** It is a system which can exchange neither mass nor energy with the surrounding. A filled thermos flank is an ideal example of an isolated system.

(ii) **Thermodynamic Variables or Coordinates :** In module–1, we have studied the motion of a body (or a system) in terms of its mass, position and velocity. To describe a thermodynamic system, we use its physical properties such on temperature (T), pressure (P), and volume (V). These are called thermodynamic variables.

(iii) **Indicator diagram :** You have learnt about displacement–time and velocity–time graphs in lesson 2. To study a thermodynamic system, we use a pressure-volume graph. This graph indicates how pressure (P) of a system varies with its volume (V) during a thermodynamic process and is known as an indicator diagram.

The indicator diagram can be used to obtain an expression for the work done. It is equal to the area under the P-V diagram (Fig. 11.1). Suppose that pressure is P at the start of a very small expansion ΔV. Then, work done by the system.

$$\Delta W = P\,\Delta V \qquad (11.1)$$

$$= \text{Area of a shaded strip ABCD}$$

Now total work done by the system when it expands from V_1 to V_2 = Area of $P_1P_2V_2V_1P_1$ Note that the area depends upon the shape of the indicator diagram.

The indicator diagram is widely used in calculating the work done in the process of expansion or compression. It is found more useful in processes where relationship between P and V is not known. The work done on the system increases its energy and work done by the system reduces it. For this reason, work done on the system is taken as negative. You must note that the area enclosed by an isotherm

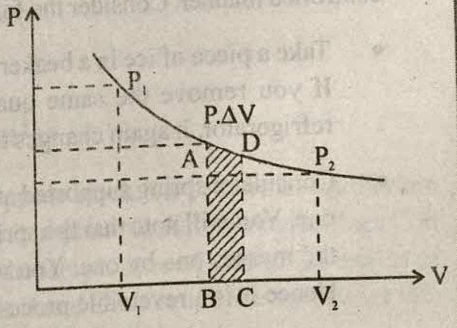


Fig. 11.1 : Indicater Diagram

(plot of p versus V at constant temperature) depends on its shape. We may conclude that work done by or on a system depends on the path. That is, work does not depend on the initial and final states.

11.2 Thermodynamic Equilibrium

Imagine that a container is filled with a liquid (water, tea, milk, coffee) at 60° C. If it is left to itself, it is common experience that after some time, the liquid attains the room temperature. We then say that water in the container has attained thermal equilibrium with the surroundings.

If within the system, there are variations in pressure or elastic stress, then parts of the system may undergo some changes. However, these changes cease ultimately, and no unbalanced force will act on the system. Then we say that it is in mechanical equilibrium. Do you know that our earth bulged out at the equator in the process of attaining mechanical equilibrium in its formation from a molten state?

If a system has components which react chemically, after some time, all possible chemical reactions will cease to occur. Then the system is said to be in chemical equilibrium.

A system which exhibits thermal, mechanical and chemical equilibria is said to be in thermodynamic equilibrium. The macroscopic properties of a system in this state do not change with time.

11.2.1 Thermodynamic Process

If any of the thermodynamic variables of a system change while going from one equilibrium state to another, the system is said to execute a thermodynamic process. For example, the expansion of a gas in a cylinder at constant pressure due to heating is a thermodynamic process. A graphical representation of a thermodynamic process is called a path.

Now we will consider different types of thermodynamic processes.

(i) **Reversible process :** If a process is executed so that all intermediate stages between the inital and final states are equilibrium states and the process can be executed back along the same equilibrium states from its final state to its initial state, it is called reversible process. A reversible process is executed very slowly and in a controlled manner. Consider the following examples :

- Take a piece of ice in a beaker and heat it. You will see that it changes to water. If you remove the same quantity of heat of water by keeping it inside a refrigerator, it again changes to ice (initial state).
- Consider a spring supported at one end. Put some masses at its free end one by one. You will note that the spring elongates (increases in length). Now remove the masses one by one. You will see that spring retraces its initial positions. Hence it is a reversible process.

As such, a reversible process can only be idealised and never achieved in practice.

(ii) **Irreversible process :** A process which cannot be retraced along the same equilibrium state from final to the initial state is called irreversible process.

All natural process are irrerersible. For example, heat poduced during friction, sugar dissolved in water, or rusting of iron in the air. It means that for irrerersible process, the intermediate states are not equilibrium states and hence such process can not be represented by a path. Does this mean that we can not analyse an irrerersible process? To do so, we use quasi-static process, which is infinitesimally close to the equilibrium state.

(iii) **Isothermal process :** A thermodynamic process that occurs at constant temperature is an isothermal process. The expansion and compression of a perfect gas in a cylinder made of perfectly conducting walls are isothermal processes. The change in pressure or volume is carried out very slowly so that any heat developed is transferred into the surroundings and the temperature of the system remains constant. The thermal equilibrium is always maintained. In such a process, ΔQ, ΔU and ΔW are finite.

(iv) **Adiabatic process :** A thermodynamic process in which no exchange of thermal energy occurs is an adiabatic process. For example, the expansion and compression of a perfect gas in a cylinder made of perfect insulating walls. The system is isolated from the surroundings. Neither any amount of heat leaves the system nor enters it from the surroundings. In this process, therefore $\Delta Q = 0$ and $\Delta U = -\Delta W$.

The change in the internal energy of the system is equal to the work done on the system. When the gas is compressed, work is done on the system. So, ΔU becomes positive and the internal energy of the system increases. When the gas expands, work is done by the system. It is taken as positive and ΔU becomes negative. The internal energy of the system decreases.

(v) **Isobaric process :** A thermodynamic process that occurs at constant pressure is an isobaric process. Heating of water under atmospheric pressure is an isobaric process.

(vi) **Isochoric process :** A thermodynamic process that occurs at constant volume is an isochoric process. For example, heating of a gas in a vessel of constant volume is an isochoric process. In this process, volume of the gas remains constant so that no work is done, i.e. $\Delta W = 0$. We therefore get $\Delta Q = \Delta U$.

In a **Cyclic Process** the system returns back to its initial state. It means that there is no change in the internal energy of the system. $\Delta U = 0$.

$$\therefore \Delta Q = \Delta W.$$

11.2.2 Zeroth Law of Thermodynamics

Let us consider three metal blocks A, B and C. Suppose block A is in thermal equilibrium with block B. Further suppose that block A is also in thermal equilibrium with block C. It means the temperature of the block A is equal to the temperature of block B as well as of block C. It follows that the temperatures of blocks B and C are equal. We summarize this result in the statement known as Zeroth Law of Thermodynamics :

If two bodies or systems A and B are separately in thermal equilibrium with a third body C, then A and B are in thermal equilibrium with each other.

Notes

Phase Change and Phase Diagram

You have learnt that at STP, matter exists in three states : solid, liquid and gas. ***The different states of matter are called its phases.*** For example, ice (solid), water (liquid) and steam (gas) are three phases of water. We can discuss these three phases using a three dimensional diagram drawn in pressure (P), temperature (T) and volume (V). It is difficult to draw three dimensional diagram. Thus, we discuss the three phases of matter by drawing a pressure-temperature diagram. This is called ***phase diagram.***

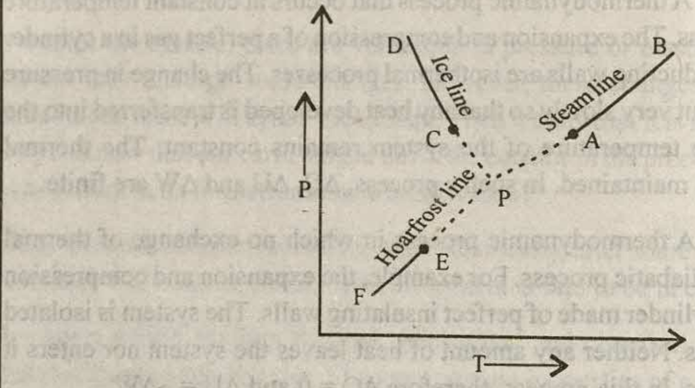


Fig. 11.3: Phase diagram of water

Refer to Fig. 11.3, which shows phase diagram of water. You can see three curves CD; AB and EF. Curve CD shows the variation of melting point of ice with pressure. It is known as a ***fusion curve.*** Curve AB shows variation of boiling point of water with pressure. It is known as ***vaporization curve.*** Curve EF shows change of ice directly to steam. It is known as a ***sublimation curve.*** This curve is also known as ***Hoarfrost Line.***

If you extend the curve AB, CD and EF (as shown in the figure with dotted lines), they meet at point P. This point is called ***triple point.*** At triple point, all three phases co-exist.

When we heat a solid, its temperature increases till it reaches a temperature at which it starts melting. This temperature is called **melting point** of the solid. During this change of state, we supply heat continuously but the temperature does not rise. The heat required to completely change unit mass of a solid into its corresponding liquid state at its melting point is called **latent heat of fusion of the solid.**

On heating a liquid, its temperature also rises till its **boiling point** is reached. At the boiling point, the heat we supply is used up in converting the liquid into its gaseous state. The amount of heat required to convert unit mass of liquid in its gaseous state at constant temperature is called **latent heat** of vaporization of the liquid.

11.2.3 Triple Point of Water

Triple point of a pure substance is a very stable state signified by precisely constant temperature and pressure values. For this reason, in kelvin's scale of thermometry, triple point of water is taken as the upper fixed point.

On increasing pressure, the melting point of a solid decreases and boiling point of the liquid increases. It is possible that by adjusting temperature and pressure, we can obtain all the three states of matter to co-exist simultaneously. These values of temperature and pressure signify the **triple point.**

Intext Questions 11.1

1. Fill in the blanks

   (i) Zeroth law of thermodynamics provides the basis for the concept of ....

   (ii) If a system A is in thermal equilibrium with a system B and B is in thermal equilibrium with another system C, then system A will also be in thermal equilibrium with system...............

   (iii) The unit of heat is

   ....

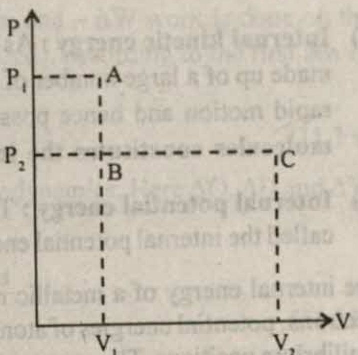


Fig. 11.2

2. Fig. 11.2 is an indicator diagram of a thermodynamic process. Calculate the work done by the system in the process :

   (a) along the path ABC from A to C

   (b) If the system is returned from C to A along the same path, how much work is done by the system.

   ....

3. Fill in the blanks.

   (i) A reversible process is that which can be .... in the opposite direction from its final state to its initial state.

   (ii) An .... process is that which cannot be retraced along the same equilibrium states from final state to the initial state.

4. State the basic difference between isothermal and adiabatic processes.

   ....

5. State one characteristic of the triple point.

   ....

11.3 Internal Energy of a System

Have you ever thought about the energy which is released when water freezes into ice ? Don't you think that there is some kind of energy stored in water. This energy is released when water changes into ice. This stored energy is called the **internal energy**. On the basis of kinetic theory of matter, we can discuss the concept of internal energy as sum of the energies of individual components/constituents. This includes kinetic energy due to their random motion and their potential energy due to interactions amongst them. Let us now discuss these.

(a) **Internal kinetic energy :** As you now know, according to kinetic theory, matter is made up of a large number of molecules. These molecules are in a state of constant rapid motion and hence possess kinetic energy. The **total kinetic energy of the molecules constitutes the internal kinetic energy of the body.**

(b) **Internal potential energy :** The energy arising due to the inter-molecular forces is called the internal potential energy.

The internal energy of a metallic rod is made up of the kinetic energies of conduction electrons, potential energies of atoms of the metal and the vibrational energies about their equilibrium positions. The energy of the system may be increased by causing its molecules to move faster (gain in kinetic energy by adding thermal energy). It can also be increased by causing the molecules to move against inter-molecular forces, i.e., by doing work on it. **Internal energy** is denoted by the letter U.

Internal energy of a system = Kinetic energy of molecules + Potential energy of molecules

Let us consider an isolated thermodynamic system subjected to an external force. Suppose W amount of work is done on the system in going from initial state i to final state f adiabatically. Let U_i and U_f be internal energies of the system in its initial and final states respectively. Since work is done on the system, internal energy of final state will be higher than that of the initial state.

According to the law of conservation of energy, we can write

$$U_i - U_f = -W$$

Negative sign signifies that work is done on the system.

We may point out here that unlike work, internal energy depends on the initial and final states, irrespective of the path followed. We express this fact by saying that U is a function of state and depends only on state variables P, V, and T. Note that if some work is done by the system, its internal energy will decrease.

11.4 First Law of Thermodynamics

You now know that the zeroth law of thermodynamics tells us about thermal equilibrium among different systems characterised by same temperature. However, this law does not tell us anything about the non-equilibrium state. Let us consider two examples : (i) Two systems at different temperatures are put in thermal contact and (ii) Mechanical rubbing between two systems. In both cases, change in their temperatures occurs but it cannot be explained by the Zeroth law. To explain such processes, the first law of thermodynamics was postulated.

The first law of thermodynamics is, in fact, the law of conservation of energy for a thermodynamic system. It states that change in internal energy of a system during a *thermodynamic process is equal to the sum of the heat given to it and the work done on it.*

Suppose that ΔQ amount of heat is given to the system and $-\Delta W$ work is done on the system. Then increase in internal energy of the system, ΔU, according to the first law of thermodynamics is given by

$$\Delta U = \Delta Q - \Delta W \tag{11.3 a}$$

This is the mathematical form of the first law of thermodynamics. Here ΔQ, ΔU and ΔW all are in SI units.

The first law of thermodynamics can also be written as

$$\Delta Q = \Delta U + \Delta W \tag{11.3 b}$$

The signs of ΔQ, ΔU and ΔW are known from the following sign conventions :

1. Work done (ΔW) by a system is taken as positive whereas the work done on a system is taken as negative. The work is positive when a system expands. When a system is compressed, the volume decreases, the work done is negative. The work done does not depend on the initial and final thermodynamic states; it depends on the path followed to bring a change.
2. Heat gained by (added to) a system is taken as positive, whereas heat lost by a system is taken as negative.
3. The increase in internal energy is taken as positive and a decrease in internal energy is taken as negative.

If a system is taken from state 1 to state 2, it is found that both ΔQ and ΔW depend on the path of transformation. However, the difference $(\Delta Q - \Delta W)$ which represents ΔU, remains the same for all paths of transformations.

We therefore say that the change in internal energy ΔU of a system does not depend on the path of the thermodynamic transformations.

11.4.1 Limitations of the First Law of Thermodynamics

The first law of thermodynamics asserts the equivalence of heat and other forms of energy. This equivalence makes the world around us work. The electrical energy that lights our houses, operates machines and runs trains originates in heat released in burning of fossil or nuclear fuel. In a sense, it is universal. It explains the fall in temperature with height; the adiabatic lapse rate in upper atmosphere. Its applications to flow process and chemical reations are also very interesting. However, consider the following processes :

- You know that heat always flows from a hot body to a cold body. But first law of thermodynamics does not prohibit flow of heat from a cold body to a hot body. It means that this law fails to indicate the direction of heat flow.
- You know that when a bullet strikes a target, the kinetic energy of the bullet is converted into heat. This law does not indicate as to why heat developed in the target cannot be changed into the kinetic energy of bullet to make it fly. It means that this law fails to provide the conditions under which heat can be changed into work.

Moreover, it has obvious limitations in indicating the extent to which heat can be converted into work.

Now take a pause and answer the following questions :

Intext Questions 11.2

1. Fill in the blanks

(i) The total of kinetic energy and potential energy of molecules of a system is called its.........................

(ii) Work done = – W indicates that work is done......................... the system.

2. The first law of thermodynamics states that ....

....

11.5 Second Law of Thermodynamics

You now know that the first law of thermodynamics has inherent limitations in respect of the direction of flow of heat and the extent of convertibility of heat into work. So a question may arise in your mind : Can heat be wholly converted into work? Under what conditions this conversion occurs? The answers of such questions are contained in the postulate of *Second law of thermodynamics.* The second law of thermodynamics is stated in several ways. However, here you will study Kelvin-Planck and Clausius statements of second law of thermodynamics.

The Kelvin-Planck's statement is based on the experience about the performance of heat engines. (Heat engine is discussed in next section.) In a heat engine, the working substance extracts heat from the source (hot body), converts a part of it into work and rejects the rest of heat to the sink (environment). There is no engine which converts the whole heat into work, without rejecting some heat to the sink. These observations led Kelvin and Planck to state the second law of thermodynamics as

It is impossible for any system to absorb heat from a reservoir at a fixed temperature and convert whole of it into work.

Clausius statement of second law of thermodynamics is based on the performance of a refrigerator. A refrigerator is a heat engine working in the opposite direction. It transfers heat from a colder body to a hotter body when external work is done on it. Here concept of external work done on the system is important. To do this external work, supply of energy from some external source is a must. These observations led Clausius to state the second law of thermodynamics in the following form.

It is impossible for any process to have as its sole result to transfer heat from a colder body to a hotter body without any external work.

Thus, the second law of thermodynamics plays a unique role for practical devices like heat engine and refrigerator.

11.5.1 Carnot Cycle

You must have noticed that when water is boiled in a vessel having a lid, the steam generated inside throws off the lid. This shows that high pressure steam can be made to do useful work. ***A device which can convert heat into work is called a heat engine.*** Modern engines which we use in our daily life are based on the principle of heat engine. These may be categorised in three types : steam engine, internal combustion engine and gas turbine. However, their working can be understood in terms of Carnot's reversible engine. Let us learn about it now.

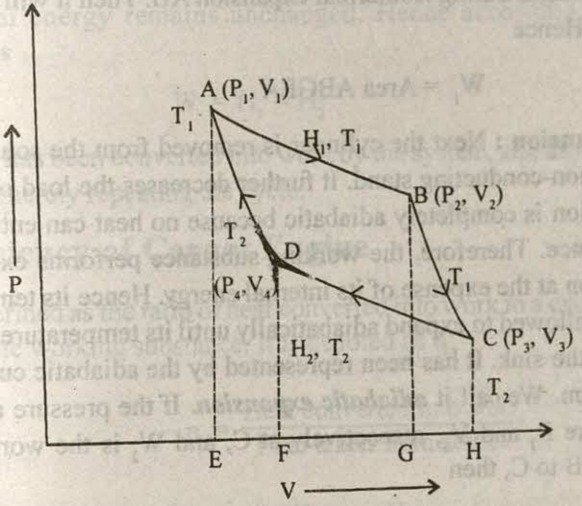


Fig. 11.4 : Indicator diagram of Carnot cycle

In Carnot cycle, the working substance is subjected to four operations : (a) isothermal expansion, (b) adiabatic expansion, (c) isothermal compression and (d) adiabatic compression. Such a cycle is represented on the P-V diagram in Fig. 11.4. To describe four operations of Carnot's cycle, let us fill one gram. mol. of the working substance in the cylinder (Fig. 11.5). Original condition of the substance is represented by point A on the indicator diagram. At this point, the substance is at temperature T_1, pressure P_1 and volume V_1.

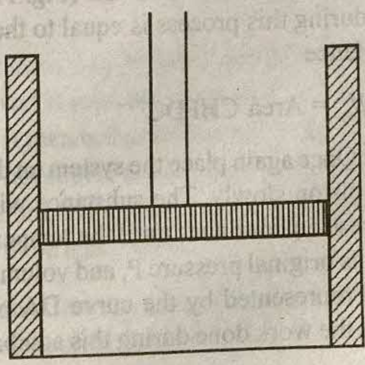


Fig. 11.5 : The cylinder with working substance

(a) **Isothermal expansion :** The cylinder is put in thermal contact with the source and allowed to expand. The volume of the working substance increases to V_2. Thus working substance does work in raising the piston. In this way, the temperature of the working substance would tend to fall. But it is in thermal contact with the source.

So it will absorb a quantity of heat H_1 from the source at temperature T_1. This is represented by the point B. At B, the values of pressure and volume are P_2 and V_2 respectively. On the indicator diagram (Fig. 11.4), you see that in going from A to B, temperature of the system remains constant and working substance expands. We call it ***isothermal expansion process.*** H_1 is the amount of heat absorbed in the isothermal expansion process. Then, in accordance with the first law of thermodynamics, H_1 will be equal to the external work done by the gas during isothermal expansion from A to B at temperature T_1. Suppose W_1 is the external work done by the gas during isothermal expansion AB. Then it will be equal to the area ABGEA. Hence

$$W_1 = \text{Area ABGEA}$$

(b) **Adiabatic expansion :** Next the cylinder is removed from the source and placed on a perfectly non-conducting stand. It further decreases the load on the piston to P_3. The expansion is completely adiabatic because no heat can enter or leave the working substance. Therefore, the working substance performs external work in raising the piston at the expense of its internal energy. Hence its temperature falls. The gas is thus allowed to expand adiabatically until its temperature falls to T_2, the temperature of the sink. It has been represented by the adiabatic curve BC on the indicator diagram. We call it ***adiabatic expansion.*** If the pressure and volume of the substance are P_3 and V_3, respectively at C, and W_2 is the work done by the substance from B to C, then

$$W_2 = \text{Area BCHGB.}$$

(c) **Isothermal compression :** Remove the cylinder from the non-conducting stand and place it on the sink at temperature T_2. In order to compress the gas slowly, increase the load (pressure) on the piston until its pressure and volume become P_4 and V_4, respectively. It is represented by the point D on the indicator diagram (Fig. 11.4). The heat developed (H_2) due to compression will pass to the sink. Thus, there is no change in the temperature of the system. Therefore, it is called an isothermal compression process. It is shown by the curve CD (Fig. 11.4). The quantity of heat rejected (H_2) to the sink during this process is equal to the work done (say W_3) on the working substance. Hence

$$W_3 = \text{Area CHFDC}$$

(d) **Adiabatic compression :** Once again place the system on the non-conducting stand. Increase the load on the piston slowly. The substance will under go an adiabatic compression. This compression continues until the temperature rises to T_1 and the substance comes back to its original pressure P_1 and volume V_1. This is an adiabatic compression process and represented by the curve DA on the indicator diagram (Fig. 11.4). Suppose W_4 is the work done during this adiabatic compression from D to A. Then

$$W_4 = \text{Area DFEAD}$$

During the above cycle of operations, the working substance takes H_1 amount of heat from the source and rejects H_2 amount of heat to the sink. Hence the net amount of heat absorbed by the working substance is

$$\Delta H = H_1 - H_2$$

Also the net work done (say W) by the engine in one complete cycle

$$W = \text{Area ABCHEA} - \text{Area CHEADC}$$

$$= \text{Area ABCD}$$

Thus, the work done in one cycle is represented on a P-V diagram by the area of the cycle.

You have studied that the initial and final states of the substance are the same. It means that its internal energy remains unchanged. Hence according to the first law of thermodynamics

$$W = H_1 - H_2$$

Therefore, heat has been converted into work by the system, and any amount of work can be obtained by merely repeating the cycle.

11.7.2 Efficiency of Carnot Engine

Efficiency is defined as the ratio of heat converted into work in a cycle to heat taken from the source by the working substance. It is denoted as η:

$$\eta = \frac{\text{Heat converted into work}}{\text{Heat taken from source}}$$

or

$$\eta = \frac{H_1 - H_2}{H_1} = 1 - \frac{H_2}{H_1}$$

It can be shown that for Carnot's engine,

$$\frac{H_2}{H_1} = \frac{T_2}{T_1}$$

Hence,

$$\eta = 1 - \frac{T_2}{T_1}$$

Note that efficiency of carnot engine does not depend on the nature of the working substance. Further, if no heat is rejected to the sink, η will be equal to one. But for H_2 to be zero, T_2 must be zero. It means that efficiency η can be100% only when $T_2 = 0$. The entire heat taken from the hot source is converted into work. This violates the second law of thermodynamics. Therefore, a steam engine can operate only between finite temperature limits and its efficiency will be less than one.

It can also be argued that the Carnot cycle, being a reversible cycle, is most efficient; no engine can be more efficient than a Carnot engine operating between the same two temperatures.

11.7.3 Limitation of Carnot's Engine

You have studied about Carnot's cycle in terms of isothermal and adiabatic processes. Here it is important to note that the isothermal process will take place only when piston moves very slowly. It means that there should be sufficient time for the heat to transfer from the working substance to the source. On the other hand, during the adiabatic process, the piston moves extremely fast to avoid heat transfer. In practice, it is not possible to fulfill these vital conditions. Due to these very reasons, all practical engines have an efficiency less than that of Carnot's engine.

Intext Questions 11.3

1. State whether the following statements are true or false.

   (i) In a Carnot engine, when heat is taken by a perfect gas from a hot source, the temperature of the source decreases.

   ....

   (ii) In Carnot engine, if temperature of the sink is decreased the efficiency of engine also decreases.

   ....

2. (i) A Carnot engine has the same efficiency between 1000K and 500K and between TK and 1000K. Calculate T.

   ....

   (ii) A Carnot engine working between an unknown temperature T and ice point gives an efficiency of 0.68. Deduce the value of T.

   ....

What You Have Learnt

- Heat is a form of energy which produces in us the sensation of warmth.
- The energy which flows from a body at higher temperature to a body at lower temperature because of temperature difference is called heat energy.
- The most commonly known unit of heat energy is calorie. 1 cal = 4.18 J and 1k cal = 10^3 cal.
- A graph which indicates how the pressure (P) of a system varies with its volume during a thermodynamic process is known as indicator diagram.
- Work done during expansion or compression of a gas is $P\Delta V = P(V_f - V_i)$.
- Zeroth law of thermodynamics states that if two systems are separately in thermal equilibrium with a third system, then they must also be in thermal equilibrium with each other.
- The sum of kinetic energy and potential energy of the molecules of a body gives the internal energy. The relation between internal energy and work is $U_i - U_f = -W$.

- The first law of thermodynamics states that the amount of heat given to a system is equal to the sum of change in internal energy of the system and the external work done.
- First law of thermodynamics tells nothing about the direction of the process.
- The process which can be retraced in the opposite direction from its final state to initial state is called a reversible process.
- The process which can not be retraced along the same equilibrium state from final to the initial state is called an irreversible process. A process that occurs at constant temperature is an isothermal process.
- Any thermodynamic process that occurs at constant heat is an adiabatic process.
- The different states of matter are called its phase and the pressure and temperature diagram showing three phases of matter is called a phase diagram.
- Triple point is a point (on the phase diagram) at which solid, liquid and vapour states of matter can co-exist. It is characterised by a particular temperature and pressure.
- According to Kelvin-Planck's statement of second law, it is not possible to obtain a continuous supply of work from a single source of heat.
- According to Clausius statement of second law, heat can not flow from a colder body to a hotter body without doing external work on the working substance.
- The three essential requirements of any heat engine are :

  (i)source from which heat can be drawn

  (ii)a sink into which heat can be rejected.

  (iii)working substance which performs mechanical work after being supplied with heat.
- Carnot's engine is an ideal engine in which the working substance is subjected to four operations (i) Isothermal expansion (ii) adiabatic expansion (iii) isothermal compression and (iv) adiabatic compression. Such a cycle is called a Carnot cycle.
- Efficiency of a Carnot engine is given only

  $\eta = 1 - \frac{H_2}{H_1}$, H_1 = Amount of heat absorbed and H_2 = Amount of heat rejected.

  $= 1 - \frac{T_2}{T_1}$, T_1 = Temperature of the source, and T_2 = Temperature of the sink.
- Efficiency does not depend upon the nature of the working substance.

Terminal Exercise

1. Distinguish between the terms internal energy and heat energy.

2. What do you mean by an indicator diagram. Derive an expression for the work done during expansion of an ideal gas.
3. Define temperature using the Zeroth law of thermodynamics.
4. State the first law of thermodynamics and its limitations.
5. What is the difference between isothermal, adiabetic, isobaric and isochoric processes?
6. State the Second law of thermodynamics.
7. Discuss reversible and irreversible processes with examples.
8. Explain Carnot's cycle. Use the indicator diagram to calculate its efficiency.
9. Calculate the change in the internal energy of a system when (a) the system absorbs 2000J of heat and produces 500 J of work (b) the system absorbs 1100J of heat and 400J of work is done on it.
10. A Carnot's engine whose temperature of the source is 400K takes 200 calories of heat at this temperature and rejects 150 calories of heat to the sink. (i) What is the temperature of the sink. (ii) Calculate the efficiency of the engine.

Answers to Intext Questions

11.1

1. (i) Temperature (ii) C (iii) Joule or Calorie
2. (a) $P_2 (V_2 - V_1)$ (b) $-P_2 (V_2 - V_1)$
3. (i) retrace (ii) irreversible
4. An isothermal process occurs at a constant temperature whereas an adiabatic process occurs at constant heat.
5. At triple point all three states of matter i.e. solid, liquid and vapour can co-exist.

11.2

1. (i) Internal energy (ii) on
2. It states that the amount of heat given to a system is equal to the sum of the change in internal energy of the system and the external energy.

11.3

1. (i) False (ii) True
2. (i) 2000 K (ii) 8583.1K

Answers to Terminal Problems

9. (a) 1500 J (b) 1500 J.

10. 300K, 25%

12

HEAT TRANSFER AND SOLAR ENERGY

In the previous lesson you have studied the laws of thermodynamics, which govern the flow and direction of thermal energy in a thermodynamic system. In this lesson you will learn about the processes of heat transfer. The energy from the sun is responsible for life on our beautiful planet. Before reaching the earth, it passes through vacuum as well as material medium between the earth and the sun. Do you know that each one of us also radiates energy at the rate of nearly 70 watt? Here we will study the **radiation** in detail. This study enables us to determine the temperatures of stars even though they are very far away from us.

Another process of heat transfer is **conduction,** which requires the presence of a material medium. When one end of a metal rod is heated, its other end also becomes hot after some time. That is why we use handles of wood or similar other bad conductor of heat in various appliances. Heat energy falling on the walls of our homes also enters inside through conduction. But when you heat water in a pot, water molecules near the bottom get the heat first. They move from the bottom of the pot to the water surface and carry heat energy. This mode of heat transfer is called **convection**. These processes are responsible for various natural phenomena, like monsoon which are crucial for existence of life on the globe. You will learn more about these processes of heat transfer in this unit.

Objectives

After studying this lesson, you should be able to :

- *distinguish between conduction convection and radiation;*
- *define the coefficient of thermal conductivity;*
- *describe green house effect and its consequenies for life on earth; and*
- *apply laws governing black body radiation.*

12.1 Processes of Heat Transfer

You have learnt the laws of thermodynamics in the previous lesson. The second law postulates that the natural tendency of heat is to flow spontaneously from a body at higher temperature to a body at lower temperature. The transfer of heat continues until the temperatures of the two bodies become equal. From kinetic theory, you may recall that temperature of a gas is related to its average kinetic energy. It means that molecules of a gas at different temperatures have different average kinetic energies.

There are three processes by which transfer of heat takes place. These are : *conduction, convection* and *radiation.* In conduction and convection, heat transfer takes place through molecular motion. Let us understand how this happens.

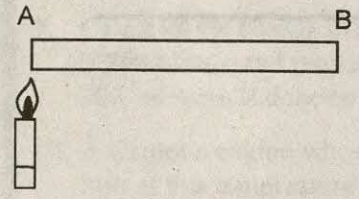


Fig. 12.1 : Heat conduction in a metal rod

Heat transfer through *conduction* is more common in solids. We know that atoms in solids are tightly bound. When heated, they can not leave their sites; they are constrained to vibrate about their respective equilibrium positions. Let us understand as to what happens to their motion when we heat a metal rod at one end (Fig.12.1). The atoms near the end A become hot and their kinetic energy increases. They vibrate about their mean positions with increased kinetic energy and being in contact with their nearest neighbouring atoms, pass on some of their kinetic energy (K.E.) to them. These atoms further transfer some K.E to their neighboures and so on. This process continues and kinetic energy is transferred to atoms at the other end B of the rod. As average kinetic energy is proportional to temperature, the end B gets hot. Thus, *heat* is *transferred from atom* to *atom by conduction. In this process, the atoms* do *not bodily move* but *simply vibrate about their mean equlibrium positions and pass energy from one to another.*

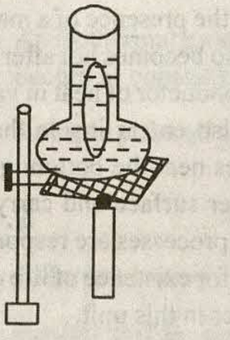


Fig. 12.2 : Convection currents are formed in water when heated

In *convection,* molecules of fluids receive thermal energy and move up bodily. To see this, take some water in a flask and put some grains of potassium permanganate ($KMnO_4$) at its bottom. Put a bunsen flame under the flask. As the fluid near the bottom gets heated, it expands. The density of water decreases and the buoyant force causes it to move upward (Fig.12.2). The space occupied by hot water is taken by the cooler and denser water, which moves downwards. Thus, a convection current of hotter water going up and cooler water coming down is set up. The water gradually heats up. These convection currents can be seen as $KMnO_4$ colours them red.

In *radiation,* heat energy moves in the form of waves. You will learn about the characteristics of these waves in a later section. These waves can pass through vacuum and do not require the presence of any material medium for their propagation. Heat from the sun comes to us mostly by radiation.

We now study these processes in detail.

Notes

12.1.1 Conduction

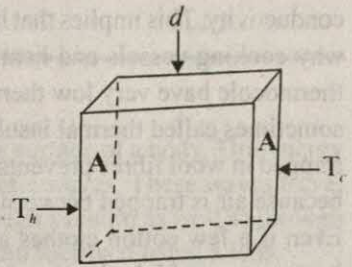


Fig. 12.3 : Heat conduction through a slab of thickness *d* and surface area *A*, when the faces are kept at temperatures T_h and T_c.

Consider a rectangular slab of area of cross-section **A** and thickness *d*. Its two faces are maintained at temperatures T_h and T_c ($<T_h$), as shown in Fig. 12.3. Let us consider all the factors on which the quantity of heat *Q* transferred from one face to another depends. We can intuitively feel that larger the area A, the greater will be the heat transferd ($Q \alpha A$). Also, greater the thickness, lesser will be the heat transfer ($Q \alpha 1/d$). Heat transfer will be more if the temperature difference between the faces, ($T_h - T_c$), is large. Finally longer the time *t* allowed for heat transfer, greater will be the value of *Q*. Mathematically, we can write

$$Q \alpha \frac{A(T_h - T_c).t}{d}$$

$$Q = \frac{KA(T_h - T_c)t}{d} \qquad (12.1)$$

where *K* is a constant which depends on the nature of the material of the slab. It is called the coefficient of thermal conductivity,or simply, **thermal conductivity** of the material. **Thermal conductivity** of a material is defined as the amount of heat transferred in one second across a piece of the material having area of cross-section $1\,m^2$ and edge 1 m when its opposite faces are maintained at a temperature difference of 1 K. The SI unit of thermal conductivity is $W\,m^{-1}\,k^{-1}$. The value of K for some materials is given in Table 12.1

Table 12.1 : Thermal Conductivity of some materials

Material	Thermal conductivity ($Wm^{-1}K^{-1}$)
Copper	400
Aluminium	240
Concrete	1.2
Glass	0.8
Water	0.60
Body talc	0.20
Air	0.025
Thermocole	0.01

Example 12.1 : A cubical thermocole box, full of ice. has side 30 cm and thickness of 5.0 cm. If outside temperature is 45°C, estimate the amount of ice melted in 6 h. (*K* for thermacole is $0.01\ J\ s^{-1}\ m^{-1}\ °C^{-1}$ and latent heat of fusion of ice is $335\ J\ g^{-1}$.

Solution : The quantity of heat transferred into the box through its one face can be obtained using Eq. (12.1) :

$$Q = \frac{KA(T_h - T_c)t}{d}$$

$$= (0.01\ J\ s^{-1} m^{-1}\ °C^{-1}) \times (900 \times 10^{-4}\ m^2) \times (45\ °C) \times (6 \times 60 \times 60\ s) / (5 \times 10^{-2} m)$$

$$= 10496\ J$$

Since the box has six faces, total heat passing into the box

$$Q = 10496 \times 6\ J$$

The mass of ice melted *m*, can be obtained by dividing *Q* by *L* :

$$m = Q/L$$

$$= \frac{10496J}{335\ Jg^{-1}} \times 6$$

$$= 313 \times 6\ g = 1878\ g$$

We can see from Table 12.1 that metals such as copper and aluminium have high thermal conductivity. This implies that heat flows with more ease through copper. This is the reason why cooking vessels and heating pots are made of copper. On the other hand, air and thermocole have very low thermal conductivities. Substances having low value of *K* are sometimes called thermal insulators. We wear woollen clothes during winter because air trapped in wool fibres prevents heat loss from our body. Wool is a good thermal insulator because air is trapped between its fibres. The trapped heat gives us a feeling of warmth. Even if a few cotton clothes are put on one above another, the air trapped in-between layers stops cold. In the summer days, to protect a slab of ice from melting, we put it in a ice box made of thermocole. Sometimes we wrap the ice slab in jute bag, which also has low thermal conductivity.

12.1.2 Convection

It is common experience that while walking by the side of a lake or a sea shore on a hot day, we feel a cool breeze. Do you know the reason? Let us discover it.

Due to continuous evaporation of water from the surface of lake or sea, the temperature of water falls. Warm air from the shore rises and moves upwards (Fig.12.4). This creates low pressure area on the shore and causes cooler air from water surface to move to the shore. The net effect of these convection currents is the transfer of heat from the shore,which is hotter, to water, which is cooler. The rate of heat transfer depends on many factors.There is no simple equation for convection as for conduction. However, the ***rate of heat transfer by convection depends on the temperature difference between the surfaces and also on their areas.***

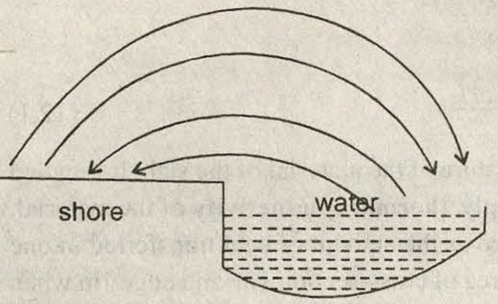


Fig. 12.4 : Convection currents. Hot air from the shore rises and moves towards cooler water. The convection current from water to the shores is experienced as cool breeze.

Now let us check how much you have learnt about the methods of heat transfer.

Intext Questions 12.1

1. Distinguish between conduction and convection.

....

2. Verify that the units of *K* are Js $^{-1}$ m^{-1} °C^{-1}.

....

3. Explain why do humans wrap themselves in woollens in winter season?

....

4. A cubical slab of surface area 1 m^2, thickness 1 m, and made of a material of thermal conductivity *K*. The opposite faces of the slab are maintained at 1°C temperature difference. Compute the energy transferred across the surface in one second. and hence give a numerical definition of *K*.

....

5. During the summer, the land mass gets very hot. But the air over the ocean does not get as hot. This results in the onset of sea breezes. Explain.

....

12.1.3 Radiation

Radiation refers to continuous emission of energy from the surface of a body. This energy is called radiant energy and is in the form of electromagnetic waves. These waves travel with the velocity of light (3×10^8 ms^{-1}) and can travel through vacuum as well as through air. They can easily be reflected from polished surfaces and focussed using a lens.

All bodies emit radiation with wavelengths that are chracteristic of their temperature. The sun, at 6000 K emits energy mainly in the visible spectrum. The earth at an ideal radiation temperature of 295 K radiates energy mainly in the far infra-red (thermal) region of electromagnetic spectrum. The human body also radiates energy in the infra-red region.

Let us now perform a simple experiment. Take a piece of blackened platinum wire in a dark room. Pass an electrical current through it. You will note that the wire has become hot. Gradually increase the magnitude of the current. After sometime, the wire will begin to radiate. When you pass a slightly stronger current, the wire will begin to glow with dull red light. This shows that the wire is just emitting red radiation of sufficient intensity to affect the human eye. This takes place at nearly 525°C. With further increase in temperature, the colour of the emitted rediation will change from dull red to cherry red (at nearly 900°C) to orange (at nearly 1100°C), to yellow (at nearly 1250°C) until at about 1600°C, it becomes white. What do you infer from this? It shows that the *temperature of a luminous body can be estimated from its colour.* Secondly, *with increase in temperature, waves of shorter wavelengths* (since red light is of longer wavelength than orange. yellow etc.) *are also emitted with sufficient intensity*. Considering in reverse order, you may argue that when the temperature of the wire is below 525°C, it emits waves longer than red but these waves can be detected only by their heating effect.

12.2 Radiation Laws

At any temperature, the radiant energy emitted by a body is a mixture of waves of different wavelengths.The most intense of these waves will have a particular wavelength (sayλ_m).At 400°C, the λ_m will be about 5×10^{-4} cm or 5 μm (1 micron (μ) $=10^{-6}$m) for a copper block.The intensity decreases for wavelengths either greater or less than this value (Fig. 12.5).

Evidently area between each curve and the horizontial axis represents the total rate of radiation at that temperature. You may study the curves shown in Fig. 12.5 and verify the following two facts.

1) The rate of radiation at a particular temperature (represented by the

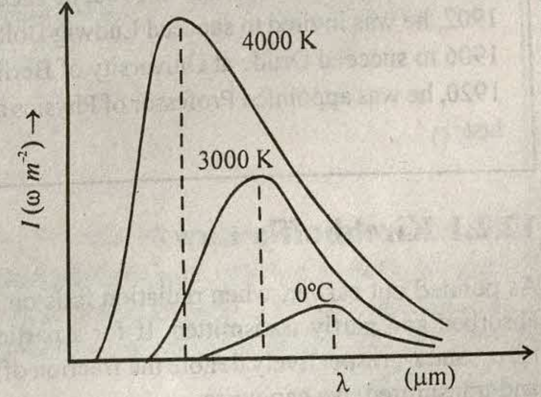


Fig. 12.5 : Variation in intensity with wavelength for a black body at different temperatures

area between each curve and the holizontal axis) increases rapidly with temperature.

2) Each curve has a definite energy maximum and a corresponding wavelength λ_m (i.e. wavelength of the most intense wave). The λ_m shifts towards shorter wavelengths with increasing temperature.

This second fact is expressed quantitatively by what is known as Wien's displacement law. It **states that** ***λ_m shifts towards shorter wavelengths as the temperature of a body is increased.*** This law is., strictly valid only for black bodies. Mathematically, ***we say that the product λ_m T is constant for a body emitting radiation at temperature*** **T:**

$$\lambda_m T = \text{constant} \qquad (12.2)$$

The constant in Eqn. (12.2) has a value 2.884×10^{-3} mK. This law furnishes us with a simple method of determining the temperature of all radiating bodies including those in space. The radiation spectrum of the moon has a peak at $\lambda_m = 14$ micron. Using Eqn. (12.2), we get

$$T = \frac{2884 \text{ micron K}}{14 \text{ micron}} = 206\text{K}$$

That is, the temperature of the lunar surface is 206K

Wilhelm Wien
(1864 – 1928)

The 1911 Nobel Leureate in physics, Wilhelm Wien, was son of a land owner in East Prussia. After schooling at Prussia, he went to Germany for his college. At the University of Berlin, he studied under great physicist Helmholtz and got his doctorate on diffraction of light from metal surfaces in 1886.

He had a very brilliant professional carrer. In 1896, he succeeded Philip Lenard as Professor of Physics at Aix-la-chappelle. In 1899, he become Professor of Physics at University of Giessen and in 1900, he succeeded W.C. Roentgen at Wurzberg. In 1902, he was invited to succeed Ludwig Boltzmann at University of Leipzig and in 1906 to succeed Drude at University of Berlin. But he refused these invitations. In 1920, he was appointed Professor of Physics at munich and he remained there till his last.

12.2.1 Kirchhoff's Law

As pointed out earlier, when radiation falls on matter, it may be partly reflected, partly absorbed and partly transmitted. If for a particular wavelength λ and a given surface, r_λ, a_λ and t_λ, respectively denote the fraction of total incident energy reflected, absorbed and transmitted, we can write

$$1 = r_\lambda + a_\lambda + t_\lambda \qquad (12.3)$$

A body is said to be perfectly black, if $r_\lambda = t_\lambda = 0$ and $a_\lambda = 1$. It means that radiations incident on black bodies will be completely absorbed. As such, perfectly black body does not exist in nature. Lamp black is the nearest approximation to a black body. It absorbs about 96% of visible light and platinum black absorbs about 98%. It is found to transmit light of long wavelength.

A ***perfectly white body***, in contrast, defined as a body with $a_\lambda = 0, t_\lambda = 0$ and $r_\lambda = 1$. A piece of white chalk approximates to a perfectly white body.

This implies that good emitters are also good absorbers. But each body must either absorb or reflect the radiant energy reaching it. So we can say that a good absorber must be a poor reflector (or good emitter).

Designing a Black Body

Kirchoff's law also enables us to design a perfectly black body for experimental purposes. We go back to an enclosure at constant temperature containing radiations between wavelength range λ and $\lambda + d\lambda$. Now let us make a small hole in the enclosure and examine the radiation escaping out of it. This radiation undergoes multiple reflections from the walls. Thus, if the reflecting power of the surface of the wall is r, and emissive power is e_λ, the total radiation escaping out is given by

$$\begin{aligned} E_\lambda &= e_\lambda + e_\lambda r_\lambda + e_\lambda r^2 + e_\lambda r_\lambda^3 + \dots \\ &= e_\lambda (1 + r_\lambda + r_\lambda^2 + r_\lambda^3 + \dots) \\ &= \frac{e_\lambda}{1 - r_\lambda} \end{aligned} \tag{12.4}$$

But from Kirchoffs Law $\dfrac{e_\lambda}{a_\lambda} = E_\lambda$

$$e_\lambda = E_\lambda a_\lambda \tag{12.5}$$

where E_λ is the emission from a black body. If now walls are assumed to be opaque (i.e. $t = 0$), from Eqn. (12.3), we can write

$$a_\lambda = 1 - r_\lambda \tag{12.6}$$

Substituting this result in Eqn. (12.5), we get

$$e_\lambda = E_\lambda (1 - r_\lambda)$$

or

$$E_\lambda = \frac{e_\lambda}{1 - r_\lambda} \tag{12.7}$$

On comparing Eqns. (12.4) and (12.7), we note that the radiation emerging out of the hole will be identical to the radiation from a perfectly black emissive surface. Smaller the hole, the more completely black the emitted radiation is. So we see that the **uniformly heated enclosure with a small cavity behaves as a black body for emission.**

Such an enclosure behaves as a perfectly black body towards incident radiation also. Any radiation passing into the hole will undergo multiple reflections internally within

Notes

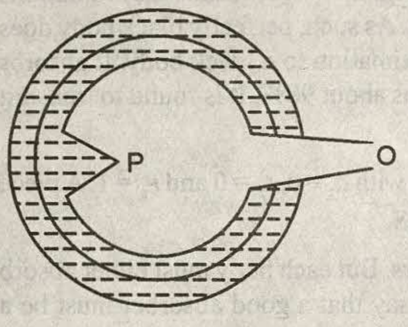


Fig. 12.6 : Fery's black body

the enclosure and will be unable to escape outside. This may be further improved by blackening the inside.Hence the enclosure is a perfect absorber and behaves as a perfectly black body.

Fig. 12.6 shows a black body due to Fery. There is a cavity in the form of a hollow sphere and its inside is coated with black material. It has a small conical opening O. Note the conical projection P opposite the hole O. This is to avoid direct radiation from the surface opposite the hole which would otherwise render the body not perfectly black.

Activity 12.1

You have studied that black surface absorbs heat radiations more quickly than a shiny white surface. You can perform the following simple experiment to observe this effect.

Take two metal plates A and B. Coat one surface of A as black and polish one surface of B. Take an electric heater. Support these on vertical stands such that the coated black surface and coated white surface face the heater. Ensure that coated plates are equidstant from the heater. Fix one cork each with wax on the uncoated sides of the plates.

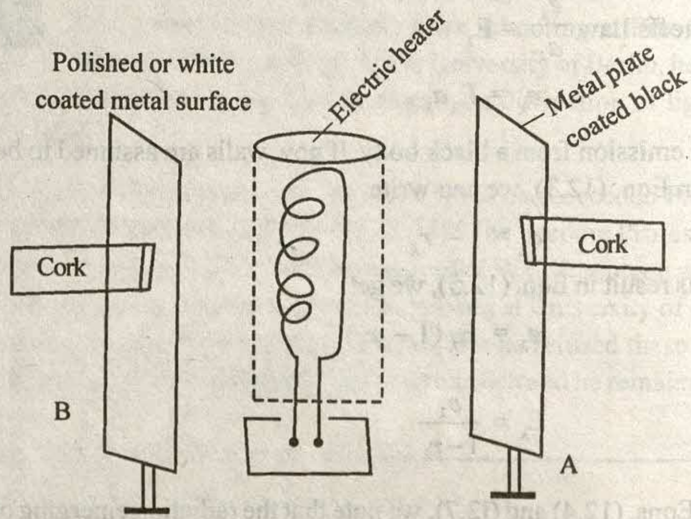


Fig. 12.7 : Showing the difference in heat absorption of a black and a shining surface

Switch on the electric heater. Since both metal plates are identical and placed at the same distance from the heater, they receive the same amount of radiation from it. You will observe that the cork on the blackened plate falls first. This is becasuse the black surface absorbs more heat than the white surface. This proves that black surfaces are good aborbers of heat radiations.

12.2.2 Stefan-Boltzmann Law

On the basis of experimental measurements, Stefan and Boltzmann concluded that the radiant energy emitted per second from a surface of area *A* is proportional to fourth power of temperature :

$$E = Ae\ \sigma\ T^4 \qquad (12.8)$$

where σ is ***Stefan-Boltzmann constant*** and has the value $5.672 \times 10^{-8} \mathrm{J\ m^{-2} s^{-1} K^{-4}}$. The temperature is expressed is kelvin, *e* is emissivity or relative emittance. It depends on the nature of the surface and temperature. The value of *e* lies between 0 and 1; being small for polished metals and 1 for perfectly black materials.

From Eqn. (12.8) you may think that if the surfaces of all bodies are continually radiating energy, why don't they eventually radiate away all their internal energy and cool down to absolute zero. They would have done so if energy were not supplied to them in some way. In fact, all objects radiate and absorb energy simultaneously. If a body is at the same temperaturture as its surroundings, the rate of emission is same as the rate of absorption; there is no net gain or loss of energy and no change in temperature. However, if a body is at a lower temperture than its surroundings, the rate of absorption will be greater than the rate of emission. Its temperature will rise till it is equal to the room temperature. Similarly, if a body is at higher temperature, the rate of emission will be greater than the rate of absorption. There will be a net energy loss. Hence, when a body at a temperature T_1 is placed in surroundings at temperature T_2, the amount of net energy loss per second is given by

$$E_{net} = Ae\sigma\ (T_1^4 - T_2^4) \ \text{ for } T_1 > T_2 \qquad (12.5)$$

Example 12.2 : Determine the surface area of the filament of a 100 W incandescent lamp at 3000 K. Given $\sigma = 5.7 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$, and emissivity *e* of the filament = 0.3.

Solution: According to Stefan-Boltzmann law

$$E = eA\sigma\ T^4$$

where *E* is rate at which energy is emitted, *A* is surface area, and T is tempeature of the surface. Hence we can rewrite it as

$$A = \frac{E}{e\sigma T^4}$$

On substituting the given data, we get

$$A = \frac{100\ \mathrm{W}}{0.3 \times (5.7 \times 10^{-8}\ \mathrm{Wm^{-2}\ K^{-4}} \times (3000\mathrm{K})^4}$$

$$= 7.25 \times 10^{-5}\ \mathrm{m}^2$$

Now it is time for you to check your understanding.

Intext Questions 12.2

1. At what wavelength does a cavity radiator at 300K emit most radiation?

....

2. Why do we wear light colour clothing during summer?

....

3. State the important fact which we can obtain from the experimental study of the spectrum of black body radiation.

....

4. A person with skin temperature 28°C is present in a room at temperature 22°C. Assuming the emissivity of skin to be unity and surface area of the person as 1.9m^2, compute the radiant power of this person.

....

12.3 Solar Energy

You have learnt in your previous classes that sun is the ultimate source of all energy available on the earth. The sun is radiating tremendous amount of energy in the form of light and heat and even the small fraction of that radiation received by earth is more than enough to meet the needs of living beings on its surface. The effective use of solar energy, therefore, may some day provide solution to our energy needs.

Some basic issues related with solar radiations are discussed below.

1. Solar Constant

To calculate the total solar energy reaching the earth, we first determine the amount of energy received per unit area in one second. The energy is called **solar constant.** Solar constant for earth is found to be 1.36 × 10^3 W m^{-2}. Solar constant multiplied by the surface area of earth gives us the total energy received by earth per second. Mathematically,

$$Q = 2\pi R_e^2 C$$

where R_e is radius of earth and C is solar constant

Note that Only half of the earth's surface has been taken into account as only this much of the surface is illuminated at one time. Therefore,

$$Q = 2 \times 3.14 \times (6.4 \times 10^6 \text{ m})^2 \times (1.36 \times 10^3 \text{ W m}^{-2})$$

$$\simeq 3.5 \times 10^{17} \text{ W}$$

$$\simeq 3.5 \times 10^{11} \text{ MW}$$

To determine solar constant for other planets of the solar system, we may make use of Stefan-Boltzman law, which gives the total energy emitted by the sun in one second :

$$\in = (4\pi r^2)\, \sigma T^4$$

where r is radius of sun and T is its temperature.

If R is radius of the orbit of the planet, then

$$E = \frac{\in}{4\pi R^2} = \left(\frac{r}{R}\right)^2 \sigma T^4 \qquad (12.6)$$

And the solar constant (E′) at any other planet orbiting at distance R′ from the sun would be

$$E' = \left(\frac{r}{R'}\right)^2 \sigma T^4 \qquad (12.7)$$

Hence
$$\frac{E'}{E} = \left(\frac{R}{R'}\right)^2 \qquad (12.8)$$

The distance of mars is 1.52 times the distance of earth from the sun. Therefore, the solar constant at mars

$$E' = E \times \left(\frac{1}{1.52}\right)^2$$

$$= 6 \times 10^2 \text{ W m}^{-2}$$

2. Greenhouse Effect

The solar radiations in appropriate amount are necessary for life to flourish on earth. The atmosphere of earth plays an important role to provide a comfortable temperature for the living organisms. One of the processes by which this is done is greenhouse effect.

In a greenhouse, plants, flowers, grass etc. are enclosed in a glass structure. The glass allows short wavelength radiation of light to enter. This radiation is absorbed by plants. It is subsequently re-radiated in the form of longer wavelength heat radiations – the infrared. The longer wavelength radiations are not allowed to escape from the greenhouse as glass is effectively opaque to heat. These heat radiations are thus trapped in the greenhouse keeping it warm.

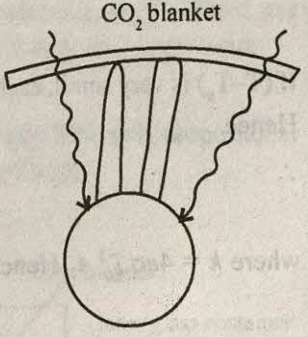


Fig. 12.8 : Green house effect

An analogous effect takes place in our atmosphere. The atmosphere, which contains a trace of carbon dioxide, is transparent to visible light. Thus, the sun's light passes through the atmosphere and reaches the earth's surface. The earth absorbs this light and subsequently emits it as infrared radiation. But carbon dioxide in air is opaque to infra-red radiations. CO_2 reflects these radiations back rather than allowing them to escape into the atmosphere. As a result, the temperature of earth increases. This effect is referred to as the ***greenhouse effect.***

Notes

Due to emission of huge quantities of CO_2 in our atmosphere by the developed as well as developing countries, the greenhouse effect is adding to global warming and likely to pose serious problems to the existence of life on the earth. A recent report by the UN has urged all countries to cut down on their emissions of CO_2, because glaciers have begun to shrink at a rapid rate. In the foreseable future, these can cause disasters beyond imagination beginning with flooding of major rivers and rise in the sea level. Once the glaciers melt, there will be scarcity of water and erosion in the quality of soil. There is a lurking fear that these together will create problems of food security. Moreover, changing weather patterns can cause droughts & famines in some regions and floods in others.

In Indian context, it has been estimated that lack of positive action can lead to serious problems in Gangetic plains by 2030. Also the sea will reclaim vast areas along our coast lie, inundating millions of people and bring unimaginable misery and devastation. How can you contribute in this historical event?

12.4 Newton's Law of Cooling

Newton's law of cooling states that ***the rate of cooling of a hot body is directly proportional to the mean excess temperature of the hot body over that of its surroundings provided the difference of temperature is small. The law can be deduced from stefan-Boltzmann law.***

Let a body at temperature T be surrounded by another body at T_0. The rate at which heat is lost per unit area per second by the hot body is

$$E = e\sigma(T^4 - T_0^4)A \qquad (12.9)$$

As $T^4 - T_0^4 = (T^2 - T_0^2)(T^2 + T_0^2) = (T - T_0)(T - T_0)(T^2 + T_0^2)$. Hence (12.10)

$$E = e\sigma(T - T_0)(T^3 + T^2 T_0 + T T_0^2 + T_0^3)A$$

If $(T - T_0)$ is very small, each of the term T^3, T^2T_0, TT_0^2 and T_0^3 may be approximated to T_0^3. Hence

$$\therefore \quad E = e\sigma(T - T_0)\,4T_0^3A$$

$$= k(T - T_0)$$

where $k = 4e\sigma T_0^3 A$. Hence,

$$E \alpha (T - T_0) \qquad (12.11)$$

This is ***Newton's law of cooling.***

Intext Questions 12.3

1. Calculate the power received from sun by a region 40m wide and 50m long located on the surface of the earth?

....

2. What threats are being posed for life on the earth due to rapid consumption of fossil fuels by human beings?

....

3. What will be shape of cooling curve of a liquid?

....

What You Have Learnt

- Heat flows from a body at higher temperature to a body at lower temperature. There are three processes by which heat is transferred : conduction, convection and radiation.
- In conduction, heat is transferred from one atom/ molecule to another atom/molecule which vibrate about their fixed positions.
- In convection, heat is transferred by bodily motion of molecules. In radiation, heat is transferred through electromagnetic waves.
- The quantity of heat transferred by conduction is given by

$$Q = \frac{K(T_h - T_c)\ At}{d}$$

- *Wien's Law.* The spectrum of energy radiated by a body at temperature T(K) has a maxima at wavelength λ_m' such that $\lambda_m T$ = constant (= 2880 μK)
- Stefan-Boltzmann Law. The rate of energy radiated by a source at T(K) is given by $E = e\sigma AT^4$

  The absorptive power a is defined as

$$\mathbf{a} = \frac{\text{Total amount of energy absorbed between } \lambda \text{ and } \lambda + d\lambda}{\text{Total amount of incident energy between } \lambda \text{ and } \lambda + d\lambda}$$

- The emissive power of a surface e_λ is the amount of radiant energy emitted per square metre area per second per unit wavelength range at a given temperature.
- The solar constant for the earth is 1.36×10^3 Jm^{-2} s^{-1}
- Newton's Law of cooling states that the rate of cooling of a body is linearly proportional to the excess of temperature of the body above its surroundings.

Terminal Exercise

1. A thermosflask (Fig.12.9) is made of a double walled glass bottle enclosed in metal container. The bottle contains some liquid whose temperature we want to maintain, Look at the diagram carefully and explain how the construction of the flask helps in minimizing heat transfer due to conduction convection and radiation.

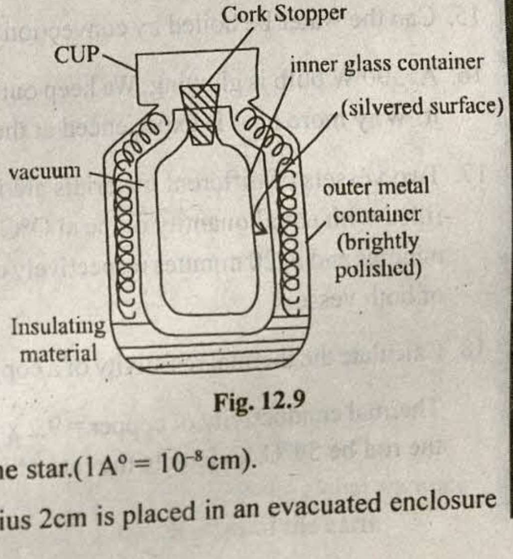


Fig. 12.9

2. The wavelength corresponding, to emission of energy maxima of a star is 4000 A°. Compute the temperature of the star.(1A° = 10^{-8} cm).
3. A blackened solid copper sphere of radius 2cm is placed in an evacuated enclosure

whose walls are kept at 1000° C. At what rate must energy be supplied to the sphere to keep its temperature constant at 127° C.

4. Comment on the statement "A good absorber must be a good emitter"

5. A copper pot whose bottm surface is 0.5cm thick and 50 cm in diameter rests on a burner which maintains the bottom surface of the pot at 110°C. A steady heat flows through the bottom into the pot where water boils at atmospheric pressure. The actual temperature of the inside surface of the bottom of the pot is 105°C. How many kilograms of water boils off in one hour?

6. Define the coefficient of thermal conductivity. List the factors on which it depends.

7. Distinguish between conduction and convection methods of heat (transfer).

8. If two or more rods of equal area of cross-section are connected in series, show that their equivalent thermal resistance is equal to the sum of thermal resistance of each rod. [Note : Thermal resistance is reciprocal of thermal conductivity]

9. Ratio of coefficient of thermal conductivities of the different materials is 4:3. To have the same thermal resistance of the two rods of these materials of equal thickness. what should be the ratio of their lengths?

10. Why do we feel warmer on a winter night when clouds cover the sky than when the sky is clear?

11. Why does a peice of copper or iron appear hotter to touch than a smilar piece of wood even when both are at the same temperature?

12. Why is it more difficult to sip hot tea from a metal cup than from a china-clay cup?

13. Why are the woollen clothes warmer than cotton clothes?

14. Why do two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double the thickness?

15. Can the water be boiled by convection inside an earth satellite?

16. A. 500 W bulb is glowing. We keep our one hand 5 cm above it and other 5 cm below it. Why more heat is experienced at the upper hand?

17. Two vessels of different materials are identical in size and in dimensions. They are filled with equal quantity of ice at O°C. If ice in both vessles metls completely in 25 minutes and in 20 minutes respectively compare the (thermal conductivities) of metals of both vessels.

18. Calculate the thermal resistivity of a copper rod 20.0 cm. length and 4.0 cm. in diamter.

Thermal conductivity of copper = 9.2 x 10^{-2} temperature different acrosss the ends of the rod be 50°C. Calculate the rate of heat flow.

Answers to Intext Questions

12.1

1. Conduction is the principal mode of transfer of heat in solids in which the particles transfer energy to the adjoining molecules.

   In convection the particles of the fluid bodily move from high temperature region to low temperature region and vice-versa.

2. $K = \frac{Qd}{t\,A(Q_2 - Q_1)}$

   $= \frac{J}{s}\frac{m}{m^2\,{}^\circ C}$

   $= J\,s^{-1}\,m^{-1}\,{}^\circ C^{-1}$

3. The trapped air in wool fibres prevents body heat from escaping out and thus keeps the wearer warm.

4. The coefficient of thermal conductivity is numerically equal to the amount of heat energy transferred in one second across the faces of a cubical slab of surface area $1m^2$ and thickness 1m, when they are kept at a temperature difference of 1°C.

5. During the day, land becomes hotter than water and air over the ocean is cooler than the air near the land. The hot dry air over the land rises up and creates a low pressure region. This causes see breeze because the moist air from the ocean moves to the land. Since specific thermal capacity of water is higher than that of sand, the latter gets cooled faster and is responsible for the reverse process during the night causing land breezes.

12.2

1. $\lambda_m = \frac{\text{Wien's constant}}{\text{Temperature}}$

   $= \frac{2880\mu K}{300K}$

   $= 9.6\mu$

2. Hint: Because light colours absorb less heat.

3. Hint: (a) $\lambda_m T = S$ (b) $t = \sigma T^4$

4. 66.4 W.

12.3

1. Solar constant x .area

   $= 2.7 \times 10^5$ W

2. Constant addition of CO_2 in air will increase greenhouse effect causing global warming due to which glaciers are likely to melt and flood the land mass of the earth.

3. Exponential decay

Answers to Terminal Problem

2. 7210 K

3. 71.6×50^{-11} W

5. 4.7×10^{5} kg

9. 3 : 4

17. 4 : 5

18. 10.9 m $^{\circ}C^{-1}$ W^{-1}, 0.298 W

SENIOR SECONDARY COURSE

PHYSICS

STUDENT'S ASSIGNMENT – 3

Maximum Marks: 50 **Time : 1½ Hours**

INSTRUCTIONS

- Answer All the questions on a seperate sheet of paper
- Give the following information on your answer sheet:
  - Name
  - Enrolment Number
  - Subject
  - Assignment Number
  - Address
- Get your assignment checked by the subject teacher at your study centre so that you get positive feedback about your performance.

Do not send your assignment to NIOS

1. At what temperature molecular motion ceases. (1)
2. What is the type (kinetic/or potential) of internal energy of an ideal gas? (1)
3. Why change in temperature of water from 14.5° C to 15.5° C is specified in defining one calorie? (1)
4. At what temperature do the Celsius and Fahrenheit scales coincide? (1)
5. What is indicated by the statement. "Internal energy is positive"? (1)
6. State two reasons due to which all practical engines have an efficiency less than the carnot's engine. (1)
7. Which diagram plays important role to explain the theory of heat engine? (1)
8. Write the dimension of coefficient of thermal conductivity. (1)
9. State two limitations of carnot's engine. (2)
10. Every gas has two specific heats where as each liquid and solid has only one specific value of specific heat, why? (2)
11. A refrigerator transfers heat from the cooling coil at low temperature to the warm surroundings. Is it against the second law of thermodynamics? Justify your answer. (2)
12. Two rods X and Y are of equal lengths. Each rod has its ends at temperature T_1 and T_2 respectively ($T_1 > T_2$). What is the condition that will ensure equal rates of flow of heat through the rods X and Y? (2)

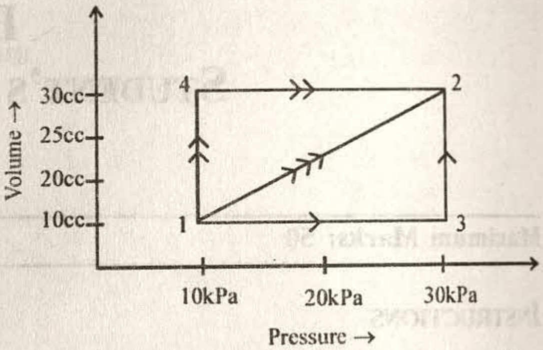
$$\left[\text{Hint}: \frac{dQ_1}{dt} = \frac{dQ_2}{dt} \Rightarrow \frac{K_1 A_1 \Delta T}{\Delta x} = \frac{K_2 A_2 \Delta T}{\Delta x} \Rightarrow \frac{A_1}{A_2} = \frac{K_2}{K_1}\right]$$ (2)

13. State first law of thermodynamics. Figure shows three paths through which a gas can be taken from the state 1 to state 2. Calculate the work done by the gas in each of the three paths. (4)

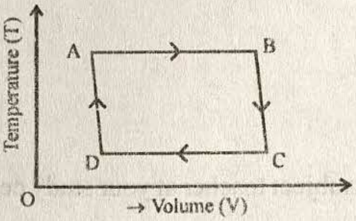
[Hint : Path 1 → 3 → 2 $w_{13} + w_{32} = 0 + p\Delta v = 0.455$

Path1 → 2 $w_{12} = \frac{1}{2}(10+3)\times 10^3 \times 15 \times 10^{-6} = 0.3t$

Path 1→ 4 ⇒ 2 $w_{14} + w_{42} = p\Delta v + 0 = 0.15t$]

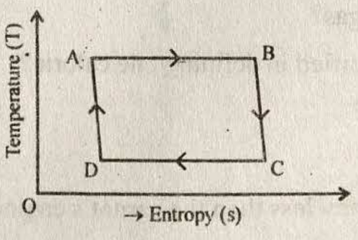


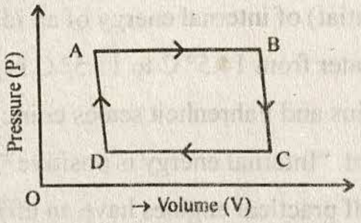
14. The P - V diagram of a certain process (carnot cycle) is reflected in figure a.Represent it on T-V and T-S diagrams. (4)



(a)

Hint :





15. Differentiate between isothermal, adiabatic, isobaric and isochoric processes. (4)

16. State Zeroth and first law of thermodynamics. Discuss the limitations of first law of thermodynamics. (4)

17. State and explain second law of thermodynamics. (4)

What do you mean by the following terms :

(*i*) thermal conductivity of a solid (*ii*) variable state of a metallic rod (*iii*) steady state of a matallic rod (*iv*) coefficient of thermal conductivity. (4)

19. Briefly describe a carnot cycle and derive an expression for efficiency of this cycle. (5)

20. What is a heat engine? Obtain an expression for its efficiency. Explain the workig principle of a referigerator. Obtain an expression for its coefficient of performance. Also obtain a relation between coefficient of performance of a refrigerator and efficiency of a heat engine. $2+2+1=(5)$

MODULE - IV
OSCILLATIONS AND WAVES

13. Simple Harmonic Motion
14. Wave Phenomena

13

SIMPLE HARMONIC MOTION

You are now familiar with motion in a straight line, projectile motion and circular motion. These are defined by the path followed by the moving object. But some objects execute motion which are repeated after a certain interval of time. For example, beating of heart, the motion of the hands of a clock, to and fro motion of the swing and that of the pendulum of a bob are localised in space and repetitive in nature. Such a motion is called periodic motion. It is universal phenomenon.

In this lesson, you will study about the periodic motion, particularly the oscillatory motion which we come across in daily life. You will also learn about simple harmonic motion. Wave phenomena – types of waves and their characteristics–form the subject matter of the next lesson.

Objectives

After studying this lesson, you should be able to :

- *show that an oscillatory motion is periodic but a periodic motion may not be necessarily oscillatory;*
- *define simple harmonic motion and represent it as projection of uniform circular motion on the diameter of a circle;*
- *derive expressions of time period of a given harmonic oscillator;*
- *derive expressions for the potential and kinetic energies of a simple harmoic oscillator; and*
- *distinguish between free, damped and forced oscillations.*

13.1 Periodic Motion

You may have observed a clock and noticed that the pointed end of its seconds hand and that of its minutes hand move around in a circle, each with a fixed period. The seconds hand completes its journey around the dial in one minute but the minutes hand takes one hour to complete one round trip. However, a pendulum bob moves to and fro about a mean position and completes its motion from one end to the other and back to its initial position in a fixed time. A motion which repeats itself after a fixed interval of time is called *periodic motion*. There are two types of periodic motion : (i) ***non–oscillatory,*** and (ii) ***oscillatory***. The motion of the hands of the clock is non-oscillatory but the to and fro motion of the pendulum bob is oscillatory. However, both the motions are periodic. It is important to note that an oscillatory motion is normally periodic but a periodic motion is not necessarily oscillatory. Remember that a motion which repeats itself in equal intervals of time is periodic and if it is about a mean position, it is ***oscillatory***.

We know that earth completes its rotation about its own axis in 24 hours and days and nights are formed. It also revolves around the sun and completes its revolution in 365 days. This motion produces a sequence of seasons. Similarly all the planets move around the Sun in elliptical orbits and each completes its revolution in a fixed interval of time. These are examples of periodic non-oscillatory motion.

Jean Baptiste Joseph Fourier
(1768 – 1830)

French Mathematician, best known for his Fourier series to analyse a complex oscillation in the form of series of sine and consine functions.

Fourier studied the mathematical theory of heat conduction. He established the partial differential equation governing heat diffusion and solved it by using infinite series of trigonometric functions.

Born as the ninth child from the second wife of a taylor, he was orphened at the age of 10. From the training as a priest, to a teacher, a revolutionary, a mathematician and an advisor to Nepolean Bonapart, his life had many shades.

He was a contemporary of Laplace, Lagrange, Biot, Poission, Malus, Delambre, Arago and Carnot. Lunar crator Fourier and his name on Eiffel tower are tributes to his contributions.

Activity 13.1

Suppose that the displacement y of a particle, executing simple harmonic motion, is represented by the equation :

$$y = a \sin \theta \tag{13.1}$$

or

$$y = a \cos \theta \tag{13.2}$$

From your book of mathematics, obtain the values of sin θ and cos θ for $\theta = 0, 30^0, 60^0, 90^0, 120^0, 150^0, 180^0, 240^0, 300^0, 330^0$ and 360^0. Then assuming that $a = 2.5$cm, determine the values of y corresponding to each angle using the relation $y = a \sin \theta$. Choose a suitable scale and plot a graph between y and θ. Similarly, using the relation $y = a \cos\theta$, plot another graph between y and θ. You will note that both graphs represents an oscillation between $+a$ and $-a$. It shows that a certain type of oscillatory motion can be represented by an expression containing sine or cosine of an angle or by a combination of such expressions.

Now check your progress by answering the following questions.

Intext Questions 13.1

1. What is the difference between a periodic motion and an oscillatory motion?

   ....

2. Which of the following examples represent a periodic motion?

   (i) A bullet fired from a gun, (ii) An electron revolving round the nucleus in an atom, (iii) A vehicle moving with a uniform speed on a road, (iv) A comet moving around the Sun, and (v) Motion of an oscillating mercury column in a U-tube.

   ....

3. Give an example of (i) an oscillatory periodic motion and (ii)Non-oscillatory periodic motion.

   ....

13.2 Simple Harmonic Motion : Circle of Reference

The oscillations of a harmonic oscillator can be represented by terms containing sine and cosine of an angle. If the displacement of an oscillatory particle from its mean position can be represented by an equation $y = a \sin\theta$ or $y = a \cos\theta$ or $y = A \sin\theta + B \cos\theta$, where a, A and B are constants, the particle executes simple harmonic motion. We define simple harmonic motion as under :

A particle is said to execute simple harmonic motion if it moves to and fro about a fixed point periodically, under the action of a force* F *which is directly proportional to its displacement* x *from the fixed point and the direction of the force is opposite to that of the displacement. We shall restrict our discussion to linear oscillations. Mathematically, we express it as

$$F = -kx$$

where k is constant of proportionality.

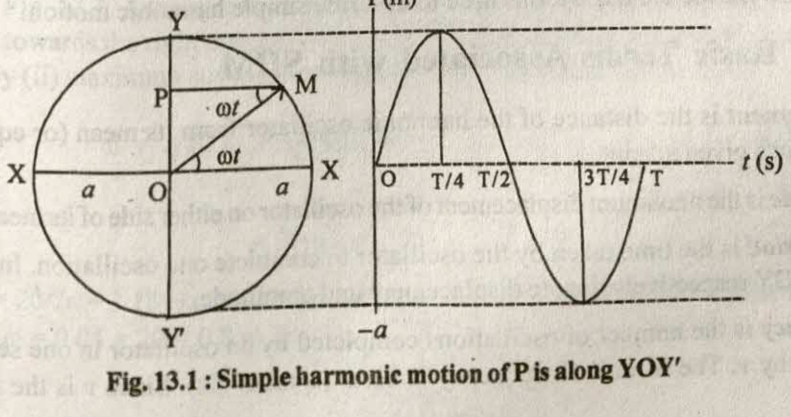


Fig. 13.1 : Simple harmonic motion of P is along YOY'

To derive the equation of simple harmonic motion, let us consider a point M moving with a constant speed υ in a circle of radius a (Fig. 13.1) with centre O. At $t = 0$, let the point be at X. The position vector OM specifies the position of the moving point at time t,. It is obvious that the position vector OM, also called the ***phaser,*** rotates with a constant angular velocity $\omega = \upsilon / a$. The acceleration of the point M is $\upsilon^2/a = a\,\omega^2$ towards the centre O. At time t, the component of this acceleration along OY $= a\omega^2 \sin \omega t$. Let us draw MP perpendicular to YOY′. Then P can be regarded as a particle of mass m moving with an acceleration $a\omega^2 \sin \omega t$. The force on the particle P towards O is therefore given by

$$F = ma\omega^2 \sin \omega t$$

But $\sin \omega t = y/a$. *Therefore*

$$F = m\omega^2 y \qquad (13.3)$$

The displacement is measured from O towards P and force is directed towards O. Therefore,

$$F = -\,m\omega^2 y$$

Since this force is directed towards O, and is proportional to displacement 'y' of P from O,' we can say that the particle P is executing simple harmonic motion.

Let us put $m\omega^2 = k$, a constant. Then Eqn. (13.3) takes the form

$$F = -\,k\,y \qquad (13.4)$$

The constant k, which is force per unit displacement, is called ***force constant***. The angular frequency of oscillations is given by

$$\omega^2 = k / m \qquad (13.5)$$

In one complete rotation, OM describes an angle 2π and it takes time T to complete one rotation. Hence

$$\omega = 2\pi/T \qquad (13.6)$$

On combining Eqns. (13.5) and (13.6), we get an expression for time period :

$$T = 2\pi \sqrt{k/m} \qquad (13.7)$$

This is the time taken by P to move from O to Y, then through O to Y′ and back to O. During this time, the particle moves once on the circle and the foot of perpendicular from its position is said to make an oscillation about O as shown in Fig.13.1.

Let us now define the basic terms used to describe simple harmonic motion.

13.2.1 Basic Terms Associated with SHM

Displacement is the distance of the harmonic oscillator from its mean (or equilibrium) position at a given instant.

Amplitude is the maximum displacement of the oscillator on either side of its mean position.

Time period is the time taken by the oscillator to complete one oscillation. In Fig. 13.1, OP, and OY respectively denote displacement and amplitude.

Frequency is the number of oscillations completed by an oscillator in one second. It is denoted by v. The SI unit of frequency is hertz (symbol Hz). Since v is the number of oscillations per second, the time taken to complete one oscillation is $1/v$. Hence $T=1/v$ or $v=(1/T)\ \text{s}^{-1}$. As harmonic oscillations can be represented by expressions containing $\sin\theta$ and or $\cos\theta$, we introduce two more important terms.

Phase ϕ is the angle whose sine or cosine at a given instant indicates the position and direction of motion of the oscillator. It is expressed in radians.

Angular Frequency ω describes the rate of change of phase angle. It is expressed in radian per second. Since phase angle ϕ changes from 0 to 2π radians in one complete oscillation, the rate of change of phase angle is $\omega = 2\pi/T = 2\pi\, v$ or $\omega = 2\pi v$.

Example 13.1 : A tray of mass 9 kg is supported by a spring of force constant k as shown in Fig. 13.2. The tray is pressed slightly downward and then released. It begins to execute SHM of period 1.0s. When a block of mass M is placed on the tray, the period increases to 2.0s. Calculate the mass of the block.

Solution: The angular frequency of the system is given by $\omega = \sqrt{k/m}$, where m is the mass of the oscillatory system. Since $\omega = 2\pi/T$, from Eqn. (13.7) we get

$$4\pi^2/T^2 = \frac{k}{m}$$

or

$$m = \frac{kT^2}{4\pi^2}$$

When the tray is empty, $m = 9\text{kg}$ and $T = 1\text{s}$. Therefore

$$9 = \frac{k(1)^2}{4\pi^2}$$

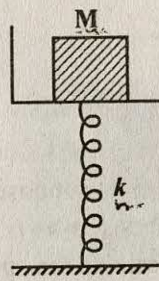


Fig. 13.2

On placing the block, $m = 9 + M$ and $T = 2\text{s}$. Therefore, $9 + M = k \times (2)^2/4\pi^2$

From the above two equations we get

$$\frac{(9+M)}{9} = 4$$

Therefore, $M = 27\text{kg}$.

Example 13.2 : A spring of force constant 1600 N m^{-1} is mounted on a horizontal table as shown in Fig. 13.3. A mass $m = 4.0$ kg attached to the free end of the spring is pulled horizontally towards the right through a distance of 4.0 cm and then set free. Calculate (i) the frequency (ii) maximum acceleration and (iii) maximum speed of the mass.

Solution :

$$\omega = \sqrt{k/m} = \sqrt{1600/4}$$

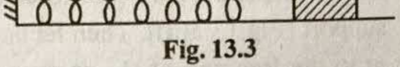


Fig. 13.3

$$= 20\text{rad s}^{-1}.$$

Therefore $v = 20/2\pi = 3.18$ Hz. Maximum acceleration $= a\,\omega^2 = 0.04 \times 400 = 16\ \text{m s}^{-2}$, and $\upsilon_{\max} = a\,\omega = 0.04 \times 20 = 0.8\ \text{m s}^{-1}$.

13.3 Examples of SHM

In order to clarify the concept of SHM, some very common examples are given below.

13.3.1 Horizontal Oscillations of a Spring-Mass System

Consider a elastic spring of force constant k placed on a smooth horizontal surface and attached to a block P of mass m. The other end of the spring is attached to a rigid wall (Fig. 13.4)). Suppose that the mass of the spring is negligible in comparison to the mass of the block.

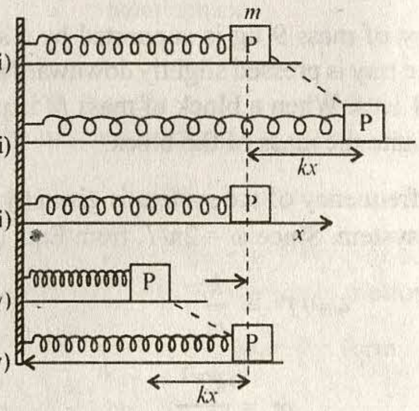


Fig.13.4 : Oscillations of a spring-mass system

Let us suppose that there is no loss of energy due to air resistance and friction. We choose x–*axis* along the horizontal direction. Initially, that is, at $t = 0$, the block is at rest and the spring is in relaxed condition [Fig.13.4(i)]. It is then pulled horizontally through a small distance [Fig. 13.4 (ii)]. As the spring undergoes an extension x, it exerts a force kx on the block. The force is directed against the extension and tends to restore the block to its equilibrium position. As the block returns to its initial position [Fig. 13.4 (iii)], it acquires a velocity v and hence a kinetic energy $K = (1/2)\, m v^2$. Owing to inertia of motion, the block overshoots the mean position and continues moving towards the left till it arrives at the position shown in Fig. 13.4 (iv). In this position, the block again experiences a force kx which tries to bring it back to the initial position [Fig. 13.4 *v*]. In this way, the block continues oscillating about the mean position. The time period of oscillation is $2\pi\sqrt{m/k}$, where k is the force per unit extension of the spring.

13.3.2 Vertical Oscillations of a Spring–Mass System

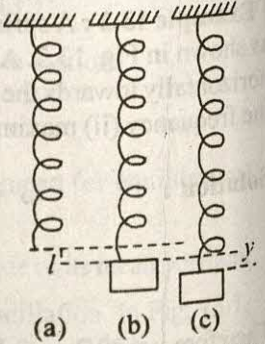


Fig.13.5: Vertical oscillations of a a spring–mass system

Let us suspend a spring of force constant k from a rigid support [Fig.13.5(a)]. Then let us attach a block of mass m to the free end of the spring. As a result of this, the spring undergoes an extension, say l [Fig.13.5(b)]. Obviously, the force constant of the spring is $k = mg/l$. Let us now pull down the block through a small distance, y (Fig.13.5 (c)]. A force ky acts on the block vertically upwards. Therefore, on releasing the block, the force *ky* pulls it upwards. As the block returns to its initial position, it continues moving upwards on account of the velocity it has gained. It overshoots the equilibrium position by a distance *y*. The compressed spring now applies on it a restoring force downwards. The block moves downwards and again overshoots the equilibrium position by almost the same vertical distance *y*. Thus, the system continues to execute vertical oscillations. The angular frequency of vertical oscillations is

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

Hence,

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (13.8)$$

This result shows that acceleration due to gravity does not influence vertical oscillations of a spring–mass system.

Galileo Galilei

(1564-1642)

Son of Vincenzio Galilei, a wool merchant in Pisa, Italy, Galileo is credited for initiating the age of reason and experimentation in modern science. As a child, he was interested in music, art and toy making. As a young man, he wanted to become a doctor. To pursue the study of medicine, he entered the University of Pisa. It was here that he made his first discovery - the isochronosity of a pendulum, which led Christian Huygen to construct first pendulum clock.

For lack of money, Galileo could not complete his studies, but through his efforts, he learnt and developed the subject of mechanics to a level that the Grand Duke of Tuscany appointed him professor of mathematics at the University of Pisa.

Galileo constructed and used telescope to study celestial objects. Through his observations, he became convinced that Copernican theory of heliocentric universe was correct. He published his convincing arguments in the form of a book, "A Dialogue On The Two Principal Systems of The World", in the year 1632. The proposition being at variance with the Aristotelian theory of geocentric universe, supported by the Church, Galileo was prosecuted and had to apologize. But in 1636, he published another book "Dialogue On Two New Sciences" in which he again showed the fallacy in Aristotle's laws of motion.

Because sophisticated measuring devices were not available in Galileo's time, he had to apply his ingenuity to perform his experiments. He introduced the idea of thought-experiments, which is being used even by modern scientists, in spite of all their sophisticated devices.

13.3.3 Simple Pendulum

A simple pendulum is a small spherical bob suspended by a long cotton thread held between the two halves of a clamped split cork in a stand,as shown in Fig. 13.6. The bob is considered a point mass and the string is taken to be *inextensible*. The Pendulum can oscillate freely about the point of suspension.

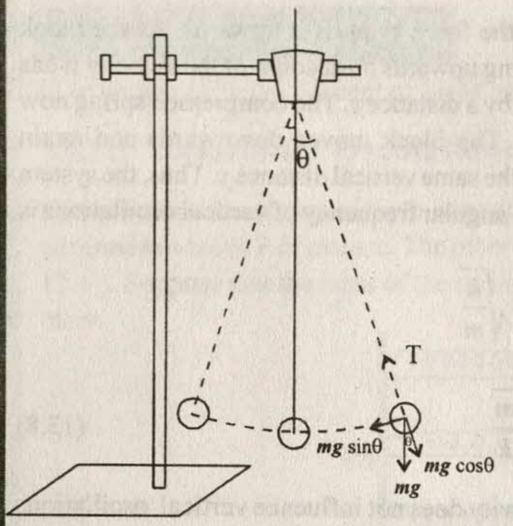


Fig.13.6 : Simple Pendulum

When the pendulum is displaced through a *small distance* from its equilibrium position and then let free, it executes angular oscillations in a vertical plane about its equilibrium position. The distance between the point of suspension and the centre of gravity of the bob defines the length of the pendulum. The forces acting on the bob of the pendulum in the displaced position shown in Fig. 13.6 are : (*i*) the weight of the bob *mg* vertically downwards, and (*ii*) tension in the string **T** acting upwards along the string.

The weight *mg* is resolved in two components : (a) *mg* cosθ along the string but opposite to **T** and (b) *mg* sinθ perpendicular to the string. The component *mg* cosθ balances the tension **T** and the component *mg* sinθ produces acceleration in the bob in the direction of the mean position. The restoring force, therefore, is *mg* sinθ. For small displacement *x* of the bob, the restoring force is $F = mg\theta = mg\ x/l$. The force per unit displacement $k = mg/l$ and hence

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/l}{m}} = \sqrt{\frac{g}{l}}$$

or

$$\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

Hence,

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (13.9)$$

Measuring Weight using a Spring

We use a spring balance to measure weight of a body. It is based on the assumption that within a certain limit of load, there is equal extension for equal load, i.e., load/extension remains constant (force constant). Therefore, extension varies linearly with load. Thus you can attach a linear scale alongside the spring and calibrate it for known load values. The balance so prepared can be used to measure unknown weights.

Will such a balance work in a gravity free space, as in a space-rocket or in a satellite?

Obviously not becuase in the absence of gravity, no extension occurs in the spring. Then how do they measure mass of astronauts during regular health check up? It is again a spring balance based on a different principle. The astronaut sits on a special chair with a spring attached to each side (Fig.13.7). The time period of oscillations of the chair with and without the astronaut is determined with the help of an electronic clock :

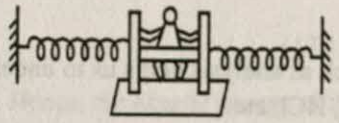


Fig. 13.7 : Spring balance for measuring the mass of an astronaut

$$T_1^2 = \frac{4\pi^2 m}{k}$$

where m is mass of the astronaut. If m_0 is mass of the chair, we can write

$$T_0^2 = \frac{4\pi^2 m_0}{k}$$

T_1 is time period of ocillation of the chair with the astronaut and T_0 without the astronaut.

On subtracting one from another, we get

$$T_1^2 - T_0^2 = \frac{4\pi^2}{k}(m - m_o)$$

$$\Rightarrow \qquad m = \frac{k}{4\pi^2}(T_1^2 - T_0^2) + m_o$$

Because the values of T_0 and k are fixed and known, a measure of T_1 itself shows the variation in mass.

Example 13.3 : Fig. 13.8 shows an oscillatory system comprising two blocks of masses m_1 and m_2 joined by a massless spring of spring constant k. The blocks are pulled apart, each with a force of magnitude F and then released. Calculate the angular frequency of each mass. Assume that the blocks move on a smooth horizontal plane.

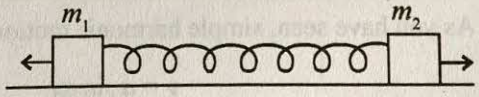


Fig. 13.8 : Oscillatory system of masses attached to a spring

Solution : Let x_1 and x_2 be the displacements of the blocks when pulled apart. The extension produced in the spring is $x_1 + x_2$. Thus the acceleration of m_1 is $k(x_1 + x_2)/m_1$ and acceleration of m_2 is $k(x_1 + x_2)/m_2$. Since the same spring provides the restoring force to each mass, hence the net acceleration of the system comprising of the two masses and the massless spring equals the sum of the acceleration produced in the two masses. Thus the acceleration of the system is

$$a = \frac{k(x_1 + x_2)}{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)} = \frac{kx}{\mu}$$

where $x = x_1 + x_2$ is the extension of the spring and μ is the reduced mass of the system. The angular frequency of each mass of the system is therefore,

Notes

$$\omega = \sqrt{k/\mu} \tag{13.10}$$

Such as analysis helps us to understand the vibrations of diatomic molecules like H_2, Cl_2, HCl, etc.

Intext Questions 13.2

1. A small spherical ball of mass *m* is placed in contact with the sunface on a smooth spherical bowl of radius *r* a little away from the bottom point. Calculate the time period of oscillations of the ball (Fig. 13.9).

....

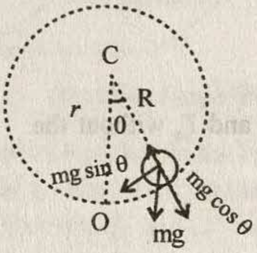


Fig. 13.9

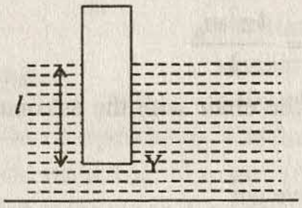


Fig.13.10

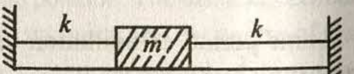


Fig. 13.11

2. A cylinder of mass *m* floats vertically in a liquid of density ρ. The length of the cylinder inside the liquid is *l*. Obtain an expression for the time period of its oscillations (Fig. 13.10).

....

3. Calculate the frequency of oscillation of the mass *m* connected to two rubber bands as shown in Fig. 13.11. The force constant of each band is *k*. (Fig. 13.11)

....

13.4 Energy of Simple Harmonic Oscillator

As you have seen, simple harmonic motion can be represented by the equation

$$y = a \sin \omega t \tag{13.11}$$

When t changes to $t + \Delta t$, y changes to $y + \Delta y$. Therefore, we can write

$$y + \Delta y = a \sin\omega\,(t + \Delta t) = a \sin\,(\omega t + \omega\Delta t)$$

$$= a\,[\sin\omega t \cos \omega\Delta t + \cos\omega t \sin \omega\Delta t]$$

As $\Delta t \to 0$, $\cos \omega\Delta t \to 1$ and $\sin \omega\,\Delta t \to \omega\,\Delta t$. Then

$$y + \Delta y = a \sin \omega t + a\,\omega\Delta t \cos \omega t. \tag{13.12}$$

Subtracting Eqn. (13.11) from Eqn. (13.12), we get

$$\Delta y = \Delta t\,\omega a \cos \omega t$$

so that $$\Delta y/\Delta t = \omega a \cos \omega t$$

or $$\upsilon = \omega a \,\text{cos}t\, \omega t \tag{13.13}$$

where $\upsilon = \Delta y/\Delta t$ is the velocity of the oscillator at time t. Hence, the ***kinetic energy*** of the oscillator at that instant of time is $K = (1/2)\, m\upsilon^2 = (1/2)\, \omega^2 a^2 \cos^2 \omega t$ (13.14)

Let us now calculate the potential energy of the oscillator at that time. When the displacement is y, the restoring force is ky, where k is the force constant. For this purpose we shall plot a graph of restoring force ky versus the displacement y. We get a straight line graph as shown in Fig. 13.12. Let us take two points P and Q and drop perpendiculars PM and QN on x–axis. As points P and Q are close to each other, trapezium PQNM can be regarded as a rectangle. The area of this rectangular strip is ($ky\ \Delta y$). This area equals the work done against the restoring force ky when the displacement changes by a small amount Δy. The area of the triangle OBC is, therefore, equal to the work done in the time displacement changes from O to OB ($= y$) = $\frac{1}{2}ky^2$. This work done against the conservative force is the ***potential energy*** U of the oscillator. Thus, the potential energy of the oscillator when the displacement is y is

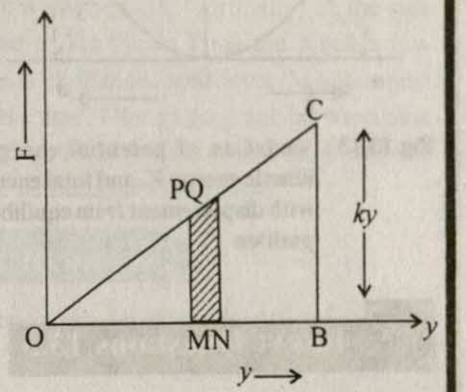


Fig.13.12: Graph between the displacement y and the restoring force ky

$$\text{U} = \frac{1}{2}ky^2$$

But $\omega^2 = k/m$. Therefore, substituting $k = m\omega^2$ in above expression we get

$$\text{U} = \frac{1}{2}m\omega^2 y^2$$

Further as $y = a \sin \omega t$, we can write

$$U = \frac{1}{2}m\omega^2 a^2 \sin^2 \omega t \tag{13.15}$$

On combining this result with Eqn. (13.14), we find that total energy of the oscillator at any instant is given by

$$E = U + K$$

$$= \frac{1}{2}m\omega^2 a^2\ (\sin^2 \omega t + \cos^2 \omega t)$$

$$= \frac{1}{2}ma^2\omega^2 \tag{13.16}$$

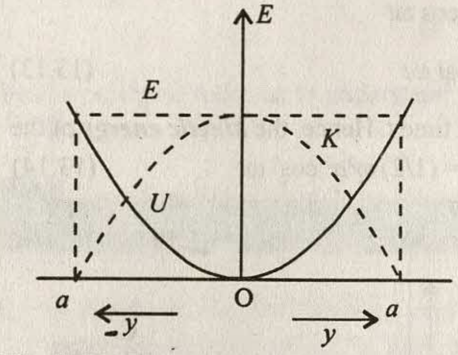


Fig.13.13 : Variation of potential energy U, kinetic energy K, and total energy E with displacement from equilibrium position

The graph of kinetic energy K, potential energy U and the total energy E versus displacement y is shown in Fig.13.13. From the graph it is evident that for $y = 0$, $K = E$ and $U = 0$. As the displacement y from the mean position increases, the kinetic energy decreases but potential energy increases. At the mean position, the potential energy is zero but kinetic energy is maximum. At the extreme positions, the energy is wholly potential. However, the sum $K + U = E$ is constant.

Intext Questions 13.3

1. Is the kinetic energy of a harmonic oscillator maximum at its equilibrium position or at the maximum displacement position? Where is its acceleration maximum?

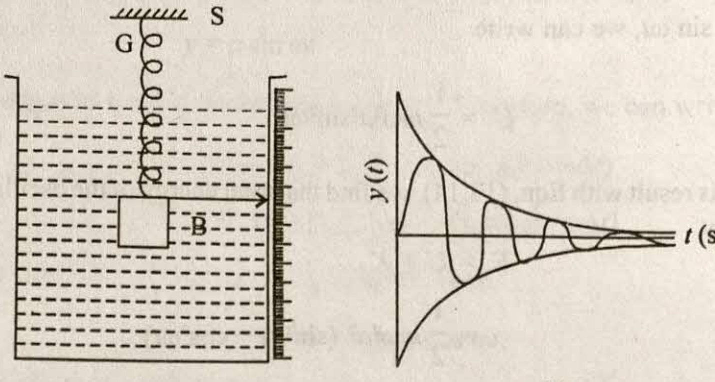
....

2. Why does the amplitude of a simple pendulum decrease with time? What happens to the energy of the pendulum when its amplitude decreases?

....

13.5 Damped Harmonic Oscillations

Every oscillating system normally has a viscous medium surrounding it. As a result in each oscillation some of its energy is dissipated as heat. As the energy of oscillation decreases the amplitude of oscillation also decreases. The amplitude of oscillations of a pendulum in air decreases continuously. Such oscillations are called damped oscillations. To understand **damped oscillations** perform activity 13.2.



(a) (b)

Fig. 13.14 : Damped vibrations : (a) experimental setup; (b) graphical representation

Notes

Activity 13.2

Take a simple harmonic oscillator comprising a metal block B suspended from a fixed support S by a spring G. (Fig. 13.14(a). Place a tall glass cylinder filled two thirds with water, so that the block is about 6 cm below the surface of water and about the same distance above the bottom of the beaker. Paste a millimetre scale (vertically) on the side of the cylinder just opposite the pointer attached to the block. Push the block a few centimetres downwards and then release it. After each oscillation, note down the uppermost position of the pointer on the millimetre scale and the time. Then plot a graph between time and the amplitude of oscillations. Does the graph [Fig. 13.14 (b)] show that the amplitude decreases with time. Such oscillations are said to be ***damped oscillations***.

13.6 Free and Forced Vibrations : Resonance

To understand the difference between these phenomena, let us perform the following activity :

Activity 13.3

Take a rigid horizontal rod fixed at both ends. Tie a loose but strong thread and hang the four pendulums A,B,C,D, as shown in Fig. 13.15. The pendulums A and B are of equal lengths, whereas C has a shorter and D has a longer length than A and B. The pendulum B has a heavy bob. Set pendulum B into oscillations. You will observe that after a few minutes, the other three pendulums also begin to oscillate. (It means that if a no. of oscillators are coupled, they transfer their energy. This has an extremely important implication for wave propagation.) You will note that the amplitude of A is larger. Why? Each pendulum is an oscillatory system with natural frequency of its own. The pendulum B, which has a heavy bob, transmits its vibrations to each of the pendulums A, C and D. As a consequence, the pendulums C and D are forced to oscillate ***not with their respective natural*** frequency ***but with the frequency of the pendulum B.*** The phenomenon is called ***forced oscillation.*** By holding the bob of any one of these pendulums, you can force it to oscillate with the frequency of C or of D. Both C and D are forced to oscillate with the frequency of B. However, pendulum A on which too the oscillations of the pendulums B are impressed, oscillates with a relatively large amplitude with ***its natural frequency.*** This phenomenon is known as **resonance.**

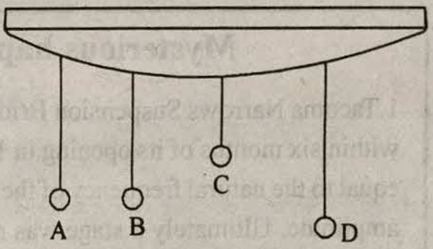


Fig. 13.15: Vibrations and resonance.

When the moving part of an oscillatory system is displaced from its equilibrium position and then set free, it oscillates to and fro about its equilibrium position with a frequency that depends on certain parameters of the system only. Such oscillations are known as ***free vibrations.*** The frequency with which the system oscillates is known as ***natural frequency.*** When a body oscillates under the influence of an external periodic force, the oscillations are called ***forced oscillations.*** In forced oscillations, the body ultimately oscillates with the frequency of the external force. The oscillatory system on which the oscillations are impressed is called driven and the system which applies the oscillating force is known as the driver. The particular case of forced oscillations in which natural frequencies of the driver and the driven are equal is known as ***resonance.*** In resonant oscillations, the driver and the driven reinforce each other's oscillations and hence their amplitudes are maximum.

Intext Questions 13.4

1. When the stem of a vibrating tuning fork is pressed against the top of a table, a loud sound is heard. Does this observation demonstrate the phenomenon of resonance or forced vibrations? Give reasons for your answer. What is the cause of the loud sound produced?

....

2. Why are certain musical instruments provided with hollow sound boards or sound boxes?

....

Mysterious happenings and resonance

1.Tacoma Narrows Suspension Bridge, Washington, USA collapsed during a storm within six months of its opening in 1940. The wind blowing in gusts had frequency equal to the natural frequency of the bridge. So it swayed the bridge with increasing amplitude. Ultimately a stage was reached where the structure was over stressed and it collapsed.

The events of suspension bridge collapse also happened when groups of marching soldiers crossed them. That is why, now, the soldiers are ordered to break steps while crossing a bridge.

The factory chimneys and cooling towers set into oscillations by the wind and sometimes get collapsed.

2.You might have heard about some singers with mysterious powers. Actually, no such power exists. When they sing, the glasses of the window panes in the auditorium are broken. They just sing the note which matches the natural frequency of the window panes.

3.You might have wondered how you catch a particular station you are interested in by operating the tuner of your radio or TV? The tuner in fact, is an electronic oscillator with a provision of changing its frequency. When the frequency of the tuner matches the frequency transmitted by the specific station, resonance occurs and the antenna catches the programme broadcasted by that station.

What You Have Learnt

- Periodic motion is a motion which repeats itself after equal intervals of time.
- Oscillatory motion is to and fro motion about a mean position. An oscillatory motion is normally periodic but a periodic motion may not *necessarily be oscillatory.*
- Simple harmonic motion is to and fro motion under the action of a restoring force, which is proportional to the displacement of the particle from its equilibrium position and is always directed towards the mean position.
- Time period is the time taken by a particle to complete one oscillation.
- Frequency is the number of vibrations completed by the oscillator in one second.
- Phase angle is the angle whose sine or cosine at the given instant indicates the position and direction of motion of the particle.
- Angular frequency is the rate of change of phase angle. Note that $\omega = 2\pi/T = 2\pi\nu$ where ω is the angular frequency in rads^{-1}, ν is the frequency in hertz (symbol : Hz) and T is the time period in seconds.
- Equation of simple harmonic motion is

  $$y = a \sin(\omega t + \phi_0)$$

  or

  $$y = a \cos(\omega t + \phi_0)$$

  where y is the displacement from the mean position at a time, ϕ_0 is the initial phase angle (at $t = 0$).
- When an oscillatroy system vibrates on its own, its vibrations are said to be free. If, however, an oscillatory system is driven by an external system, its vibrations are said to be forced vibrations. And if the frequency of the driver equals to the natural frequency of the driven, the phenomenon of resonance is said to occur.

Terminal Exercise

1. Distinguish between a periodic and an oscillatory motion.
2. What is simple harmonic motion?
3. Which of the following functions represent (i) a simple harmonic motion (ii) a periodic but not simple harmonic (iii) a non periodic motion? Give the period of each periodic motion.

   (1) $\sin \omega t + \cos \omega t$ (2) $1 + \omega^2 + \omega t$

   (3) $3 \cos(\omega t - \frac{\pi}{4})$
4. The time period of oscillations of mass 0.1 kg suspended from a Hooke's spring is 1s. Calculate the time period of oscillation of mass 0.9 kg when suspended from the same spring.

5. What is phase angle? How is it related to angular frequency?

6. Why is the time period of a simple pendulum independent of the mass of the bob, when the period of a simple harmonic oscillator is $T = 2\pi\sqrt{m/k}$?

7. When is the magnitude of acceleration of a particle executing simple harmonic motion maximum? When is the restoring force maximum?

8. Show that simple harmonic motion is the projection of a uniform circular motion on a diameter of the circle. Obtain an expression for the time period of a simple harmonic oscillator in terms of mass and force constant.

9. Obtain expressions for the instantaneous kinetic energy potential energy and the total energy of a simple harmonic oscillator.

10. Show graphically how the potential energy U, the kinetic energy K and the total energy E of a simple harmonic oscillator vary with the displacement from equilibrium position.

11. The displacement of a moving particle from a fixed point at any instant is given by $x = a\cos\omega t + b\sin\omega t$. Is the motion of the particle simple harmonic? If your answer is no, explain why? If your answer is yes, calculate the amplitude of vibration and the phase angle.

12. A simple pendulum oscillates with amplitude 0.04m. If its time period is 10s, calculate the maximum velocity.

13. Imagine a ball dropped in a frictionless tunnel cut across the earth through its centre. Obtain an expression for its time period in terms of radius of the earth and the acceleration due to gravity.

14. Fig. 13.16 shows a block of mass $m = 2$kg connected to two springs, each of force constant $k = 400\text{Nm}^{-1}$. The block is displaced by 0.05m from equilibrium position and then released. Calculate (a) The angular frequency ω of the block, (b) its maximum speed; (c) its maximum acceleration; and total energy dissipated against damping when it comes to rest.

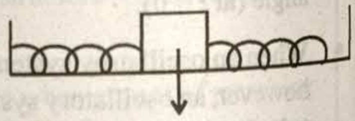


Fig.13.16

Answers to Intext Questions

13.1

1. A motion which repeats itself after some fixed interval of time is a periodic motion. A to and fro motion on the same path is an oscillatory motion. A periodic motion may or may not be oscillatory but oscillation motion is perodic.

2. (ii), (iv), (v);

3. (i) To and fro motion of a pendulum.

   (ii) Motion of a planet in its orbit.

13.2

1. Return force on the ball when displaced a distance x from the equilibrium position is $mg \sin\theta = mg\,\theta = mg\,x/r$. $\therefore\ \omega = \sqrt{g/r}$.

2. On being pushed down through a distance y, the cylinder experiences an upthrust $y\alpha\rho g$. Therefore $\omega^2 = \frac{\alpha\rho g}{m}$ and $m = \alpha p\rho$. From the law of flotation m = mass of black. Hence, $\omega^2 = g/l$ or $T = 2\pi\sqrt{l/g}$.

3. $\omega^2 = k/m$ and hence $v = 1/2\pi\sqrt{k/m}$. Note that when the mass is displaced, only one of the bands exerts the restoring force.

13.3

1. K.E is maximum at mean position or equilibrium position; acceleration is maximum when displacement is maximum.

2. As the pendulum oscillates it does work against the viscous resistance of air and friction at the support from which it is suspended. This work done is dissipated as heat. As a consequence the amplitude decreases.

13.4

1. When an oscillatory system called the driver applies is periodic of force on another oscillatory system called the driven and the second system is forced to oscillate with the frequency of the first, the phenomenon is known as forced vibrations. In the particular case of forced vibrations in which the frequency of the driver equals the frequency of the driven system, the phenomenon is known as resonance.

2. The table top is forced to vibrate not with its natural frequency but with the frequency of the tuning fork. Therefore, this observation demonstrates forced vibrations. Since a large area is set into vibrations, the intensity of the sound increases.

3. The sound board or box is forced to vibrate with the frequency of the note produced by the instrument. Since a large area is set into vibrations, the intensity of the note produced increases and its duration decreases.

Answers to Terminal Problems

4. 3s

11. $A = \sqrt{a^2 + b^2}, \theta = \tan^{-1}\left(\frac{a}{b}\right)$

12. $\frac{2}{\pi} \times 10^{-3}\,\text{ms}^{-1}$

14. (*a*) 14.14 s^{-1} (*b*) 0.6 ms^{-1} (*c*) 0.3 ms^{-2} (*d*) 0.5J

14

WAVE PHENOMENA

You would have noticed that when a stone is dropped into still water in a pond, concentric rings of alternate elevations and depressions emerge out from the point of impact and spread out on the surface of water. If you put a straw piece on the surface of water, you will observe that it moves up and down at its place. Here the particles of water are moving up and down at their places. But still there is something which moves outwards. We call it a *wave.* Waves are of different types : Progressive and stationary, mechanical and electro-magnetic. These can also be classified as longitudinal and trnsverse depending on the direction of motion of the material particles with respect to the direction of propagation of wave in case of mechnical waves and electric and magnetic vectors in case of e.m. waves. Waves are so intimate to our existance.

Sound waves travelling through air make it possible for us to listen. Light waves, which can travel even through vacuum make us see things and radio waves carrying different signals at the speed of light connect us to our dear ones through differents forms of communication. In fact, wave phenomena is universal.

The working of our musical instruments, radio, T.V require us to understand wave phenomena. Can you imagine the quality of life without waves? In this lesson you will study the basics of waves and wave phenomena.

Objectives

After studying this lesson, you should be able to :

- *explain propagation of transverse and longitudinal waves and establish the relation* $\upsilon = \nu\lambda$*;*
- *write Newton's formula for velocity of longitudinal waves in a gas and explain Laplace's correction;*
- *discuss the factors on which velocity of longitudional waves in a gas depends;*
- *explain formation of transverse waves on stretched strings;*
- *derive the equation of a simple harmonic wave;*

- *explain the phenomena of beats, interference and phase change of waves on the basis of principle of superposition*
- *explain formation of stationary waves and discuss harmonics of organ pipes and stretched strings;*
- *discuss Doppler effect and apply it to mechanical and optical systems;*
- *explain the properties of em waves, and*
- *state wavelength range of different parts of em spectrum and their applications.*

14.1 Wave Propagation

From the motion of a piece of straw, you may think that waves carry energy; these do not transport mass. A vivid demonstration of this aspect is seen in tidel waves. Do you recall the devastation caused by Tsunami, waves which hit Indonesia, Thailand, Sri lanka and India caused by a deep sea quake waves of, 20m height were generated and were responsible for huge loss of life.

To understand how waves travel in a medium let us perform an activity.

Activity 14.1

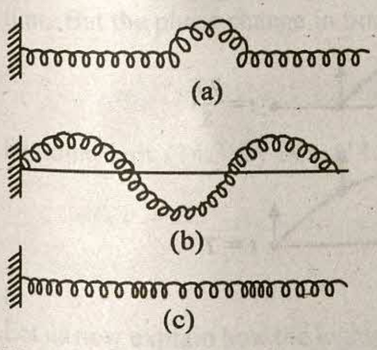


Fig. 14.1 : Wave motion on a slinky (a) pulse on a slinky, (b) transverse wave, and (c) longitudinal Wave

Take a long coil spring, called slinky, and stretch it along a smooth floor or bench, keeping one end fixed and the other end free to be given movements . Hold the free end in your hand and give it a jerk side–ways.[Fig 14.1(a)]. You will observe that a kink is produced which travels towards the fixed end with definite speed . This kink is a wave of short duration. Keep moving the free end continuously left and right. You will observe a train of pulses ravelling towards the fixed end. This is a *transverse wave* moving through the spring [Fig. 14.1 (b)].

There is another type of wave that you can generate in the slinky . For this keep the slinky straight and give it a push along its length . A pulse of compression thus moves on the spring. By moving the hand backwards and forwards at a constant rate you can see ulternate compressions and rarefactions travelling along its length . These are called *longitudinal waves* [Fig. 14.1(c)].

14.1.1 Propagation of Transverse Waves

Refer to Fig 14.2. It shows a mechanical model for wave propapation. It comprises a row of spherical balls of equal masses, evenly spaced and connected together by identical springs. Let us imagine that by means of suitable device, ball-1, from left, is made to execute S.H.M. in a direction perpendicular to the row of balls with a period T. All the balls, owing to inertia of rest will not begin to oscillate at the same time. The motion is

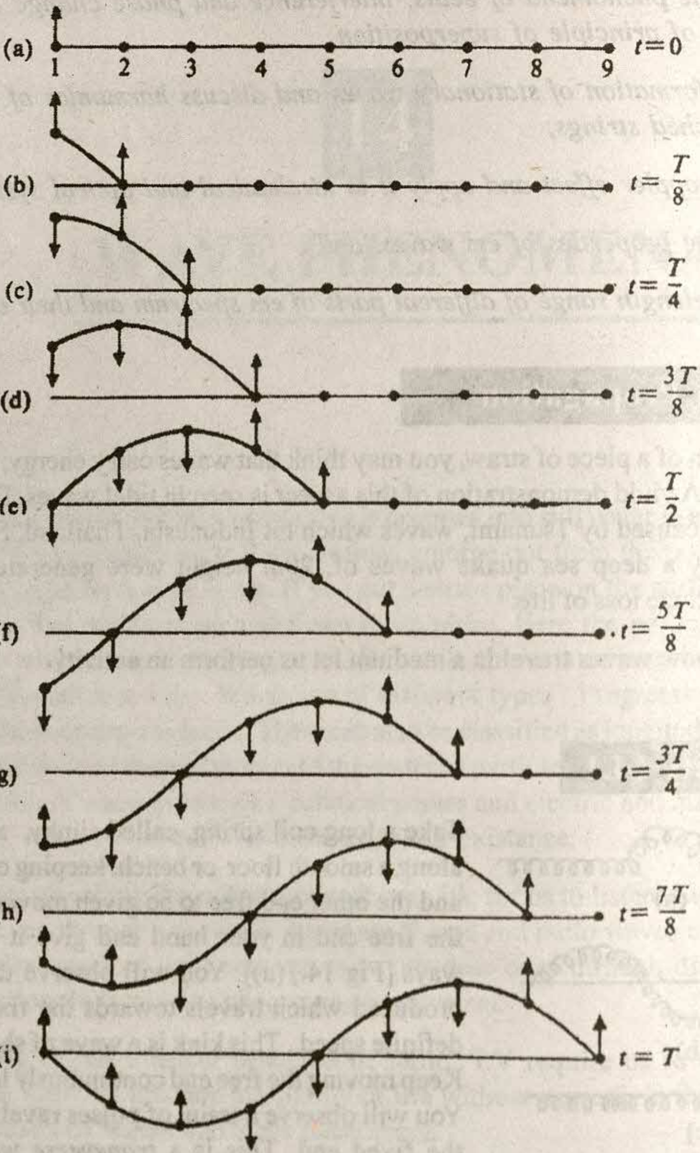


Fig. 14.2 : Instantaneous profiles at intervals of T/8 when a transverse wave is generated on a string.

passed on from one ball to the next one by one. Let us suppose that the time taken by the disturbance to travel from one ball to the next is $T/8$s. This means that in the interval $T/8$s, the disturbance propagates from the particle at mark 1 to the particale at mark 2. Similarly, in the next $T/8$ interval, the disturbance travels from the particle at mark 2 to the particle at mark 3 and so on. In parts (a)—(i) in Fig. 14.2 we have shown the instantaneous positions of particles at all nine marked positions at intervals of $T/8$. (The arrows indicate the directions of motion along which particles at various marks are about to move.) You will observe that

(i) At $t = 0$, all the particles are at their respective mean positions.

(ii) At $t = T$, the first, fifth and ninth particles are at their respective mean positions. The first and ninth particles are about to move upward whereas the fifth particle is about to move downward. The third and seventh particles are at position of maximum displacement but on opposite sides of the horizontal axis. The envelop joining the instantaneous positions of all the particles at marked positions in Fig. 14.2(a) are similar to those in Fig. 14.2(i) and represents a *transverse wave.* The positions of third and seventh particles denote a *trough and a crest,* respectively.

The important point to note here is that *while the wave moves along the string, all particles of the string are oscillating up and down about their respective equilibrium positions with the same period* (T) *and amplitude* (A). This wave remains *progressive* till it reaches the fixed end.

In a wave motion, the distance between the two nearest particles vibrating in the same phase is called a wavelength. *It is denoted by λ.*

It is evident that time taken by the wave to travel a distance λ is T. (See Fig. 14.2). Therefore, the velocity of the wave is

$$v = \frac{\text{Distance}}{\text{Time}} = \frac{\lambda}{T} \tag{14.1}$$

But, 1/T= ν, the frequency of the wave. Therefore,

$$v = \nu\lambda \tag{14.2}$$

Further, if two consecutive particles in same state of motion are separated by a distance λ, the phase difference between them is 2π. Therefore, the phase change per unit distance

$$k = \frac{2\pi}{\lambda} \tag{14.3}$$

We call k the propagation constant. You may recall that ω denotes phase change per unit time. But the phase change in time T is 2π hence

$$\omega = \frac{2\pi}{T} = 2\pi\nu \tag{14.4}$$

Dividing Eqn. (14.3) by Eqn. (14.4), we get an expression for the wave velocity :

$$v = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda}$$

or

$$v = \nu\lambda \tag{14.5}$$

Let us now explain how the logitudinal waves propagate.

14.1.2 Propagation of a Longitudinal Wave

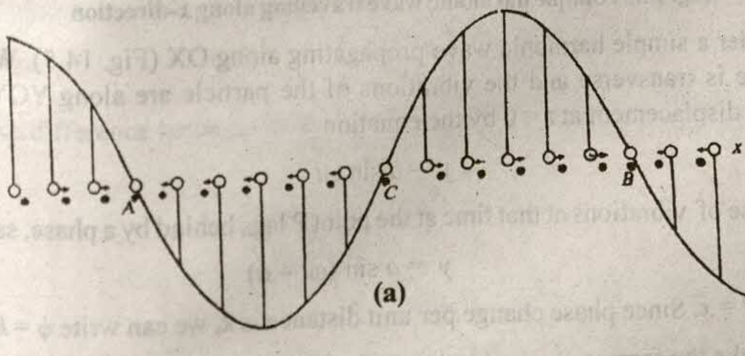


Fig. 14.3 : Graphical representation of a longtudinal wave.

In longitudinal waves, the displacement of particles is along the direction of wave propagation In Fig. 14.3, the hollow circles represent the mean positions of equidistant particles in a medium. The arrows show their (rather magnified) longitudinal displacements at a given time. You will observe that the arrows are neither equal in length nor in the same direction. This is clear from the position of solid circles, which describe instantaneous positions of the particles corresponding to the heads of the arrows. The displacements to the right are shown in the graph towards + y-axis and displacements to the left towards the $-y$-axis.

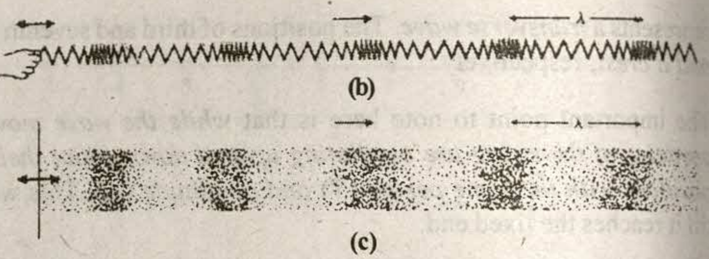


Fig. 14.4 : Longitudinal waves on a spring are analogous to sound waves.

For every arrow directed to the right, we draw a proportionate line upward. Similarly, for every arrow directed to the left, a proportionate line is drawn downward. On drawing a smooth curve through the heads of these lines, we find that the graph resembles the displacement-time curve for a transverse wave. If we look at the solid circles, we note that around the positions A and B, the particles have crowded together while around the position C, they have separated farther. These represent regions of *compression and rarefaction.* That is, there are alternate regions where density (pressure) are higher and lower than average.A sound wave propagating in air is very similar to the longitudinal waves that you can generate on your spring (Fig. 14.4).

Let us now derive equation of a simple harmonic wave.

14.1.3 Equation of a Simple Harmonic Wave in One Dimension

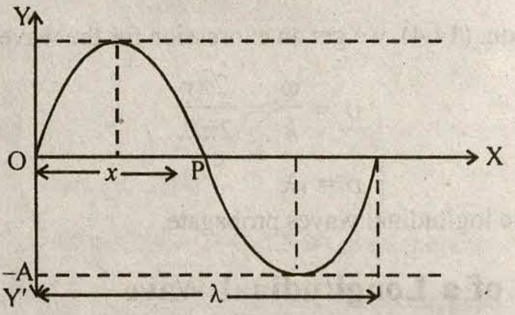


Fig. 14.5 : Simple harmonic wave travelling along x-direction

Let us consider a simple harmonic wave propagating along OX (Fig. 14.5). We assume that the wave is transverse and the vibrations of the particle are along YOY′. Let us represent the displacement at $t = 0$ by the equation

$$y = a \sin \omega t \tag{14.6}$$

Then the phase of vibrations at that time at the point P lags behind by a phase, say ϕ. Then

$$y = a \sin (\omega t - \phi) \tag{14.7}$$

Let us put OP = x. Since phase change per unit distance is k, we can write $\phi = kx$. Hence, Eqn. (14.7) take the form

$$y(x, t) = a \sin (\omega t - kx) \tag{14.8}$$

Further as $\omega = 2\pi/t$ and $k = 2\pi/\lambda$, we can rewrite Eqn (14.8) as

$$y\ (x,\ t) = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \tag{14.9}$$

In terms of wave velocity ($\upsilon = \lambda/T$), this equation can be expressed as

$$y = a \sin \frac{2\pi}{\lambda}\ (\upsilon t - x) \tag{14.10}$$

In deriving Eqn. (14.8) we have taken initial phase of the wave at O as zero. However, if the initial phase angle at O is ϕ_0 the equation of the wave would be

$$y\ (x,t) = a \sin\ [(\omega t - kx) + \phi_0] \tag{14.11}$$

Phase difference between two points on a wave

Let us consider two simple harmonic waves travelling along OX and represented by the equations

$$y = a \sin\ (\omega t - kx) \tag{14.8}$$

and

$$y = a \sin\ [\omega t - k\ (x + \Delta x)] \tag{14.12}$$

The phase difference between them is

$$\Delta\phi = k\Delta x = \frac{2\pi}{\lambda} .\Delta x = -\frac{2\pi}{\lambda}\ (x_2 - x_1) \tag{14.13}$$

where Δx is called the path difference between these two points. Here the negative sign indicates that a point positioned later will acquire the same phase at a later time.

Phase difference at the same position over a time interval Δt :

We consider two waves at the same position at a time interval Δt. For the first wave, phase ϕ, is given by

$$\phi_1 = \frac{2\pi}{T} t_1 - \frac{2\pi}{\lambda} x$$

and for the another wave phase

$$\phi_2 = \frac{2\pi}{T} t_2 - \frac{2\pi}{\pi} x.$$

The phase difference between them

$$\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{T}\ (t_2 - t_1)$$

$$= 2\pi\nu\ (t_2 - t_1). \tag{14.13(a)}$$

$$= 2\pi\nu(\Delta t)$$

Example 14.1 : A progressive harmonic wave is given by $y = 10^{-4} \sin(100\pi t - 0.1\pi x)$. Calculate its (i) frequency, (ii) wavelength and (iii) velocity y and x are in metre.

Solution: comparing with the standard equation of progressive wave

$$y = A \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

we get (i) $2\pi \nu = 100\,\pi \Rightarrow \nu = 50$ Hz

(ii) $\frac{2\pi}{\lambda} = 0.1\,\pi \Rightarrow \lambda = 20$ m

(iii) $\upsilon = \nu\lambda = 1000\ \text{ms}^{-1}$

14.1.4 Transverse and Longitudinal Waves

We now consider transverse and longitudinal waves and summerise the difference between them.

	Transverse waves		Longitudinal waves
(i)	Displacements of the particles are perpendicular to the direction of propagation of the wave.	(i)	Displacements of the particles are along the direction of propagation of the wave.
(ii)	Transverse waves look as crests and troughs propagating in the medium.	(ii)	Longitudinal waves give the appearance of alternate compressions and rarefaction moving forward.
(iii)	Transverse waves can only be transmitted in solids or on the surface of the liquids.	(iii)	Longitudinal waves can travel in solids, liquids and gases.
(iv)	In case of a transverse wave, the displacement - distance graph gives the actual picture of the wave itself.	(iv)	In case of longitudinal waves, the graph only represents the displacement of the particles at different points at a given time.

Essential properties of the medium for propagation of longitudinal and transverse mechanical waves are: (i) the particles of the medium must possess mass, (ii) the medium must possess elasticity. Longitudinal waves for propagation in a medium require volume elasticity but transverse waves need modulus of rigidity. However, light waves and other electromagnetic waves, which are transverse, do not need any material medium for their propagation.

Intext Questions 14.1

1. State the differences between longitudinal and transverse waves?

....

2. Write the relation between phase difference and path difference.

....

3. Two simple harmonic waves are represented by equations $y_1 = a \sin(\omega t - kx)$ and

$y_2 = a \sin[(\omega t - kx) + \phi]$. What is the phase difference between these two waves?

....

14.2 Velocity of Longitudinal and Transverse Waves in an Elastic Medium

14.2.1 Newton's Formula for Velocity of Sound in a Gas

Newton to derive a relation for the velocity of sound in a gaseous medium, assumed that compression and rarefaction caused by the sound waves during their passage through the gas take place under isothermal condition. This means that the changes in volume and pressure take place at constant temperature. Under such conditions, Newton agreed that the velocity of sound wave in a gas is given by

$$\upsilon = \sqrt{\frac{P}{\rho}} \qquad (14.15)$$

For air, at standard temperature and pressure $P = 1.01 \times 10^5\ \text{Nm}^{-2}$ and $\rho = 1.29\ \text{kg m}^{-3}$. On substituting these values in Eqn.(14.15) we get

$$\upsilon = \sqrt{1.01\times10^5/1.29} = 280\ \text{ms}^{-1}$$

Clouds collide producing thunder and lightening, we hear sound of thunder after the lightening. This is because the velocity of light is very much greater than the velocity of sound in air. By measuring the time interval between observing the lightening and hearing the sound, the velocity of sound in air can be determined. Using an improved technique, the velocity of sound in air has been determined as 333 ms^{-1} at 0°C. The percent error in the value predicted by Newton's formula and that determined experimentally is $\frac{333-280}{333} \times 100\%$ $= 16\%$. This error is too high to be regarded as an experimental error. Obviously there is something wrong with Newton's assumption that during the passage of sound, the compression and the rarefaction of air take place isothermally.

14.2.2 Laplace's Correction

Laplace pointed out that the changes in pressure of air layers caused by passage of sound take place under adiabatic condition owing to the following reasons.

(i) Air is bad conductor of heat and

(ii) Compression and rarefactions caused by the sound are too rapid to permit heat to flow out during compression and flow in during rarefaction.

Under adiabatic conditions

$$E = \gamma P,$$

Where $\gamma = \frac{C_p}{C_v}$

Hence, $$\upsilon = \sqrt{\frac{\gamma P}{\rho}} \qquad (14.16)$$

For air, $\gamma = 1.4$. Therefore, at STP, speed of sound is given by

$$\upsilon = \sqrt{1.4 \times 1.01 \times 10^5 / 1.29}$$

$$= 333\text{ms}^{-1}$$

This value is very close to the experimentally observed value.

14.2.3 Factors affecting velocity of sound in a gas

(i) Effect of Temperature

From Laplace's formula

$$\upsilon = \sqrt{\frac{\gamma P}{\rho}}$$

Since density is ratio of mass perumit volume, this expression takes the form

$$= \sqrt{\frac{\gamma PV}{M}}$$

Using the equation of state PV = *n*RT, where *n* is number of moles in mass *m* of the gas

$$\upsilon = \sqrt{\frac{\gamma RT}{\frac{M}{n}}}$$

$$= \sqrt{\frac{\gamma RT}{m}} \qquad (14.17\text{ a})$$

Where *m* denotes the gram molecular mass. This result shows that

$$\upsilon \; \alpha \; \sqrt{T}$$

$$\Rightarrow \qquad \upsilon = \upsilon_0 \left(1 + \frac{t}{2 \times 273}\right) + \ldots\ldots\ldots\ldots$$

$$\simeq 333 + \frac{333}{546} t$$

$$\simeq 333 + 0.61t \qquad (14.17\text{b})$$

Note that for small temperature variations, velocity of sound in air increases by 0.61 ms^{-1} with every degree celsius rise in temperature.

***(ii)* Effect of pressure**

When we increase pressure on a gas, it gets compressed but its density increases in the same proportion as the pressure i.e. P/ρ remains constant. It means that, pressure has no effect on the velocity of sound in a gas.

***(iii)* Effect of density**

If we consider two gases under identical conditions of temperature and pressure, then

$$v \;\alpha\; \frac{1}{\sqrt{\rho}}$$

If we, compare the velocities of sound in oxygen and hydrogen, we get

$$\frac{v_{\text{oxygen}}}{v_{\text{hydrogen}}} = \sqrt{\frac{\rho_{\text{hydrogen}}}{\rho_{\text{oxygen}}}} = \sqrt{\frac{M_{\text{hydrogen}}}{M_{\text{oxygen}}}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

This shows that velocity of sound in hydrogen is 4 times the velocity of sound in oxygen under identical conditions of temperature and pressure. Is this result valid for liquids and solids as well. You will discover answer to this question in the next sub–section.

(*iv*) Effect of humidity on velocity of sound in air

As humidity in air increases (keeping conditions of temperature and pressure constant), its density decreases and hence velocity of sound in air increases.

Example 14.2 : At what temperature is the speed of sound in air double of its value at S.T.P.

Solution : We know that $\frac{v}{v_0} = \sqrt{\frac{T}{m}} = 2 = \sqrt{\frac{T}{273}}$

On squaring both sides and rearranging terms, we get

$$\therefore \qquad T = 273 \times 4 = 1092\text{k}$$

14.2.4 Velocity of Waves in Stretched Strings

The velocity of a transverse wave in a stretched string is given by

$$v = \sqrt{\frac{F}{m}} \qquad (14.18\text{ a})$$

Where F is tension in the string and *m* is mass per unit length of the wire. The velocity of longtudinal waves in an elastic medium is given by

$$v = \sqrt{E/\rho} \qquad (14.18\text{b})$$

where E is elasticity. It may be pointed out here that since the value of elasticity is more in solids, the velocity of longitudinal waves in solids is greater than that in gases and liquids. In fact, $v_g < v_l < v_s$.

Intext Questions 14.2

1. What was the assumption made by Newton in deriving his formula?

....

2. What was wrong with Newton's formula?

....

3. Show that for every 1°C rise in temperature, the velocity of sound in air increases by 0.61 ms^{-1}.

....

4. Calculate the temperature at which the velocity in air is (3/2) times the velocity of sound at 7°C?

....

5. Write the formula for the velocity of a wave on stretched string?

....

6. Let λ be the wavelength of a wave on a stretched string of mass per unit length m and n be its frequency. Write the relation between n, λ, F and m? Further if $\lambda = 2\ell$, what would be the relation between n, l, F and m?

....

14.3 Superposition of Waves

Suppose two wave pulses travel in opposite directions on a slinky. What happens when they meet? Do they knock each other out? To answer these questions, let us perform an activity.

Activity 14.2

Produce two wavecrests of different amptitudes on a stretched slinky, as shown in Fig. 14.6 and watch carefully. The crests are moving in the opposite directions. They meet and overlap at the point midway between them [Fig. 14.6(b)] and then separate out. Thereafter, they continue to move in the same direction in which they were moving before crossing each other. Moreover, their shape also does not change [Fig. 14.6(c)].

Now produce one crest and one trough on the slinky as shown in Fig. 14.6(d). The two are moving in the opposite direction. They meet [Fig. 14.6(e)], overlap and then separate out. Each one moves in the same direction in which it was moving before crossing and each one has the same shape as it was having before crossing. Repeat the experiment again and observe carefully what happens at the spot of overlapping of the two pulses [(Fig. 14.6(b) and (e)]. You will note that when crests overlap, the resultant is more and when crest overlaps the through, the resultant is on the side of crest but smaller size. We may summarize this result as : *At the points where the two pulses overlap, the resultant displacement is the vector sum of the displacements due to each of the two wave pulses.* ***This is called the principle of superposition.***

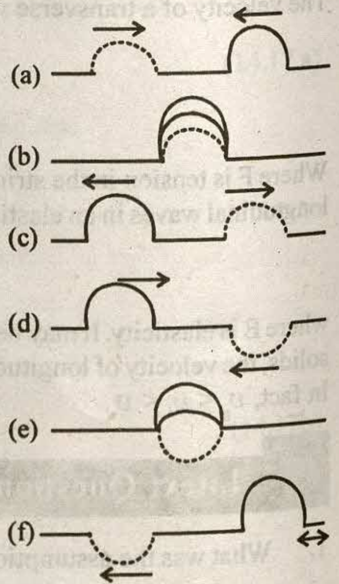


Fig. 14.6 : Illustrating principle of superpositionof waves

This activity demonstrates not only the principle of superposition but also shows that two or more waves can traverse the same space independent of each other. Each one travels as if the other were not present. This important property of the waves enable us to tune to a particular radio station even though the waves broadcast by a number of radio stations exist in space at the same time. We make use of this principle to explain the phenomena of *interference of waves, formation of beats and stationary or standing waves.*

14.3.1 Reflection and Transmission of Waves

We shall confine our discussion in respect of mechanical waves produced on strings and springs. What happens and why does it happen when a transverse wave crest propagates towards the fixed end of a string? Let us perform the following activity to understand it.

Activity 14.3

Fasten one end of a slinky to a fixed support as shown in (Fig. 14.7 (a). Keeping the slinky horizontal, give a jerk to its free end so as to produce a transverse wave pulse which travells towards the fixed end of the slinky (Fig. 14.7(a)). You will observe that the pulse bounces back from the fixed end. As it bounces back, the crest becomes a trough travels back in the opposite direction. Do you know the reason? As the pulse meets the fixed end, it exerts a force on the support. The equal and opposite reaction not only reverses the direction of propagation of the wave pulse but also reverses the direction of the displacement of the wave pulse (Fig. 14.7(b)). The support being much heavier than the slinky, it can be regarded as a denser medium. The wave pulse moving in the opposite direction is called the reflected wave pulse. So, we can say that ***when reflection takes place from a denser medium, the wave undergoes a phase change of π, that is, it suffers a phase reversal.***

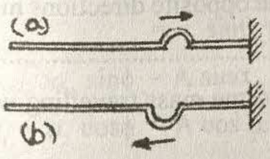


Fig. 14.7 : **Reflection from a denser medium : a phase reversal.**

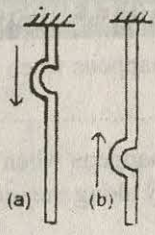


Fig.14.8(a) : A pulse travelling down towards the free end, (b) on reflection from the free end direction of its displacement remains unchanged

Let us now see what happens on reflection from a rarer medium. For this we perform another activity.

Activity 14.4

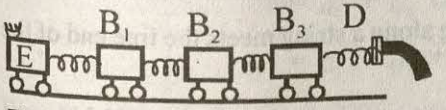


Fig. 14.9 : **Longitudinal waves are reflected from a denser medium without change of type but with change of sign**

Suspend a fixed rubber tube from a rigid support (Fig. 14.8 a). Then generate a wave pulse travelling down the tube. On reflection from the free end, the wave pulse travels upward but without any change in the direction of its displacements i.e. crest returns as crest. Why? As the wave pulse reaches the free end of the tube, it gets reflected from a rarer boundary. (Note that air is rarer than the rubber tube.) Hence there is no change in the direction of displacement of the wave pulse. ***Thus on reflection from a rarer medium, no phase change takes place.***

You may now raise the question : Do longitudinal waves also behave similarly? Refer to Fig. 14.9, which shows a row of bogies. Now suppose that the engine E moves a bit towards the right. The buffer spring between the engine E and the first bogie gets compressed and pushes bogie B_1 towards the right. It then tries to go back to its original shape. As this compressed spring expands, the spring between the 1st and the 2nd bogie gets compressed. As the second compressed spring expands, it moves a bit towards the 3rd bogie. In this manner the compression arrives at the last buffer spring in contact with the fixed stand D. As the spring between the fixed stand and the last bogie expands, only the last bogie moves towards the left. As a result of this, the buffer spring between the next two bogies on left is compressed. This process continues, till the compression reaches between the engine and the first bogie on its right. Thus, a compression returns as a compression. But the bogies then move towards the left. In this mechanical model, the buffer spring and the bogies form a medium. The bogies are the particles of the medium and the spring between them shows the forces of elasticity.

Thus, when reflection takes place from a denser medium, the longitudinal waves are reflected without change of type but with change in sign. And on reflection from a rare medium, a longitudinal wave is reflected back without change of sign but with change of type. By 'change of type' we mean that rarefaction is reflected back as compression and a compression is reflected back as rarefaction.

Intext Questions 14.3

1. What happens when two waves travelling in the opposite directions meet?

   ....

2. What happens when two marbles each of the same mass travelling with the same velocity along the same line meet?

   ....

3. Two similar wave pulses travelling in the opposite directions on a string meet. What happens (i) when the waves are in the same phase? (ii) the waves are in the opposite phases?

   ....

4. What happens when a transverse wave pulse travelling along a string meets the fixed end of the string?

   ....

5. What happens when a wave pulse travelling along a string meets the free end of the string?

   ....

6. What happens when a wave of compression is reflected from (i) a rarer medium (ii) a denser medium?

   ....

14.4 Superposition of Waves Travelling in the Same Direction

Superposition of waves travelling in the same direction gives rise to two different phenomena (i) interference and (ii) beats depending on their phases and frequencies. Let us discuss these phenomena now.

14.4.1 Interference of waves

Let us compute the ratio of maximum and minimum intensities in an interference pattern obtained due to superposition of waves. Consider two simple harmonic waves of amplitudes a_1 and a_2 each of angular frequency ω, both propagating along x – *axis*, with the same velocity $\upsilon = \omega/k$ but differing in phase by a constant phase angle ϕ. These waves are represented by the equations

$$y_1 = a_1 \sin(\omega t - kx)$$

and

$$y_2 = a_2 \sin[(\omega t - kx) + \phi]$$

where $\omega = 2\pi/T$ is angular frequency and $k = \dfrac{2\pi}{\lambda}$ is wave number.

Since, the two waves are travelling in the same direction with the same velocity along the same line, they overlap. According to the principle of superposition, the resultant displacement at the given location at the given time is

$$y = y_1 + y_2 = a_1 \sin(\omega t - kx) + a_2 \sin[(\omega t - kx) + \phi]$$

If we put $(\omega t - kx) = \theta$, then

$$y = a_1 \sin\theta + a_2 \sin(\theta + \phi)$$

$$= a_1 \sin\theta + a_2 \sin\theta \cos\phi + a_2 \sin\phi \cos\theta$$

Let us put $\quad a_2 \sin\phi = A \sin\alpha$

and $\quad a_1 + a_2 \cos\phi = A \cos\alpha$

Then

$$y = A \cos\alpha \sin\theta + A \sin\alpha \cos\theta$$

$$= A \sin(\theta + \alpha)$$

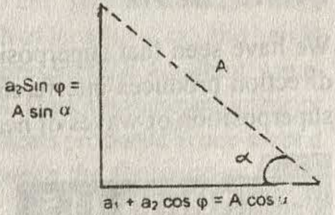


Fig. 14.10 : Calculating resultant amplitude A

Substituting for θ we get

$$y = A \sin[(\omega t - kx) + \alpha]$$

Thus, the resultant wave is of angular frequency ω and has an amplitude A given by

$$A^2 = (a_1 + a_2 \cos\phi)^2 + (a_2 \sin\phi)^2$$

$$= a_1^2 + a_2^2 \cos^2\phi + 2a_1a_2 \cos\phi + a_2^2 \sin^2\phi$$

$$A^2 = a_1^2 + a_2^2 + 2\,a_1a_2\cos\phi \qquad (14.18)$$

In Eqn. (14.18), ϕ is the phase difference between the two superposed waves. If path difference, between the two waves corresponds to phase difference ϕ, then

$\phi = \dfrac{2\pi p}{\lambda}$, where $\dfrac{2\pi}{\lambda}$ is the phase change per unit distance.

When the path difference is an even multiple of $\frac{\lambda}{2}$, *i.e.*, $p = 2m\frac{\lambda}{2}$, then phase difference is given by $\phi = (2\pi/\lambda) \times (2m\,\lambda/2) = 2m\pi$. Since $\cos 2\pi = +1$, from Eqn. (14.18) we get

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

That is, when the collinear waves travelling in the same directions are in phase, the amplitude of the resultant wave on superposition is equal to sum of individual amplitudes.

As intensity of wave at a given position is directly proportional to the square of its amplitude, we have

$$I_{\max} \alpha (a_1 + a_2)^2$$

When $p = (2m + 1)\,\lambda/2$, then $\phi = (2m + 1)\,\pi$ and $\cos\phi = -1$. Then from Eqn. (14.18), we get

$$A^2 = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$

This shows that when phases of two collinear waves travelling in the same direction differ by an odd integral multiple of π, the amplitude of resultant wave generated by their superposition is equal to the difference of their individual amplituds.

Then $I_{\min} \alpha (a_1 - a_2)^2$

Thus
$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} \qquad (14.19)$$

If $a_1 = a_2$, the intensity of resultant wave is zero. These results show that interference is essentially redistribution of energy in space due to superposition of waves.

14.4.2 Beats

We have seen that superposition of waves of same frequency propagating in the same direction produces interference. Let us now investigate what would be the outcome of superposition of waves of nearly the same frequency. First let us perform an activity.

Activity 14.5

Take two tuning forks of same frequency 512 Hz. Let us name them as A and B. Load the prong of the tuning fork B with a little wax. Now sound them together by a rubber hammer. Press their stems against a table top and note what you observe. You will observe that the intensity of sound alternately becomes maximum and minimum. These alternations of maxima and minima of intensity are called beats. One alternation of a maximum and a minimum is one beat. On loading the prong of B with a little more wax, you will find that no. of beats increase. On further loading the prongs of B, no beats may be heard. The reason is that our ear is unable to hear two sounds as separate produced in an interval less than one tenths of a second. Let us now explain how beats are produced.

(a) Production of beats : Suppose we have two tuning forks A and B of frequencies N and N + n respectively; n is smaller than 10. In one second, A completes N vibrations but B completes N + n vibrations. That is, B completes n more vibrations in one second than the tuning fork A. In other words, B gains n vibrations over A in 1s and hence it gains 1 vib. in (1/n) s. and half vibration over A in (1/2n) s. Suppose at $t = 0$, i.e. initially, both the tuning forks were vibrating in the same phase. Then after (1/2n)s, B will gain half vibration over A. Thus after $\frac{1}{2n}s$ it will vibrate in oposite phase. If A sends a wave of compression then B sends a wave of rarefaction to the observer. And, the resultant intensity received by the ear would be zero. After (1/n)s, B would gain one complete vibration. If now A sends a wave of compression, B too would send a wave of compression to the observer. The intensity observed would become maximum. After (3/2n)s, the two forks again vibrate in the opposite phase and hence the intensity would again become minimum. This process would continue. The observer would hear 1 beat in $(1/n)$s, and hence n beats in one second. Thus, the *number of beats heard in one second equals the difference in the frequencies of the two tuning forks.* If more than 10 beats are produced in one second, the beats are not heard as separate. The beat frequency is n and beat period is $1/n$.

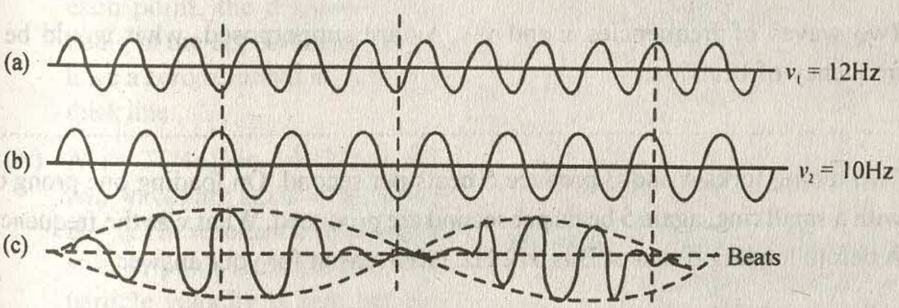


Fig.14.11 : (a) Displacement time graph of frequency 12 Hz. (b) displacement time graph of frequency 10 Hz. Superposition of the two waves produces 2 beats per second.

(b) Graphic method : Draw a 12 cm long line. Divide it into 12 equal parts of 1 cm. On this line draw 12 wavelengths each 1 cm long and height 0.5 cm. This represents a wave of frequency 12 Hz. On the line (b) draw 10 wavelengths each of length 1.2 cm and height 0.5 cm. This represents a wave of frequency 10Hz. (c) represents the resultant wave. Fig, 14.11 is not actual waves but the displacement time graphs. Thus, the resultant intensity alternately becomes maximum and minimum. The number of beats produced in one second is Δv. Hence, the beat frequency equals the difference between the frequencies of the waves superposed.

Example 14.3 : A tuning fork of unknown frequency gives 5 beats per second with another tuning of 500 Hz. Determine frequency of the unknown fork.

Solution : $v' = v \pm n = 500 \pm 5$

⇒ The frequency of unknown tuning fork is either 495 Hz or 505 Hz.

Example 14.4 : In an interference pattern, the ratio of maximum and minimum intensities is 9. What is the amplitude ratio of the superposing waves?

Solution : $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 \Rightarrow 9 = \left(\frac{1+r}{1-r}\right)^2$, where $r = \frac{a_2}{a_1}$.

Hence, are can write

$$\frac{1+r}{1-r} = 3$$

You can easily solve it to get $r = \frac{1}{2}$, i.e., amplitude of one wave is twice that of the other.

Intext Questions 14.4

1. If the intensity ratio of two waves is 1:16, and they produce interference, calculate the ratio I_{max}/I_{min}?

   ....

2. Waves of frequencies ν and $\nu + 4$ emanating from two sources of sound are superposed. What will you observe?

   ....

3. Two waves of frequencies ν and $\nu + \Delta\nu$ are supperposed, what would be the frequency of beats?

   ....

4. Two tuning forks A and B produce 5 beats per second. On loading one prong of A with a small ring, again 5 beats per second are produced. What was the frequency of A before loading if that of B is 512 Hz. Give reason for your answer.

   ....

14.5 Superposition of Waves of Same Frequency Travelling in the Opposite Directions

So far we have discussed superposition of collinear waves travelling in the same direction. In such waves, crests, and troughs or rarefactions and compressions in a medium travel forward with a velocity depending upon the properties of the medium. Superposition of progressive waves of same wavelength and same amplitude travelling with the same speed along the same line in a medium in opposite direction gives rise to stationary or standing waves. In these waves crests and troughs or compressions and rarefactions remain stationary relative to the observer.

14.5.1 Formation of Stationary (Standing) Waves

To understand the formation of stationary waves, refer to Fig. 14.12 where we have shown the positions of the incident, reflected and resultant waves, each after T/4s, that is, after quarter of a period of vibration.

(i) Initially, at $t = 0$, [Fig. 14.12(i)], the incident wave, shown by dotted curve, and the reflected wave, shown by dashed curve, are in the opposite phases. Hence the resultant displacement at each point is zero. All the particles are in their respective mean positions.

(ii) At $t = T/4$s [Fig. 14.12(ii)], the incident wave has advanced to the right by $\lambda/4$, as shown by the shift of the point P and the reflected wave has advanced to the left by $\lambda/4$ as shown by the shift of the point P′. The resultant wave form has been shown by the thick continuous curve. It can be seen that the resultant displacement at each point is maximum. Hence the particle velocity at each point is zero and the strain is maximum

(iii) At t = T/2s [Fig. 14.12(iii)], the incident wave advances a distance λ/2 to the right as shown by the shift of the point P and the reflected wave advances a distance λ/2 to the left as shown by the shift of the point P′. At each point, the displacements being in the opposite directions, have a zero resultant shown by a thick line.

(iv) At t = 3T/4s [Fig. 14.12(iv)], the two waves are again in the same phase. The resultant displacement at each point is maximum. The particle velocity is zero but the strain is maximum possible.

(v) At t = 4T/4s [Fig. 14.12(v)], the incident and reflected waves at each point are in the opposite phases. The resultant is a straight line (shown by an unbroken thick line). The strain $\Delta y/\Delta x$ at each point is zero.

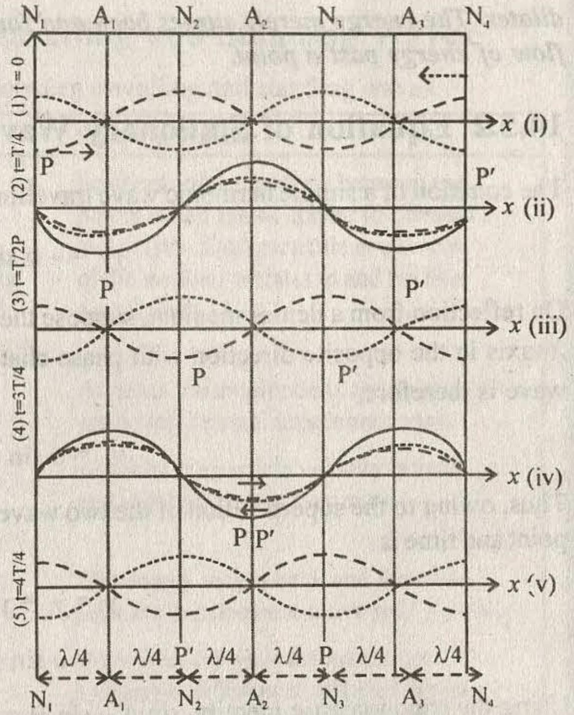


Fig. 14.12 : Showing formation of stationary waves due to superposition of two waves of same wave length, same amplitude travelling in opposite direction along the same line.

Note that

- at points N_1, N_2, N_3 and N_4, the amplitude is zero but the strain is maximum. Such points are called **nodes;**
- at points A_1, A_2 and A_3, the amplitude is maximum but the strain is minimum. These points are called **antinodes;**
- the distance between two successive nodes or between two, successive antinode is λ/2;
- the distance between a node and next antinode is λ/4;
- the time period of oscillation of a stationary wave equals the time period of the two travelling waves whose superposition has resulted in the formation of the stationary wave; and
- the energy alternately surges back and forth about a point but on an average, the energy flow past a point is zero.

Superposition of two identical collinear waves travelling with the same speed in opposite directions leads to formation of stationary waves. They are called stationary waves, because the wave form does not move forward, but alternately shrinks and

dilates. The energy merely surges back and forth and on an average, there is no net flow of energy past a point.

14.5.2 Equation of Stationary Wave

The equation of a simple harmonic wave travelling with velocity $v = \omega/k$ in a medium is

$$y_1 = -a \sin(\omega t - kx)$$

On reflection from a denser medium, suppose the wave travels along the same line along X–axis in the opposite direction with phase change of π. The equation of the reflected wave is therefore,

$$y_2 = a \sin(\omega t + kx)$$

Thus, owing to the superposition of the two waves, the resultant displacement at a given point and time is

$$y = y_1 + y_2$$

$$= a \sin(\omega t + kx) - a \sin(\omega t - kx)$$

Using the trigonometric identity. $\sin A - \sin B = 2 \operatorname{Sin} (A - B)/2, \cos (A + B)/2$, above expression simplifies to

$$y = +2a \sin kx \cos \omega t \qquad (14.20)$$

Let us put $+2a \sin kx = A$. Then we can write

$$y = A \cos \omega t$$

Eqn. (14.20) represents a resultant wave of angular frequency ω and amplitude $2a \sin kx$ This is the equation of stationary wave. The amplitude of the resultant wave, oscillates in space with an angular frequency ω, which is the phase change per metre. At such points where $kx = m\pi = m\lambda/2$, $\sin kx = \sin m\pi = 0$. Hence $A = 0$,

The points where the amplitude is zero are referred to as **nodes.** At these points $\Delta y/\Delta x$ = maximum, that is strain is maximum. Obviously the spacing between two nearest points is $\lambda/2$.

At those points where $kx = (2m + 1)\pi/2$ or $x = (2m + 1)\lambda/2 \times \lambda/2\pi = (2m + 1)\lambda/4$

$\operatorname{Sin} kx = \sin(2m + 1)\pi/2 = \pm 1$.

Hence A is maximum. At these points the strain $\Delta y/\Delta x$ is zero. Obviously the spacing between two such neighbouring points is $\lambda/2$. These points where the amplitude is maximum but strain is zero are referred to as **antinodes.**

It may be pointed out here that at nodes, the particle velocity is zero and at antinodes, particle velocity $\Delta y/\Delta t$ is maximum. Therefore, it follows that the average flow of energy across any point is zero. The energy merely surges back and forth. That is why, these waves have been named stationary or standing waves.

14.5.3 Distinction between Travelling and Standing Waves

Let us summarise the main differences between travelling and standing waves.

Travelling Waves	Standing Waves
1. Particular conditions of the medium namely crests and troughs or compressions and rarefactions appear to travel with a definite spped depending on density and elasticity (or tension) of the medium.	Segments of the medium between two points called nodes appear to contract and dialate. Each particule or element of the medium vibrates to and fro like a pendulum.
2. The amplitude of vibration of all the particles is the same.	At nodes the amplitude is zero but at antinodes the amplitude is maximum.
3. All the particles pass through their mean positions with maximum velocity one after the other.	At nodes the particle velocity is zero and at antinodes it is maximum.
4. Energy is transferred from particle to particle with a definite speed.	The energy surges back and forth in a segment but does not move past a point.
5. In a travelling wave the particles attain their maximum velocity one after the other.	In a stationary wave the maximum velocity is different at different points. It is zero at nodes but maximum at antinodes. But all the particles attain their respective maximum velocity simultaneously.
6. In a travelling wave each region is subjected to equal strains one after the other.	In case of standing waves strain is maximum at nodes and zero at antinodes.
7. There is no point where there is no change of density.	Antinodes are points of no change of density but at nodes there is maximum change of density.

Intext Questions 14.5

1. Does energy flow across a point in case of stationary waves? Justify your answer.

....

2. What is the distance between two successive nodes, and between a node and next antinode?

....

3. Pressure nodes are displacement antinodes and pressure antinodes are displacement nodes. Explain.

....

4. Stationary waves of frequency 170Hz are formed in air. If the velocity of the waves is 340 ms^{-1}, what is the shortest distance between (i) two nearest nodes (ii) two nearest antinode (iii) nearest node and antinode?

....

14.6 Characteristics of Musical Sound

The characteristics of musical sounds help us to distinguish one musical sound from another. These are pitch, londness and quality. We will now discuss these briefly.

14.6.1 Pitch

The term pitch is the characteristic of musical notes that enables us to classify a note as 'high' or 'low'. It is a subjective quantity which cannot be measured by an instrument. It depends on frequency. However, there does not exist any one-to-one correspondence between the two. A shrill, sharp or acute sound is said to be of high pitch. But a dull, grave and flat note is said to be of low pitch. Roaring of lion, though of high intensity, is of low pitch. On the other hand, the buzzing of mosquito, though of low intensity, is of high pitch.

14.6.2 Loudness

The loudness of sound is a subjective effect of intensity of sound received by listeners ear. The *intensity of waves is the average amount of energy transported by the wave per unit area per second normally across a surface at a given point.* There is a large range of intensities over which the ear is sensitive. As such, logarithmic scale rather than arithmetic intensity scale is more convenient.

Threshold of hearing and Intensity of Sound

The intensity level β of a sound wave is defined by the equation.

$$\beta = 10 \log I/I_0 \qquad (14.21)$$

where I_0 is arbitrarily chosen reference intensity, taken as $10^{-12}\ Wm^{-2}$. This value corresponds to the faintest sound that can be heard. Intensity level is expressed in decibels, abbreviated *db*. If the intensity of a sound wave equals I_0 or $10^{-12}\ Wm^{-2}$, its intensity level is then $I_0 = 0$ *db*. Within the range of audibility, sensitivity of human ear varies with frequency. ***The threshold audibility at any frequency is the minimum intensity of sound at that frequency, which can be detected.***

The standard of perceived loudness is the ***sone.*** A sone is the loudness experienced by a listener with normal hearing when 1kilo hertz tone of intensity 40db is presented to both ears.

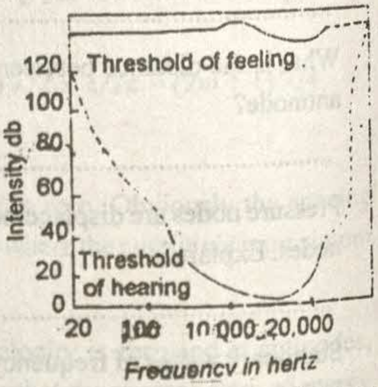


Fig. 14.13 : Auditory area between threshold of hearing and threshold of feeling

The range of frequencies and intensities to which ear is sensitive have been represented in a diagram in Fig. 14.13, which is in fact a graph between frequency in hertz versus intensity level 1 in decibels. This is a graph of auditory area of good hearing. The following points may be noted.

- The lower part of the curve shows that the ear is most sensitive for frequencies between 2000 Hz to 3000 Hz, where the threshold of hearing is about 5db. Threshold of hearing in general, is zero decibel.
- At intensities above those corresponding to

Notes

the upper part of the curve, the sensation changes from one of hearing to discomfort and even pain. This curve represents the threshold of feeling.

- Loudness increases with intensity, but there is no definite relation between the two.
- Pure tones of same intensity but different frequencies do not necessarily produce equal loudness.
- The height of the upper curve is constant at a level of 120 db for all frequencies.

The intensity of sound waves depends on the following factors :

- ***Amplitude of vibration :*** *$I\ \alpha\ a^2$ where a* is amplitude of the wave.
- ***Distance between the observer and the Source :*** $I\ \alpha\ 1/r^2$ where r is the distance of the observer from the source (provided it is a point source).

3. ***Intensity is directly proportional to the square of frequency of the wave*** ($I\ \alpha\ v^2$).
4. ***Intensity is directly proportional to the density of the medium*** ($I\ \alpha\ \rho$).

14.6.3 Quality

It is the *characteristic of sound waves which enables us to distinguish between two notes of the same pitch and intensity but sounded by two different instruments.* No instrument, except a tuning fork, can emit a pure note; a note of one particular frequency. In general, when a note of frequency n is sounded, in addition to it, notes of higher frequencies $2n$, $3n$, $4n$ ... may also be produced. These notes, have different amplitudes and phase relations. The resultant wave form of the emitted waves determines the quality of the note emitted. Quality, like loudness and pitch is a subjective quantity. It depend on the resultant wave form.

14.6.4 Organ Pipes

It is the simplest form of a wind instrument. A wooden or metal pipe producing musical sound is known as organ pipe. Flute is an example of organ pipe. If both the ends of the pipe are open, we call it an ***open pipe.*** However, if one end is closed, we call it a ***closed pipe.*** When we blow in gently, almost a pure tone is heard. This pure tone is called a ***fundamental note***. But, when we blow hard, we also hear notes of frequencies which are integral multiple of the frequency of the fundamental note. You can differentiate between the sounds produced by water from a tap into a bucket. These frequencies are called **overtones.**

Note that

- At the closed end of a pipe, there can be no motion of the air particles and the closed end must be node.
- At the open end of the pipe, the change in density must be zero since this end is in communication with atmosphere. Further, since the strain is zero, hence this end must be an antinode.

(a) Open pipe : The simplest mode of vibrations of the air column called fundamental

Notes

mode is shown in Fig.14.14 (a). At each end, there is an antinode and between two antinodes, there is a node. Since the distance between a node and next antinode is λ/4, the length l of the pipe is

$$l = (\lambda/4) + (\lambda/4) = \lambda/2 \text{ or } \lambda = 2l.$$

The frequency of the note produced is

$$n_1 = \upsilon/\lambda = \upsilon/2l$$

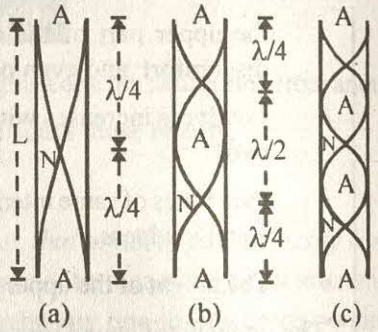


Fig. 14.14 : Harmoniscs of an open Organ pipe. The curves represent the wave of the longitudinal standing waves

The next mode of vibration of the air column is shown in Fig.14.14 (b). One more node and one more antinode has been produced. In this case

$$\lambda = (\lambda/4) + (\lambda/4) + (\lambda/4) + (\lambda/4) = l$$

The frequency of the note is

$$n_2 = \upsilon/\lambda = \upsilon/l = 2\upsilon/2l$$

$$n_2 = 2\upsilon/2l$$

That is $n_2 = 2n_1$

The note produced is called second harmonic or 1st ***overtone***. To get the second harmonic you have to blow harder. But if you blow still harder one more node and one more antinode is produced [Fig.14.14{c)]. Thus, in this case

$$l = \frac{\lambda}{2} + \frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{4}$$

$$\lambda = \frac{2l}{3}$$

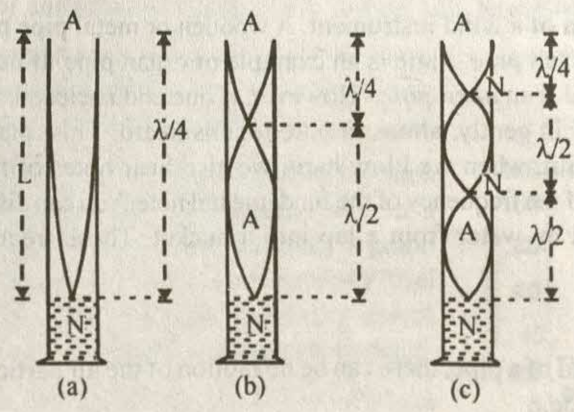


Fig. 14.15 : Harmonics of a closed organ pipe. The curves represented wave form of the longitudinal standing waves.

Therefore, the frequency of the note emitted is

$$n_3 = \frac{\upsilon}{\lambda} = \frac{3\upsilon}{2l} = 3n_1$$

Notes

The note produced is called the 3rd harmonic or 2nd overtone.

(b) Closed pipe : The simplest manner in which the air column can vibrate in a closed pipe is shown in Fig. 14.15(a). There is an antinode at the open end and a node at the closed end. The wave length of the wave produced is given by

$$l = \lambda/4 \text{ or } \lambda = 4l$$

Therefore, the frequency of the note emitted is

$$n_1 = \upsilon/\lambda = \upsilon/41$$

The note produced is called *fundamental* note. On blowing harder one more node and antinode will be produced (Fig. 14.15(b)). The wavelength of the note produced is given by

$$l = \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{3\lambda}{4} \text{ or } \lambda = \frac{4l}{3}$$

The frequency of the note emitted will be

$$n_2 = \frac{\upsilon}{\lambda} = \frac{3\upsilon}{4l} = 3n_1$$

The note produced is called the first overtone or the 3rd harmonic of the fundamental, blowing still harder one more node and one more antinode will be produced Fig. 14.15(C). The wavelength of the note produced is then given by

$$l = \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{5\lambda}{4} \text{ or } \lambda = \frac{4l}{5}$$

The frequency of the note emitted then will be

$$n_3 = \frac{\upsilon}{\lambda} = \frac{5\upsilon}{4l} = 5n_1$$

The note produced is called the second overtone or the 5th harmonic of the fundamental. On comparison with the notes emitted by the open and closed pipe, you will find that the open pipe is richer in overtones. In closed pipe, the even order harmonics are missing.

Example 14.5 : Two organ pipes – one open and the other closed – are of the same length. Calculate the ratio of their fundamental frequencies.

Solution : $\frac{\text{Frequency of open pipe}}{\text{Frequency of closed pipe}} = \frac{\upsilon/2\ell}{\upsilon/4\ell} = 2$

∴ Frequency of note produced by open pipe = 2 × frequency of fundamental note produced by closed pipe.

Intext Questions 14.6

1. How pitch is related to frequency?

....

2. What is that characteristic of musical sounds which enables you distinguis between two notes of the same frequency, and same intensity but sounded by two different instruments?

....

3. Name the characteristic of sound which helps you identify the voice of your friend.

....

4. Out of open and closed organ pipes, which one is richer in overtones?

....

5. What is the ratio of the frequencies of the notes emitted (1) by an open pipe and (ii) by a closed pipe of the same length.

....

6. What will be the effect of temperature, if any, on the frequency of the fundamental note of an open pipe?

....

Noise Pollution

When the sensation of sound changes from one of hearing to discomfort, it causes noise pollution and if it persits for a long time, it has harmful effects on certain organ of human beings. Noise is also one of the by-products of industrialisation and misuse of modern amentities provided by science to human beings. We summarise here under the sources or description of noises and their effects as perceived by the human beings.

Table 14.1 : Sources of Noise and their Effects

Source	Intensity Level in decibels	Perceived Effect by human being
Threshold of hearing	0 (=10^{-12} Wm^{-2})	Just audible
Rustle of leaves	10	Quiet
Average whisper	20	Quiet
Radio at low volume	40	Quiet
Quiet automobile	50	moderately loud
Ordinary conversation	65	do
Busy street traffic	70 to 80	loud
Motor bike and heavy vehicles	90	very loud
Jet engine about 35m away	105	Uncomfortable
Lightening	120 (=1 Wm^{-2})	do
Jet plane at take off	150	Painful sound

(a) Effect of Noise Pollution

1. It causes impairment of hearing. Prolonged exposure of noise at 85 or more than 85db causes severe damage to the inner part of the ear.
2. It increases the rate of heart beat and causes dilation of the pupil of eye.
3. It causes emotional disturbance, anxiety and nervousness.
4. It causes severe headache leading to vomiting.

(b) Methods of Reducing Noise Pollution

1. Shifting of old industries and setting new ones away from the dwellings.
2. Better maintenance of machinery, regular oiling and lubrication of moving parts.
3. Better design of engines and machines.
4. Restriction on use of loudspeakers and amplifiers.
5. Restricting the use of fire crackers, bands and loud speakers during religious, political and marriage processions.
6. Planting trees on roads for intercepting the path of sound.
7. Intercepting the path of sound by sound absorbing materials.
8. Using muffs and cotton plugs.

Shock Waves

When a source of waves is travelling faster than the sound waves, shock waves are produced. The familiar example is the explosive sound heared by an observer when a supersonic plane flies past over the head of the observer. It may be pointed out that the object which moves with a speed greater than the speed of sound is itself a source of sound.

14.7 Electromagnetic Waves

You know that light is an e.m. wave. It has wavelength in the range 4000°A to 7500°A. A brief description of em waves is given below.

14.7.1 Properties of e.m. waves

The following properties of e.m. waves may be carefully noted.

(*i*) e.m. waves are transverse in nature

(*ii*) They consist of electric (**E**) and magnetic fields (**B**) oscillating at right angles to each other and perpendicular to the direction of propagation (k). Also $\mathbf{E} = c\mathbf{B}$. [see figures 14.16]

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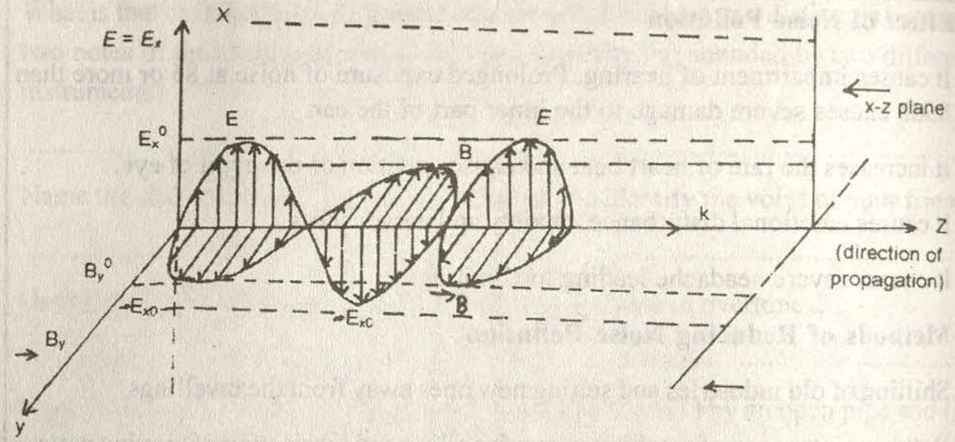


Fig. 14.16 : Electrical and Magnetic fields in em waves

(iii) They propagate through free space (in vacuum) with a uniform velocity = $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ = 3×10^8 ms^{-1} = c (velocity of light). For a medium of permeability μ $(= \mu_0 . \mu_r)$ and permittivity ε $(= \varepsilon_0 . \varepsilon_r)$ the velocity becomes

$$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} < c$$

(iv) The nature and action of these waves depends on their frequency (or wavelength). Maxwell's theory placed no restriction on possible wavelengths for e.m. waves and hence e.m. waves of wavelengths ranging from 6×10^{-13} m have been successfully produced. There is no limit to very long wavelengths which correspond to radio broadcast waves. The whole range of e.m. waves from very long to very short wavelengths constitutes the *electromagnetic spectrum.*

James Clark Maxwell
(1831 – 1879)

Scottish Mathematician and physicist Maxwell is famous for his theories of electromagnetic fields. Through his equations of electromagnetic principles he showed that they implicitly indicated the existence of em waves which travelled with the speed of light, thus relating light and electromagnetism.

With clausius he developed the kinetic theory of gases. He developed a statistical theory of heat. A man of varied interests, he derived the theorem of equipartition of energy, showed that viscosity varies directly with temperature and tried to explain the rings of saturn.

14.7.2 Electromagnetic Spectrum

Maxwell gave the idea of e.m. waves while Hertz, J.C. Bose, Marconi and others successfully produced such waves of different wavelengths experimentally. However, in all the methods, *the source of e.m. waves is the accelerated charge.*

Electromagnetic waves are classified according to the method of their generation and are named accordingly. Overlapping in certain parts of the spectrum by different classes of e.m waves is also observed. This tells that the e.m. waves of wavelenths in the overlapping region can be produced by two different methods. It is ***important to remember that the physical properties of e.m. waves are determined by the frequencies or wavelengths and not by the method of their generation.*** A suitable classification of e.m. waves is called the electromagnetic spectrum.

There is no sharp dividing point between one class of e.m. waves and the next. The different parts are as follows :

(i) **The low frequency radiations** $\left\{\begin{matrix} v = 60\text{Hz to } 50\text{Hz} \\ \lambda = 5\times10^{6}\,m \text{ to } 6\times10^{6}\,m \end{matrix}\right\}$: generated from a.c. circuits are classified as power frequencies or power waves or electric power utility e.m. waves. These *weaves have the lowest* frequency.

(ii) **Radio Waves** $\left\{\begin{matrix} \lambda = 0.3\text{m to } 10^{6}\text{m} \\ v = 10^{9}\,\text{Hz } to \text{ } 300\text{Hz} \end{matrix}\right\}$: Radio waves are generated when charges are accelerated through conducting wires. They are generated in such electronic devices as LC oscillators and are *used extensively in radio and television communications.*

(iii) **Microwaves** $\left\{\begin{matrix} \lambda = 10^{-3}\text{m to } 0.3\text{m} \\ v = 10^{11}\,\text{Hz } to \text{ } 10^{9}\,\text{Hz} \end{matrix}\right\}$: These are generated by oscillating currents in special vacuum tubes. Because of their short wavelengths, they are well suited for the radar system used in aircraft navigation, T.V. communication and for studying the atomic and molecular properties of matter. Microwave ovens use these radiations as heat waves. It is suggested that solar energy could be harnessed by beaming microwaves to Earth from a solar collector in space.

(iv) **Infra-red waves** $\left\{\begin{matrix} \lambda = 7\times10^{-7}\text{m to } 10^{-3}\text{m} \\ v = 4.3\times10^{14}\,\text{Hz to } 3\times10^{11}\,\text{Hz} \end{matrix}\right\}$: Infra-red waves, also called heat waves, are produced by hot bodies and molecules. These are readily absorbed by most materials. The temperature of the body, which absorbs these radiations, rises. Infrared radiations have many practical and scientific applications including physical therapy infrared photography etc. These are detected by a thermopile.

(v) **Visible light** $\left\{\begin{matrix} \lambda = 4\times10^{-7}\text{m to } 7\times10^{-7}\text{m} \\ v = 7.5\times10^{14}\,\text{Hz to } 4.3\times10^{14}\,\text{Hz} \end{matrix}\right\}$: These are the e.m. waves that human eye can detect or to which the human retina is sensitive. It forms a very small portion of the whole electromagnetic spectrum. These waves are produced by the rearrangement of electrons in atoms and molecules. When an electron-jumps from outer orbit to inner orbit of lower energy, the balance of energy is radiated in the form of visible radiation. The various wavelengths of visible lights are classified with colours, ranging from violet ($\lambda = 4 \times 10^{-7}$m) to red ($\lambda = 7 \times 10^{-7}$). Human eye is most sensitive to yellow-green light ($\lambda = 5 \times 10^{-7}$m). Light is the basis of our communitation with the world around us.

(vi) **Ultraviolet** $\left\{\begin{matrix} \lambda = 3\times10^{-9}\,\text{m to } 4\times10^{-7}\,\text{m} \\ \nu = 10^{-17}\,\text{Hz to } 7.5\times10^{14}\,\text{Hz} \end{matrix}\right\}$: Sun is the important source of ultraviolet radiations, which is the main cause of suntans. Most of the ultraviolet light from Sun is absorbed by atoms in the upper atmosphere i.e. stratosphere, which contains ozone gas. This ozone layer then radiates out the absorbed energy as heat radiations. Thus, the lethal (harmful to living beings) radiations get converted into useful heat radiations by the ozone gas, which warms the stratosphere. These ultraviolet rays are used in killing the bacteria in drinking water, in sterilisation of operation theatres and also in checking the forgery of documents.

(vii) **X-rays** $\left\{\begin{matrix} \lambda = 4\times10^{-13}\,\text{m to } 4\times10^{-8}\,\text{m} \\ \nu = 7.5\times10^{20}\,\text{Hz to } 7.5\times10^{15}\,\text{Hz} \end{matrix}\right\}$: These are produced when high energy electrons bombard a metal target (with high melting point) such as tungsten. X-rays find their important applications in medical diagnostics and as a treatment for certain forms of cancer. Because, they destroy living tissues, care must be taken to avoid over-exposure of body parts. X-rays are also used in study of crystal-structure. They are detected by photographic plates.

(viii) **Gamma rays** $\left\{\begin{matrix} \lambda = 6\times10^{-17}\,\text{m to } 10^{-10}\,\text{m} \\ \nu = 5\times10^{24}\,\text{Hz to } 3\times10^{18}\,\text{Hz} \end{matrix}\right\}$: These are emitted by radioactive nuclei such as cobalt (60) and ceasium (137) and also during certain nuclear reactions in nuclear reactors. These are highly penetrating and cause serious damage when absorbed by living tissues. Thick sheets of lead are used to shield the objects from the lethal effects of gamma rays.

The energy (E) of e.m. waves is directly proportional to their frequency ν $\left(E = h\nu = \frac{hc}{\lambda}\right)$ and inversely proportional to their wave-length (λ). Thus gamma rays are the most energetic and penetrating e.m. waves, while the power frequencies, and the A.M. radio waves are the weakest radiations. Gamma rays are used to detect metal flaws in metal castings. They are detected by Geiger tube or scintillation counter.

Depending on the medium, various types of radiations in the spectrum will show different characteristic behaviours. For example, while whole of the human body is opaque to visible light, human tissues are transparent to X-rays but the bones are relatively opaque. Similarly Earth's atmosphere behaves differently for different types of radiations.

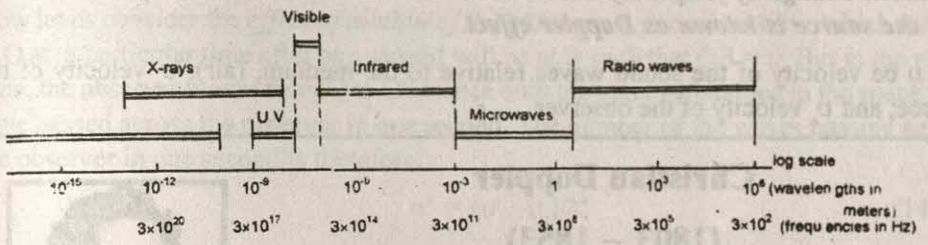


Fig. 14.17 : Electromagnetic spectrum

Intext Questions 14.7

1. Fill in the blanks:

   (i)..............................are generated by oscillating currents in special vacuum tubes.

   (ii) Human eye is most sensitive to....................................color light.

   (iii)............................is the important source of ultraviolet radiation.

   (iv)............................are used as the diagnostic tool in medical,

   (v) Infrared radiations can be detected by a..................................

   ....

2. Which of the e.m. waves are more energetic?

   (i) Ultraviolet or infrared.

   (ii) x-rays or γ-rays

   ....

3. Which of the e.m. waves are used in aircraft navigation by radar?

   ....

4. Which gas in the atmosphere absorbs ultraviolet radiations from the Sun before reaching the earth's surface?

   ....

5. How are the electric field and magnetic field oriented with respect to each other in an e.m. wave?

   ....

14.8 Doppler Effect

While waiting on a railway platform for the arrival of a train, you might have observed that the pitch of the whistle when the engine approaches you and when the engine moves away from you are different. You will note that the pitch is higher when the engine approaches but is lower when the engine moves away from you. Similarly, the pitch of the horn of a bus going up a hill changes constantly.

Apparent change of frequency observed due to the relative motion of the observer and the source is known as Doppler effect.

Let υ be velocity of the sound waves relative to the medium, (air), υ_s velocity of the source; and υ_o velocity of the observer.

Christian Doppler

(1803 – 1853)

C.J. Doppler, an Austrian physicist and mathematician, was born on Nov., 29, 1803 in a family of stone mesons. A pale and frail person, he was not considered good enough for his family business. So on recommendation of the professor of mathematics at Salzburg Lycousin, he was sent to Vienna Polytechnic from where he graduated in 1825.

A struggler through out his life, Doppler had to work for 18 months as a book-keeper at a cotton spinning factory. He could think of marrying in 1836 only when he got a permanent post at the technical secondary school at prague. He was once reprimanded for setting too harsh papers in maths for polytechnique students. But he pushed his way through all odds and finally got succes in getting the position of the first director of the new Institute of Physics at Vienna University.

The Doppler effect discovered by him made him famous overnight, because the effect had far reaching impact on acoustics and optics. The RADAR, the SONAR, the idea of expanding universe there are so many developments in science and technology which owe a lot to Doppler effect. He died on March 17, 1853 in Venice, Italy.

It is important to note that the wave originated at a moving source does not affect the speed of the sound. The speed υ is the property of the medium. The wave forgets the source as it leaves the source. Let us suppose that the source, the observer and the sound waves travel from left to right. Let us first consider the ***effect of motion of the source.*** A particular note which leaves the sources at a given time after one second arrives at the point A such that SA = υ. In this time, the source moves a distance υ_s. Hence all the n waves that the source had emitted in one second are contained in the space $x = \upsilon - \upsilon_s$. Thus length of each wave decreased to

$$\lambda' = \frac{\upsilon - \upsilon_s}{n} \qquad ...(14.22)$$

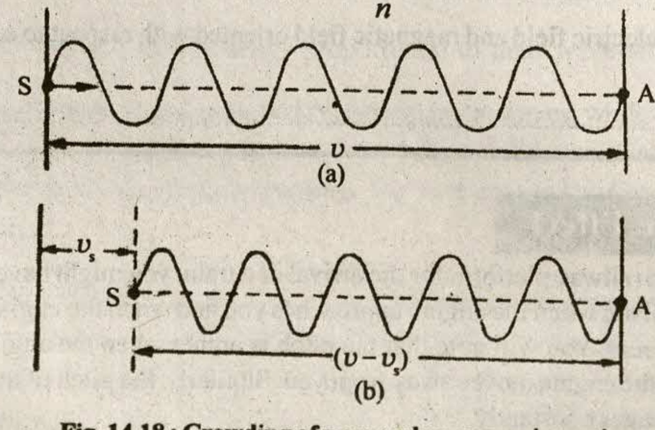


Fig. 14.18 : Crowding of waves when source is moving

Now let us consider the ***effect of motion of the observer.*** A particular wave which arrives at O at a particular time after one second will be at B such that OB = υ. But in the mean time, the observer moves from O to O′. Hence only the waves contained in the space O′B have passed across the observer in one second. The number of the waves passing across the observer in one second is therefore,

$$n' = (\upsilon - \upsilon_0)/\lambda' \qquad (14.23)$$

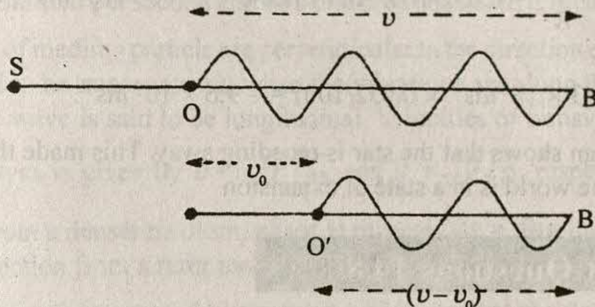


Fig. 14.19 : Waves received by a moving listner

Substituting for λ′ from Eqn. (14.22) we get

$$n' = \frac{\upsilon - \upsilon_0}{\upsilon - \upsilon_s} n \qquad (14.24)$$

where n' is the observed frequency when both observer and source are moving in the direction from the source to the observer.

In using Eqn.(14.24) ***the velocity of sound is taken positive in the direction from the source to the observer. Similarly***, υ_0 and υ_s are taken positive if these are in the direction of υ and vice versa.

The utility of Doppler's effect arises from the fact that it is applicable to light waves as much as to sound waves. In particular, it led us to the concept of expansion of the universe.

The following examples will help you to understand this application of Doppler's effect.

Example 14.6 : The light from a star, on spectroscopic analysis, shows a shift towards the red end of the spectrum of a spectral line. If this shift, called the red shift, is 0.032%, calculate the velocity of recession of the star.

Solution : In this case, the source of waves is the star. The observer is at rest on the Earth. We have shown that in such a case

$$\lambda' = \frac{\upsilon - \upsilon_s}{n}$$

But $n = \upsilon/\lambda$ Therefore, $\lambda' = \dfrac{\upsilon - \upsilon_s}{\upsilon/\lambda}$

$$= \lambda \frac{(\upsilon - \upsilon_s)}{\upsilon}$$

$$= \lambda \left(1 - \frac{\upsilon_s}{\upsilon}\right)$$

On rearranging terms, we can write

$$\frac{\lambda' - \lambda}{\lambda} = -\frac{\upsilon_s}{\upsilon}$$

or

$$\frac{\Delta\lambda}{\lambda} = \frac{\upsilon_s}{\upsilon}$$

we are told that $\frac{\Delta\lambda}{\lambda} = 0.032/100$. And since $\upsilon = c = 3 \times 10^8$ ms^{-1}, we get

$$\upsilon_s = \upsilon\frac{\Delta\lambda}{\lambda} = -(3 \times 10^8 \text{ ms}^{-1} \times 0.032/100) = -9.6 \times 10^4 \text{ ms}^{-1}.$$

The negative sign shows that the star is receding away. This made the astrophysists to conclude that the world is in a state of expansion

Intext Questions 14.8

1. A SONAR system fixed in a submarine operates at frequency 40.0kHz. An enemy submarine moves towards it with a speed of 100ms^{-1}. Calculate the frequency of the sound reflected by the sonar. Take the speed of sound in water to be 1450 ms^{-1}.

   ....

2. An engine, blowing a whistle of frequency 200Hz moves with a velocity 16ms^{-1} towards a hill from which a well defined echo is heard. Calculate the frequency of the echo as heard by the driver. Velocity of sound in air is 340ms^{-1}.

   ....

Constancy of Speed of Light

Aristotle, believed that light travels with infinite velocity. It was for the first time in September, 1876 that the Danish astronomer, Roemer, indicated in a meeting of Paris Academy of Sciences that the anomalous behaviour of the eclipse, times of Jupiter's inner satellite, Io, may be due to the finite speed of light. Feazeu, Focult, Michelson and many other scientists carried out experiments to determine the speed of light in air with more and more precision.

Albert Einstein, in his 1905 paper, on special theory of relativity, based his arguments on two postulates. One of the postulates was the constancy of speed of light in vacuum, irrespective of the wavelength of light, the velocity of the source or the observer. In 1983, the velocity of light in vacuum, was declared a universal constant with a value 299792458 ms^{-1}.

However, the Autralian researcher Barry Setterfield and Trevn Norwah have studied. the data of 16 different experiments on the speed of light in vacuum, carried out over the last 300 years, by different scientists at different places. According to them, the speed of light in vacuum is decreasing with time. If this hypothesis is sustained and coroborated by experiements, it will bring in thorough change in our world view. Major areas in which this change will be enormous are : Maxwell's laws, atomic structure, radioactive decay, gravitation, concepts of space, time and mass etc.

What You Have Learnt

- The distance between two nearest points in a wave motion which are in the same phase is called wavelength.
- The equation of a simple harmonic wave propagating along x–axis is $y = a \sin (\upsilon t - kx)$.
- The energy transmitted per second across a unit area normal to it is called intensity..
- If the vibrations of medium particle are perpendicular to the direction of propagation, the wave is said to be transverse but when the vibrations are along the direction of propagation the wave is said to be longitudinal. Velocities of transverse wave and longitudinal waves is given by $\upsilon = \sqrt{T/m}$ and $\upsilon = \sqrt{E/\rho}$ respectively.
- On reflection from a denser medium, phase is reversed by π. But there is no phase reversal on reflection from a rarer medium.
- When two waves are superposed, the resultant displacement at any point is vector sum of individual displacements at that point. Superposition of two colliner waves of same frequency but differing phases, when moving in the same direction results in redistribution of energy giving rise to interference pattern.
- Superposition of two collinear waves of the same frequency and same amplitude travelling in the opposite directions with the same speed results in the formation of stationary waves. In such waves, waveform does not move.
- In a stationary wave, the distance between two successive nodes or successive antinodes is $\lambda/2$. It is, therefore, obvious that between two nodes, there is an antinode and between two antinodes there is a node.
- The displacement is maximum at antinodes and minimum at nodes.
- Intensity level is defined by the equation $\beta = 10 \log (I/I_0)$, where I_0 is an arbitrarily chosen reference intensity of 10^{-12} W m^{-2}. Intensity level is expressed in decibels (Symbol. db)
- Quality of a note is the characteristic of musical sounds which enable us to distinguish two notes of the same pitch and same loudness but sounded by two different instruments.
- Electromagnetic waves are transverse in nature, and do not require any medium for their propagation.
- Light is an e.m. wave with wavelength in the range 4000 Å – 7500 Å.
- The frequency of e.m. waves does not change with the change in the medium.
- e.m. waves are used for wireless radio communication, TV transmission, satellite communication etc.

Terminal Exercises

1. How will you define a wave in the most general form?
2. Explain using a suitable mechanical model, the propagation of (i) transverse waves

(ii) longitudinal wave. Define the term wavelength and frequency.

3. Define angular frequency ω and propagation constant k and hence show that the velocity of the wave propagation is $v = \omega/k = n\lambda$.

4. Derive the equation of a simple harmonic wave of angular frequency of (i) transverse (ii) longitudinal waves.

5. What are the essential properties of the medium for propagation of (i) transverse waves (ii) longitudinal waves.

6. Derive an expression for the intensity of the wave in terms of density of the medium, velocity of the wave, the amplitude of the wave and the frequency of the wave.

7. Write Newton's formula for the velocity of sound in a gas and explain Laplace's correction.

8. When do two waves interfere (i) constructively (ii) destructively?

9. Show using trigonometry that when two simple harmonic waves of the same angular frequency ω and same wavelenght λ but of amplitudes a_1 and a_2 are superposed, the resultant amplitude is $A = \sqrt{a_2^2 + a_2^2 + 2a_1a_2 \cos\theta}$, where θ is the phase difference between them. What would be the value of A, for $\theta = 0$, (ii) for $\theta = 2\pi$, and (iii) for $\theta = (2m + 1)\pi$?

10. What are beats? How are they formed? Explain graphically.

11. Discuss graphically the formation of stationary waves. Why are these wave called stationary waves? Define nodes and antinodes.

12. State three differences between stationary and travelling waves.

13. Derive the equation of a stationary wave and show that displacement nodes are pressure antinodes and displacement antinodes are pressure nodes?

14. What are the characteristics of musical sounds. Explain.

15. What is a decibel (symbol) db)? What is meant by 'threshold of hearing' and 'threshold of feeling'?

16. What is meant by quality of sound? Explain with examples?

17. Discuss the harmonics of organ pipes. Show that an open pipe is richer in harmonics.

18. Show that (i) the frequency of open organ pipes. is two times the frequency of the fundamental note of a closed pipe of same length (ii) to produce a fundamental note of same frequency, the length of the open pipe must be two times the length of the closed pipe.

19. Describe an experiment to demonstrate existence of nodes and antinodes in an organ pipes?

20. State the causes of noise pollution, its harmful effects and methods of minimising it.

21. Explain Doppler's effect and derive an expression for apparant frequency. How does this equation get modified if the medium in which the sound travels is also moving.

22. Discuss the applications of Doppler's effect in (i) measuring the velocity of recession of stars, (ii) velocity of enemy plane by RADAR and (iii) velocity of enemy boat by SONAR?

23. Calculate the velocity of sound in a gas in which two waves of wavelengths 1.00m and 1.01m produce 10 beats in 3 seconds.

24. What will be the length of a closed pipe if the lowest note has a frequency 256Hz at 20C. Velocity of sound at 0C = 332 ms^{-1}.

25. The frequency of the sound waves emitted by a source is 1 kHz. Calculate the frequency of the waves as perceived by the observer when (a) the source and the observer are stationary, (b) the source is moving with a velocity of 50ms^{-1} towards the observer, and (c) the source is moving with a velocity of 50ms^{-1} away from the observer. Velocity of sound in air is 350ms^{-1}.

26. Write the characteristic properties of e.m. waves which make them different from sound waves.

27. How does the velocity of e.m. waves depend upon the permeability μ and permittivity ε of the medium through which they pass?

28. Give the range of wavelengths of the following e.m. waves:

    (i) Radio Waves (ii) Microwaves : (iii) Ultraviolet; (iv) x-rays.

29. How are x-rays produced?

30. Can e.m. waves of all frequencies propagate through vacuum?

31. Fill in the blanks.

    (i) A changing electric field produces a____________ in the adjacent region.

    (ii) ____________ are more harmful to our eyes than x-rays.

    (iii) ____________ are emitted from radio active nuclei of cobalt.

    (iv) Infra red rays are less energies than____________

    (v) In an e.m. wave propagating along z-direction, if the E field oscillates in the X,Z plane then the B field will oscillate in the ____________ plane.

    (vi) The ratio $\frac{\mathbf{E}}{\mathbf{H}}$ in free space of e.m. wave is called ____________.

    (vii) The frequency range of F.M. band is ____________.

    (viii)____________signal is frequency modulated in T.V. broadcasting.

Answers to Intext Questions

14.1

1. See section 14.1.4.

2. If p be the path difference, then the phase difference is $\theta = \frac{2\pi}{\lambda}p$.

3. ϕ

14.2

1. Newton assumed that compression and rarefaction caused by sound waves takes place under isothermal condition.

3. Newton assumed that isothermal conditions instead of adiabatic conditions for sound propagation.

4. 357°C.

5. $v = \sqrt{\dfrac{T}{m}}$

6. Therefore, $n = \dfrac{1}{\lambda}\sqrt{\dfrac{T}{m}}$

Further, for the simplest mode of vibration, at the two ends of the string, there are nodes and in between the two nodes is an antinode. Therefore, $l = l/2$ or $\lambda = 2l$, hence $n = \lambda/2l \sqrt{\dfrac{T}{m}}$. If the string vibrates in p segments, the $\lambda = p\, l/2$ or $\lambda = 2l/p$. Then $n = (p/2l)\sqrt{\dfrac{T}{m}}$.

14.3

For answers to all questions see text.

14.4

1. 25/9.
2. Beats with frequency 4Hz are produced.
3. Frequency of beat is Δv.
4. 517, on loading the frequency of A decreases from 517 to 507.

14.5

1. No energy swings back and forth in a segment.

2. Distance between two successive nodes is $\lambda/2$, and between a node and antinode is $\lambda/4$.

4. (i) 1m, (ii) 1m, (iii) 1/4m.

14.6

1. Pitch increases with increase in frequency.
2. Timbre
3. Timbre
4. Open pipe
5. For a closed pipe in case of fundamental note $l = \lambda/4$ or $\lambda = 4l$, therefore $n = \upsilon/\lambda = \upsilon/4l$.

For an open pipe $\ell = \lambda/2$. Therefore $n' = \upsilon/2l$.

Comparing (i) and (ii) we find that $n' = 2n$

6. $n = \frac{\upsilon}{2\ell}$. As υ increases with increase in temperature n also increases.

14.7

(*i*) microwaves.

(*ii*) yellow–green ($\lambda = 5 \times 10^{-7}$ m)

(*iii*) Sun.

(*iv*) X – rays.

(*v*) thermopile.

2. (*i*) ultra voilet

   (ii) r – rays.

3. Microwaves

4. Ozone.

5. Perpendicular to each other.

14.8

1. $n' = n\,\frac{c - \upsilon_0}{c}$

$$= 40 \times 10^3 \times \frac{1450-100}{1450}$$

$$= 40 \times \frac{135}{145} \times 10 = 37.2 \text{ KHz.}$$

2. $n' = 200 \times \frac{340+16}{340-16}$

$$= 200 \times \frac{356}{224} = 220 \text{ Hz.}$$

Answer to Terminal Problems

23. 337 ms^{-1}

24. ~ 30 cm.

25. (*a*) 1 kHz

    (*b*) 857 Hz

    (*c*) 1143 Hz.

SENIOR SECONDARY COURSE
PHYSICS
STUDENT'S ASSIGNMENT – 4

Maximum Marks: 50 **Time : 1½ Hours**

INSTRUCTIONS

- Answer All the questions on a seperate sheet of paper
- Give the following information on your answer sheet:
  - Name
  - Enrolment Number
  - Subject
  - Assignment Number
  - Address
- Get your assignment checked by the subject teacher at your study centre so that you get positive feedback about your performance.

Do not send your assignment to NIOS

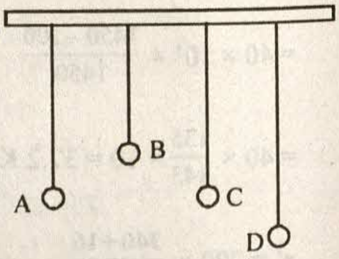
1. Which of the following represent simple harmonic motion (1)

   (*a*) $y = 1 + \omega t$.

   (*b*) $y = \sin.\omega t + \cos.\omega t$.

   (*c*) $y = \sin.\omega t + \cos \omega t$.

2. Four simple pendulum A, B, C and D are suspended from the same support. If any out of the pendulums is set into vibration all the four start oscillating. Which two of these pendulums will oscillate with the same frequency. Why? (1)



3. A mass *m* when made to oscillate on a spring of force constant *k* oscillates with a frequency *v*. The spring is then cut into two identical parts and the same mass is made to oscillate on half of the spring. What is the new frequency of oscillation of mass *m*. (1)
4. Give an example of a motion which is periodic but not oscillatory. (1)
5. Draw a graph showing the variation of velocity of sound in air with pressure. (1)
6. Is there a deviation in the direction of propagation of a sound wave in passing from air to water? Explain. (1)
7. What happens when a transverse wave pulse travelling on a string meets the fixed end of the string?(1)
8. What happens to the speed of *em* waves as they enter from vacuum to a material medium. (1)
9. Draw reference circle for the SHM represented by –

   $x = 3 \sin\left(2\pi r + \frac{\pi}{4}\right)$

Indicate the initial position of the particle, the radius of the circle and the angular speed of the rotating particle. For simplicity, the sense of rotation may be taken to be anticlockwise. In the given expression x is in *cm* and t is in seconds. (2)

10. Two waves having intensities in the ratio 1 : 9, superpose to produce interference pattern on a screen. Find the ratio of maximum and minimum intensities in the interference pattern. (2)

11. Two tuning forks A and B are marked 480 *hz* each. When they are sounded together they give 5 beats s^{-1}. What can you say about the frequency marked on the tuning forks. How can you find the ratio v_A/v_B? (2)

12. (a) Name the *em* waves used in aircraft navigation by radar?

    (b) Which gas in atmosphere absorbs *u-v* radiation? (2)

13. Write Laplace's formula for the speed of sound in air. Using the formula explain why the speed of sound in air (*a*) increases with temperature (*b*) increases with humidity. (4)

14. A transverse harmonic wave on a string is described by

    $y(x_1 t) = 3.0 \sin(36t + 0.018x)$

    find (*i*) amplitude of particle velocity. (*ii*) wave velocity. (4)

15. A bat emits urasonic waves of frequency 10^3 KHz in air. If the waves strike a water surface, find the difference in the wave lengths of transmilted sound and reflected sound. (speed of sound in air is 350 ms^{-1} and is water 1500 ms^{-1}). (4)

16. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 50 Hz. The mass of the wire is 3.5×10^{-2} *kg* and its linear density is 4.0×10^{-2} $kg\ m^{-1}$. What is (a) the speed of a transverse wave on the string and (b) the tension in the string? (4)

17. A pipe 20 *cm* long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open. (4)

18. Explain why :

    (*i*) solids can support both transverse as well as longitudinal waves, however, only lougitudinal can propagate in gases.

    (*ii*) the shape of a pulse get distorted during propagation in a dispersive medium.

    (*iii*) in a sound wave displacement node is pressure antinode and vice-versa.

    (*iv*) a note played on voilin and sitar has the same frequency but the two may still be distinguished from each other. (4)

19. Discuss the applications of doppler effect in measuring

    (*i*) the velocity of recession of stars (*ii*) velocity of enemy boat by SONAR. (5)

20. The transverse displacement of a string of length 1.5 *m* and mass .03 kg which is clamped at both ends, is given by (5)

    $$y = 0.068m\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$$

    when x and y are in m and t in s.

    (*i*) Does it represent travelling wave or stationary wave?

    (*ii*) Interpret the wave as a result of superposition of two waves.

    (*iii*) Determine the frequency, wavelength and speed of each super posing wave.

Curriculum

Rationle

Physics is a fundamental science because it deals with the basic features of the world, such as, time, space, motion, charge, matter and radiation. Every event that occurs in the natural world has some features that can be viewed in these terms. Study of physics need not necessarily be taken as a means of becoming a physicist; it is a means of rationally understanding nature. Physics lies behind all technological advancements, such as, computer, internet, launching of rockets and satellites, radio and T.V communications, lasers, etc. It also finds applications in such simple activities of men as lifting a heavy weight or making a long jump. Physics is thus an all pervading science and its study helps us in finding answers to whys and hows of our day to day happenings.

Keeping in view the issues highlighted in the National Curriculum Framework (NCF) for School Education, present Physics curriculum has been so designed that it not only focusses on the basic concepts of Physics but relates them to the daily life activities. The application of the laws of Physics and their effects on daily life have been reflected in the curriculum. The basic themes of Physics which would be of interest to all, particularly to those who are interested in pursuing Physics as a career in life have been selected to form core content of the curriculum. Besides, the curriculum also includes such emerging areas as electronics, communication, nuclear physics, photography and Audio - videography, which find immense applications in daily life.

Though mathematics is basic to the understanding of most of the problems of physics, in the present course, stress has been given to avoid rigour of mathematics like intergration and differentiation. The focus has been to teach concepts of physics rather than mathematical calculations.

Course Objectives

The basic objectives of the sr. secondary level Physics course are to enable the learner to :

- acquire knowledge and develop understanding of concepts, fundamental laws, principles and processes in the area of physics so that relationship between causes and effects of physical phenomnon can be understood;
- appreaciate the contributions of physics towards improving quality of life;
- promote interest in physics and foster a spirit of enquiry; and
- improve competencies of individuals in work skills required in their profession.

As a part of this process, the course also aims at developing the following abilities in the learner:

- experimental skills like taking observations, manipulation of equipment, and communicative skills such as reporting of observations and experimental results;
- problem solving ability e.g analyzing a situation or data, establishing relationship between cause and effect;
- scientific temper of mind by making judgment on verified facts and not opinions, by showing willingness to accept new ideas and discoveries; and
- awareness of the dangers inherent in the possible misuse of scientific knowledge.

Course Structure

The physics curriculum at sr. secondary level consists of both theory and practical components.

(i) The theoretical part of the Physics curriculum includes two parts – core modules and optional modules.

1. **Core modules :** The core modules comprise of the essential concepts and phenomena of physics which

a student at this level should know. There are eight core modules which contain predominantly the subject matter of mechanics, electricity, light and other areas of physics representing the minimum knowledge required to progress into the more advanced areas and to develop appreciation for the fact that physics plays a significant role in most situations.

Core Modules	Marks	Minimum Study Time (hours).
1..Motion, Force and Energy	14	45
2. Mechanics of Solids and Fluids	06	20
3. Thermal Physics	08	25
4. Oscillations and Waves	05	20
5.Electricity and Magnetism	14	45
6. Optics and Optical Inctruments	07	25
7. Atoms and Nuclei	07	25
8. Semiconductors and their Applications	07	205
Total	68	230 hours

2. Optional Modules : The optional modules are in the application oriented specific fields like Electronics and Communication and Photography and Audi– Videography. In the optional modules there is a choice to opt any one of the given modules. Each modules carries a weightage of 12 makrs which makes 15% of total theory marks.

Modules	Marks	Minimum Study Time (in hours)
1. Electronics and Communication	12	30
2. Photography and Audio – Videography	12	30

(ii) Paractical in physics

There is a compulsory component of practicals in Physics. It carries a weightage of 20% marks in the term end examination. A list of experiments and suggested activities to be performed by the students is given at the end of theory syllabus.

Module 1 : Motion, Force and Energy

Approach : Besides highlighting the importance of universal standard units of measurement, applications of dimensions and vectors in the study of physics to be described in this module. The concept of motion and rest, cause of motion and different types of motion have been described with the help of daily life examples. Significance of gravitation, concept of work and energy are to be highlighted. The basics of the motion of a rigid body and the significance of rotational motion in day to day life has been explained.

Unit 1.1.: Units, Dimensions and Vectors

- Units of measurement – fundamental and derived units
- Dimensions of physical quantities
- Applications of dimensions
- Vectors and scalars

Supprotive Video programme

1. Application of Vector in our daily life

- Vectors and their graphical representation
- Addition and subtraction of vectors
- Resolution of vectors into rectangular components (two dimensions)
- Unit vector
- Scalar and Vector products

2. Unit 1.2 motion in a Straight line

- Distance and displacement
- Speed, velocity and acceleration
- Average & instantaneous velocities.
- Uniformly accelerated motion
- Position – time and velocity – time graphs
- Equations of motion with constant acceleration including motion under gravity
- Relative motion

Supportive Video Programme

1. Motion and Rest

3. Unit 1.3: Newton's laws of motion

- Concept of force and inertia
- First law of motion
- Concepts of momentum
- Second law of motion
- Third law of motion
- Impulse
- Conservation of linear momentum
- Friction – static and kinetc, factors affecting friction
- Importance of friction and methods of reducing fiction
- Free body diagram technique
- Elementary idea of inertial and non – inertial frames of references.

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1. Laws of Motion
2. Frictional force

Unit 1.4: Motion in a Plane

- Projectile motion (time of fligtht, range and maximum height)
- Trajectory of a projectile
- Uniform circular motion
- Centripetal acceleration
- Circular motion in daily life

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1. Planetary Motion
2. Circular Motion

Unit 1.5: Gravitation

- Universal law of gravitation
- Acceleration dute to gravity and its variation with height, depth and latitude (only formula), value of g at moon
- Kepler's laws of planetary motion (no derivation)
- Motion of planets, orbital and escape velocity
- Satellites – geostationary and polar
- Achievements of India in the field of space exploration
- Applications of satellites

Supprotive Video Programme

1. Planetary Motion
2. Satellites and Their Applications

6. Unit 1.6 : Work, Energy and Power

- Work done by a constant force
- Work done by a varying force (graphical method) With example of spring
- Work – energy theorem
- Conservative and non – conservative forces
- Mechanical Energy (kinetic and potential energies) With examples.
- Conservation of energy (spring pendulum, etc)
- Elastic and inelastic collisions
- Power and its units.

Supportive Video Programme

1. Work and Power
2. Mechanical Energy

7. Unit 1.7 : Motion of a Rigid Body

- Rigid body motion, center of mass, couple and torque
- Moment of inertia, radius of gyration and its significance
- Theorems of parallel and perpendicular axes concerning moment of inertia and their uses in simple cases (no derivation)
- Equations of motion for a uniformly rotating rigid body (no derivation)
- Angular momentum and law of conseration of angular momentum with simple applications
- Rotational and transnational motions with examples (motion of ball, cylinder, flywheel on an incline plane)
- Rotational energy

Supportive Video Programme

1. Rotational Motion

Module 2 : Mechanics of Solids and Fluids

Approach : The classification of the substances into solids, liquids and gases is done on the basis of intermolecular forces. This module explains the elastic behaviour of the solids and highlights source of elastic behaviour of solids. The mechanical properties of the fluids like buoyancy, surface tention, capillary action etc. have been explained with the help of daily like examples and their applications have been highlighted.

8. Unit 2.1 : Elastic Properties of Solids

- Elastic behaviour and Hooke's law, stress – strain curve
- Inter –molecular forces
- Young's modulous, bulk modulous, modulous of rigidity and compressibility
- Some applications of elastic behaviour of solids like cantilever, girder etc.

Supportive Video Porgramme

1. Elastic Behaviour of solids

9. Unit 2.2: Properties of Fluids

- Hydrostatic pressure and buoyancy
- Pascal's law and its applications.
- Forces of cohesion and adhesion
- Surface tension and surface energy
- Angle of contact and capillary action
- Application of surface tension, drops, bubbles and detergents
- Types of liquid flow – laminar and turbulent, Reynold's number,
- Viscosity and Stoke's law
- Terminal velocity
- Bernoulli's theorem (no derivation) and its applications

Supportive Video Porgramme

1. Hydrostatic Pressure
2. Surface Tension
3. Viscosity and Berinoulli's Theorem

Module 3 : Thermal Physics

Approach : Behaviour of gases and the gas laws have been described with the help of kinetic theory of gases. The concept of temperature is to be explained by thermal equilibrium. Laws of thermodynamics and their applications in our day to day life are to be explained in this module. Working of heat engines and refrigerators will be explained. Different modes of transfer of heat and their applications in different situations are to be emphasized. The concept of thermal pollution and the issue of green house effect will also be dealt with in this module.

10. Unit 1.1 kinetic Theory of Gases

- Kinetic Theory of gases
- Deduction of the relation $PV = \frac{1}{3} mn \overline{c}^2$
- Ideal gas equation of state

Supportive Video Programme

- K.E. and temperature relationship
- Degree of freedom and law of equipartition of energy
- Specific heats of gases & ralationship between C_p & C_v

11. Unit 3.2 Laws of Thermodynamics

- Thermal equilibrium – Zeroth law of thermo dynamics and concept of temperature
- Thermodynamic variables and thermodynamic equilibrium
- Thermodynamic processes - isothermal, adiabatic, reversible, irreversible and cyclic process.
- First law of thermodynamics – internal energy
- Phase change, Phase diagram, latent heat and triple Point carnot's cycle and its efficiency – second law of thermodynamics, heat engine and refrigerator
- Limitations of Carnot's engine

Supportive Video Programme

1. Thermodynamic Processes
2. Heat Engine and Refrigerator

12. Unit 3.3 : Heat Transfer and Solar Energy

- Modes of transfer of heat – conduction, convection and radiation
- Newton's law of cooling.
- Green house effect
- Solar energy

Supportive Video Programme

1. Transfer of Heat

Module 4 : Oscilations and Waves

Approach : Besides explaining the terms associated with periodic motion, the harmonic motion will be described with the help of common examples. A qualitative idea of forced oscillations,resonance and damped oscillations will also be given in the module.

13. Unit 4.1: Simple Harmonic Motion

- Periodic motion – amplitude, period, frequency and phase
- Reference circle and equation of simple harmonic motion
- Examples of spring mass system and simple pendulum
- Forced oscillations and resonance (no derivation)
- Damped oscillations (no derivation)

Supportive Video Programme

1. Simple Harmonic Motion

14. Unit 4.2: Wave Phenomena

- Formation and propagation of waves

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1. Formation and Propagation of Waves

- Wavelenth, frequency, speed and their relationship, amplitude of wave and wave equation.
- Longitudinal and transverse waves
- Wave motion in a medium and formula for its speed
- Reflection of waves from rigid boundary
- Superposition of waves – interference of waves
- Standing waves and beats (only qualitative treatment with equations)
- Characteristics of musical sound (overtones and harmonics)
- Threshold of hearing, intensity of sound and noise pollution
- Doppler effect and its application. (qualitative only)
- Electromagnetic waves and their properties
- Em – waves spectra
- Constancy of speed of light (non – evaluative in a box)

2. Sound Waves

Module 5 Electricity and Magnetism

Approach : The basic concept of electrostatics and frictional electricity will be described in the module. The electric field and electric potential due to a point charge will be explained. Different types of capacitors, their compbinations and applications will be explained. The electric current and thermal and magnetic effects of current are explained in the module. Significance of maganetic effedct of current and electromagnetic induction has been emphasized. The generation and transmission of current power and the problems of low voltage and load shedding have been explained.

15. Unit 5.1 : Electric Charge and Electric Field

- Frictional electricity – electric charges and their conservation
- Coulomb's law
- Superposition principle
- Electric field and field intensity due to a point charge (through diagram)
- Force on a charged particle in an electric field
- Electric field of a dipole in uniform electric field
- Electric flux and Gauss theorem in electrostatics (no derivation)
- Uses of Gauss's theorem to determine electric field of a point charge, long wire, plane sheet.

Supportive Video Programme

1. Coublomb's Law or Frictional Electricity

16. Unit 5.2 : Electric Potential and Capacitors

Supportive Video Programme

- Electric potential due to a point charge

- Electric potential at a point due to a dipole (axial and equatorial).
- Electric Potential energy of a system of point charges
- Relation between electric field and potential – equipotent surface,
- Conductors and electric field inside a conductor
- Electrostatic shielding
- Capacitors and capacitance of a parallel plate capacitor.
- Different type of capacitors and their applications
- Capacitors in series and parallel combinations
- Energy stored in a capacitor
- Dielectrics and their polarization
- Effects of dialectics on capacitance

17. Unit 5.3 : Electric Current

- Electric current in a conductor
- Concept of drift velocity of electrons
- Ohm's law, ohmic and non – ohmic resistances –
- Colour coding of resistors.
- Free and bound electrons
- Combination of resistances (series and parlalle)
- Kirchoff's laws and their application to electrical circuits
- Wheatstone bridge principle and its application
- Electromotive force and potential difference
- Potentiometer and its applications.
- Heating effect of electric current – Joule's law of heating

Supportive Video Programme

1. Ohm's Law
2. Heating Effect of Electric Current

18. Unit 5.4 Magnectism and Magnetic Effect of Electric Current

- Bar Magnet and its magnetc field
- Magnetic effect of electric current
- Bio – Savart's law and its application to find magnetic field at the center of a coil carrying current (qualitative treatment)
- Ampere's circuital law and its application in finding magnetic field of a wire, circular loop (at the center), and solenoid.

Supportive Video Programme

1. Magnetism

- Force on a charged particle in a magnetic field; Lorentz force
- Force on a current carrying wire in a uniform magnetic fiedd
- Current loop as a Magnectic dipole and its magnetic moments
- Torque on a current loop in magnetc field
- Moving coil galvanometer and its conversion into ammeter and voltmeter
- Earth's magnetic field
- Ferro magnetic materials – domain theory (qualitative)

19. Unit 5.5 : Electromagnetic Induction and Alternating Current

- Feraday's law of electro – magnetic induction
- Lenz's law
- Self and mutual inducton – choke coil
- Alternating current and voltage illustrating with Phase diagram – peak and rms values
- Circuits containing only R, L or C separately – phase relationship between I & V
- LCR series combination (using phaser diagram only) and resonance
- Generators – AC and DC
- Transformers and their applications
- Transmission of electric power
- Problem of low voltage and load shedding (concepts of stabilizer and inverters)

Supportive Audio/Video Promgramme

1. Generation and Transmission of of Electric Current

Module 6 : Optics and Optical Instruments

Approach : After giving a brief introduction of reflection of light, the basic concepts like refraction, total internal reflection, dispersion, scattering, of light will be described in the module. The wave properties of light like interference, diffraction and polarization are also to be described in a qualitative manner. Further applications of the properties of light have been described to construct various types of optical instruments.

20. Unit 6.1 Reflection and Refraction of Light

- Reflection of light from spherical mirrors, sign convention and mirror formulae
- Refraction of light, Snell's law of refraction
- Total Internal Reflection and its applications in fibre optics

Supportive Video Programme

1. Reflection of light
2. Refraction of light

- Refraction through single curved surface and lenses
- Lens maker's formula and magnification
- Power of a lens
- Combination of lenses
- Defects of vision and their correction (myopia and hypermetropia)

21. Unit 6.2 : Dispersion and Seattering of light

- Dispersion of light, angle of deviation
- Rainbow and its formation
- Defects of image formation–spherical and chromatic aberration (qualitative only)
- Scattering of light in atmosphere.

Supportive Video Programme

1. Rainbow

22. Unit 6.3 : Wave Phenomena of light

- Huygen's wave theory and wave propagation.
- Interference–Young's double slit experiment
- Diffraction of light at a single slit (qualitative)
- Polarization-Brewster's law and its application in daily life

Supportive Video Programme

23. Unit 6.4 : Optical Instruments

- Simple and Compound microscopes and their magnifying power
- Telescopes–reflecting and refracting
- Resolving power and Rayleigh's criterion
- Applications in astronomy

Supportive Video Programme

1. Optical Instruments

Module 7 : Atoms and Nuclei

APPROACH : Different atomic models describing the structure of atom have been described and the limitations of these and their modifications have been systematically presented in the module. Nuclei and radio activity have been explained along with their applications. The peaceful uses of nuclear energy have been described highlighting the latest trends.

24. Unit 7.1 : Structure of Atom

- Alpha-Particle scattering and Rutherford's atomic model
- Bohr's model of hydrogen atom and energy levels
- Hydrogen spectrum
- Emission and absorption spectra

Supportive Video Programme

1. Atomic Structure

25. Unit 7.2 : Dual nature of Radiation and Matter

- Work function and emission of electrons
- Photoelectric effect and its explanation

Supportive Video Programme

1. Photo electric Effect and its Applications

- Photo electric tube and its applications
- Matter waves-Davisson and Germer experiment
- Electron microscope (non evaluative box)

26. Unit 7.3 : Nuclei and Radioactivity

- Atomic mass unit, mass number, size of nucleus
- Isotopes and isobars
- Nuclear forces, mass-energy equivalence
- Mass defect and binding-energy curve
- Radioactivity-alpha, beta decay and gamma emission
- Half life and decay constant of nuclei
- Applications of radioactivity

Supportive Video Programme

1. Radioactivity and its Applications

27. Unit 7.4 : Nuclear Fission and Fusion

- Nuclear reactions
- Nuclear fission and chain reaction
- Nuclear-fusion-energy in stars
- Misuses of nuclear energy-atom bomb and hydrogen bomb (non-evaluative in a box
- Peaceful uses of Nuclear Energy (including latest trends)
- Hazards of nuclear radiation and safety measures

Supportive Video Programme

1. Nuclear Energy

Module 8 : Semiconductors and their Applications

APPROACH : Semiconductors find a very significant place in almost all the electronic devices. Besides highlighting the basis of semiconductors, different types of semiconductor devices and their applications have been explained in the module.

28. Unit 8.1 : Semiconductors and Semiconductor Devices

- Intrinsic and extrinsic semiconductors
- Pn-junction-its formation and properties
- Biasing of pn-junction diode
- Characterstics of pn-junction diode
- Types of diodes-zanier diode, LED, Photo diode and solar cell
- Transistors-pnp and npn
- Characteristic curves of a transistor

Supportive Video Programme

29. Unit 8.9 : Applications of Semiconductor Devices

- pn-junction diode as a rectifier
- Zener diode as a voltage regulator
- Transistor as an amplifier (common emitter)

Supportive Video Programme

Semiconductor Devices and their application

- Transistor as a switching device
- Logic gates and their realization (OR, AND, NOT, NAND, NOR)
- Introduction to integrated circuits (IC)

OPTIONAL MODULES

Optional Module–1 : Electronics and Communication

APPROACH : In the present age of information and communication technology, it is essential for all to know the basic of electronics and communication technology. Working principles of different electronic devices used in daily life have been explained. Besides explaining communication systems, the communication techniques and media have been explained in the module.

30. Unit 1 : Electronics in Daily Life

- Power supply – SMPS, inverters, UPS
- Circuit Breaker – MCB
- Timer – digital clock
- Processor – calculator
- LCD
- transducers and control system – Burglar alarm/fire alarm

Supportive Video Programme

1. Electronics in Daily Life

31. Unit 2 : Communication Systems

- Communication system model –
- Components of communication systems like transmitter, receiver media of communication and antenna
- Types of signals – analogue & digital
- Electromagnetic waves in communication

Supportive Video Programme

1. Communication systems

32. Unit 3 : Communication Techniques and Devices

- Sampling
- Modulation – Analogue AM and FM, digital (p)
- Demodulation
- Role of tuner
- Common communication devices–radio/TV/Fax/Modern etc.

Supportive Video Programme

1. Modulation and Demodulation

33. Unit 4 : Communication Media

- Guided Media – transmission lines and optical fibre
- Unguided Media and antenae–ground wave Communication, sky wave communication, space wave communication and satellite communication.
- Communication application to modern day communication
- EDUSAT

Supportive Video Programme

Communication media

Optional Module – 2 : Photography and Audio–Video–Graphy

APPROACH : The basic principles of physics used in the field of photography and audio–videography have been described in different units of the module. Working principle of camera, types of camera, film exposing and processing have been explained the basic principles of audio and video recording both on tape and on compact disc have be described.

30. Unit 1 : Photography – Camera

- Camera – an introduction, parts of a camera, camera eye (lens), shutters, special lenses.
- Types of camera – their basic principle, constructions and working
- Principle of video camera.
- Choosing a camera, picture size.
- Choice of lens – angle of view and resolving power, aperture and focusing system.

Supportive Video Programme

1. Camera & its Working

31. Unit 2 : Film Exposing and Processing

- Constituents of photographic films and types of films.
- Characteristics of film
- Film exposure, aperture and speed
- Processing the film – developing, fixing and washing
- Printing of the photo

Supportive Video Programme

1. Film Exposing and Processing

32. Unit 3 : Audio–Video Recording

- Basic principle of recording
- Conversion of audio signal into electrical signals,
- Conversion of video signal into electrical signals.
- Storage of audio–video signals on tapes.
- Quality of recording, sound recording on cine films.
- Tape characteristics, structure and composition, tape format, tape speeds, important tape parameters,
- Presentation of tapes, storage techniques, precautions during handling and transportation.

Supportive Video Programme

1. Audio–Video Recording

33. Unit 4 : Compact Disc for Audio–Video Recording

- Limitations of traditional audio–video recording systems,
- Compact Disc
- Need for compact disc, advantages of compact disc.
- CD for audio recording,
- Basic principle of audio recordings,
- Methods of CD – audio-recording,

Supportive Video Programme

1. Compact Disc for Audio–Video Recording

- CD for video-recording,
- Basic principle for video recording
- Methods of CD – video recording
- General operating and installation precautions,
- CD – players, operating principle,
- Quality of reproduction.

List of Practicals

Section-A

1. Measurement of physical quantities using single scales like metre scale, graduated cylinder, thermometer, spring balance, stopwatch, ammeter, voltmeter,
2. Measurement of physical quantities using about scales like vernier callipers, screw gauge, barometer, travelling microscope etc.
3. Plotting and interpreting graphs of physical quantities like (i) L-T, L-T^2 for a simple pendulum, (ii) load-extension for spring balance, (iii) θ-t for a cooling body, (iv) I-V Characteristics for a resistor, (v) i-δ relation for a glass prism.

Section-B

1. Study the variation of time period (T) of a simple pendulum with lengths (L). Plot the L-T^2 graph and use it to determine (a) the length of a second's pendulum, (b) the value of acceleration due to gravity.
2. Determine the weight of a given body using parallelogram law of forces. Also, calculate, the mass of the body.
3. Draw the cooling curve of a body and calculate the rate of cooling at three different points of the curve.
4. Determine the specific heat capacity of a liquid using the method of mixtures.
5. Study the extension of a spring under different loads and calculate its spring constant (static method).
6. Determine the spring constant of a spring by dynamic method.
7. Study the rate of flow of a liquid as a function of pressure head using a burette.
8. Study the fall of a spherical body in various liquids of different viscosities. Determine the terminal velocity of the body in a viscous liquid and determine the coefficient of viscosity of that liquid.

Section-C

1. Study the formation of stationary waves in (a) stretched strings and (b) air columns. Determine the frequency of the tuning fork and comment on the result.
2. Investigate formation of images with mirrors and lenses. Determine the focal length of (a) convex lens, and (b) concave mirror.
3. Determine the internal resistance of a cell using a potentiometer.
4. Determine the resistance of a moving coil galvanometer by half deflection method. Convert the galvanometer into a voltmeter of suitable range and verify it.
5. Determine the resistivity of the material of a given wire using a metre bridge.

6. Study the I-V characteristics of (a) a resistor, and (b) a *P-n* juncton.

7. Study the characteristics of an *npn* transistor in common emitter configuration. Determine current and voltage gains.

Home Activities (Suggestive)

1. Determine the refractive index of a transparent liquid using cancave mirror and single pin.

2. Draw a graph between the angle of incidence and the angle of deviation for a glass prism. Determine the refractive index of the glass using the graph.

3. Study the relationship between the angle of rotation of a plane mirror and the change in angle of reflection.

4. Draw magnetic filed line due to a bar magnet keeping (i) North pole pointing north, and (ii) North pole pointing south. Locate the neutral points.

Physics

भौतिकी

(312)

Assignment - I

मूल्यांकन पत्र - I

(Lessons 1-12)

(पाठ 1 से 12 तक)

Max. Marks: 25
कुल अंक : 25

Note: (i) All questions are compulsory. Each question carries equal marks.
टिप्पणी: सभी प्रश्नों का उत्तर देना अनिवार्य है। प्रत्येक प्रश्न के अंक समान हैं।

(ii) Write your name, enrolment number, AI name and subject etc. on the top of the first page of the answer sheet.
उत्तर पुस्तिका के प्रथम पृष्ठ पर ऊपर की ओर अपना नाम, अनुक्रमांक, अध्ययन केन्द्र का नाम, विषय आदि स्पष्ट शब्दों में लिखिए।

1. Answer any **two** of the following questions in about 60-80 words each.
निम्नलिखित में से किन्हीं **दो** के उत्तर दीजिए। प्रत्येक उत्तर 60-80 शब्दों में होना चाहिए।

(a) Assuming that the frequency of vibration, f, of a wire/string depends on the length l, tension T and mass per unit length m, establish a formula for f in terms of l, T and m.
यह मानकर कि एक तार के कम्पन की आवृत्ति f उसके तार की लम्बाई l, तनाव T एवं तार की प्रति इकाई लम्बाई के द्रव्यमान m पर निर्भर करती है, कम्पन की आवृत्ति f के लिए l, T व m के पदों में सूत्र स्थापित कीजिए।

(b) A player throws a ball upwards with initial speed of 29.4 ms^{-1}. What are the velocity and acceleration of the ball at the highest point of its motion? To what height does the ball rise and after how much time does the ball return to the player's hands?
एक खिलाड़ी एक गेंद को प्रारंभिक चाल 29.4 ms^{-1} से उपर की तरफ फेंकता है। गेंद के पथ के उच्चतम बिंदु पर गेंद के वेग व त्वरण का मान कितना होगा? गेंद कितनी ऊँचाई तक उठती है तथा कितने समय के बाद गेंद खिलाड़ी के हाथों में वापस आ जाती है?

(c) Two cars A and B are moving in two concentric circular paths at their maximum permissible speeds. If the speed of the car A is more than the speed of car B, then which of them is moving in the inner circular path? Justify your answer.
दो कारें A व B दो समकेन्द्रीय वृत्ताकार पथों पर एक–समान गति से घूम रही हैं। यदि कार A की चाल कार B की चाल से अधिक हो तो उनमें से कौन सी कार अन्दर वाले वृत्ताकार पथ पर चल रही है?

2. Answer any **two** of the following questions in about 60-80 words each.
निम्नलिखित में से किन्हीं **दो** के उत्तर दीजिए, प्रत्येक उत्तर 60-80 शब्दों में होना चाहिए।

(a) Two objects of masses 6 kg and 2 kg are connected by a light cord passing over a frictionless fixed pulley. Calculate (i) the acceleration of the system, (ii) tension in the cord.

6 kg व 2 kg द्रव्यमान की दो वस्तुएं एक हल्की रस्सी द्वारा एक–दूसरे से जुड़ी हुई हैं और रस्सी एक घर्षण रहित स्थिर पुली के ऊपर से गुजर रही है। इस प्रक्रम का त्वरण कितना है तथा रस्सी में तनाव कितना है?

(b) You can easily move a brick lying on a floor by gently pushing it by your foot. But you hurt your foot when you try to kick the brick. Explain why?

किसी चिकने फर्श पर पड़ी ईंट को आप पैर से धीरे से धकेल कर आसानी से खिसका सकते हैं, लेकिन यदि आप उसे ठोकर मारें तो आपके पैर में चोट लग जाएगी। ऐसा क्यों होता है, व्याख्या कीजिए।

(c) A shell at rest explodes into three fragments of equal masses. If two of them fly off at right angles to one another, each with a speed of 1 ms^{-1}, calculate the speed of the third fragment.

एक स्थिर गोला विस्फोटित होकर बराबर द्रव्यमान के तीन टुकड़ों में बंट जाता है। यदि इसके दो टुकड़े एक दूसरे से समकोण बनाते हुए प्रत्येक 1 ms^{-1} की चाल से दूर चले जाते हैं, तो तीसरे टुकड़े की चाल की गणना कीजिए।

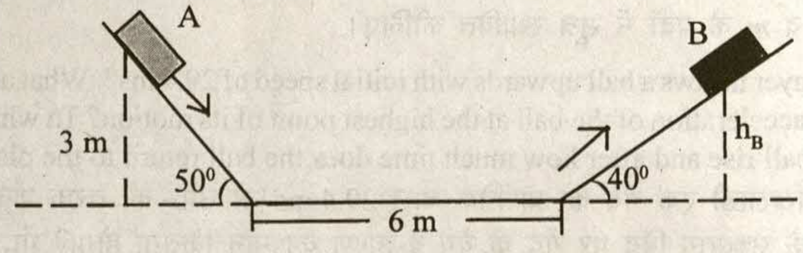
3. A block, as shown in the figure below, is released from rest at position A. To what height h_B will it rise before momentarily coming to rest at position B, if

(i) both the planes are frictionless, and (ii) the planes have coefficient of friciton $\mu_k = 0.15$?

नीचे चित्र में दिखाया गया ब्लॉक विरामावस्था में बिन्दु A से छोड़ा जाता है। बिन्दु B पर क्षण भर के लिए विरामावस्था में आने से पहले यह ब्लॉक कितनी ऊँचाई (h_B) तक जाएगा, यदि

(i) दोनों तल घर्षण रहित हों, तथा

(ii) दोनों तलों का घर्षण नियतांक $\mu_k = 0.15$ हो



OR (अथवा)

A cylindrical tank has water filled in it upto a height of 4.00 m. If a small hole is made in the wall of the tank near its bottom, calculate the speed of efflux of water. Take value of g = 9.8 ms^{-2}. Will the speed of efflux change if the hole is made 1.00 m above the bottom of the tank?

एक बेलनाकार टैंक में 4.00 मीटर ऊँचाई तक पानी भरा हुआ है। यदि इसकी दीवार में पेंदी के पास एक छोटा सा छेद कर दिया जाए तो उससे निकलने वाले पानी के बहाव का वेग कितना होगा? g = 9.8 ms^{-2} है। यदि दीवार में छेद पेंदी से एक मीटर ऊपर किया जाए तो क्या पानी के बहाव के वेग में कोई परिवर्तन होगा?

4. Three gas molecules chosen at random are found to have speeds 500 ms^{-1}, 1000 ms^{-1} and 1500 ms^{-1} respectively. Calcualte the ratio of their root mean square speed and average speed.

यादृच्छिक रूप से चुने गए तीन गैस अणुओं की चाल क्रमशः 500 ms^{-1}, 1000 ms^{-1} तथा 1500 ms^{-1} है। उनके वर्ग माध्य मूल वेग तथा औसत वेग का अनुपात ज्ञात कीजिए।

OR (अथवा)

A cylinder fitted with a movable piston contains 1 mole of oxygen at a temperature of 27^0 C. The gas is now heated to a temperature of 127^{0}C, keeping its pressure at 1 atm.

(i) Draw a P-V diagram representing the process.

(ii) How much work is done by the gas in this process?

(iii) What is that on which this work is done?

(iv) What is the change in the internal energy of the gas?

(v) How much heat was supplied to the gas?

एक बेलन में 27^0 C पर 1 मोल आक्सीजन गैस है। बेलन में एक घर्षण रहित पिस्टन है, जो गैस में 1 atm दाब बनाए रखता है। गैस को 127^0 C ताप तक गरम किया जाता है, तो

(i) इस प्रक्रिया को दर्शाते हुए एक P-V ग्राफ बनाइए।

(ii) इस प्रक्रिया में गैस द्वारा कितना कार्य किया जाता है?

(iii) किस पर यह कार्य किया जाता है?

(iv) गैस की आंतरिक ऊर्जा में कितना परिवर्तन होता है?

(v) गैस को कितनी ऊष्मा प्रदान की जाती है?

5. Take two different types of metallic springs. One by one fix one end of the spring with a rigid support on a wall and place a metre scale along its side. Place a load of 100 g at the free end. Note down extension of the spring on the metre scale. Now increase the load to 200 g and again note down the extension. Repeat this process for 5 different loads and record your observations. Plot graphs between load and extension for each spring. Compare and interpret the graphs. Verify the Hooke's law for both the springs. अलग–अलग तरह के दो धात्विक स्प्रिंग लीजिए। एक–एक करके इन स्प्रिंगों को इनके एक सिरे से दीवार में लगे किसी दृढ़ अवलम्ब के साथ लटकाइए। तथा दीवार पर स्प्रिंगों के साथ एक मीटर पैमाना भी लगाइए। अब एक स्प्रिंग के दूसरे सिरे पर 100 g का भार लटकाइए। इससे स्प्रिंग की लम्बाई में हुई वृद्धि को मीटर पैमाने पर मापिए। अब स्प्रिंग पर 200 g का भार लटकाइए और पुनः स्प्रिंग की लम्बाई में हुई वृद्धि को मापिए। इसी प्रक्रिया को 5 विभिन्न भारों के लिए दोहराइए तथा उनके कारण हुई लम्बाई में वृद्धि को मापिए। यही प्रक्रिया दूसरी स्प्रिंग के लिए दोहराइए। प्रत्येक स्प्रिंग के लिए भार व लम्बाई में वृद्धि के बीच ग्राफ खींचिए। दोनों ग्राफों की तुलना कीजिए तथा व्याख्या कीजिए। दोनों स्प्रिंगों के लिए हुक के नियम का सत्यापन कीजिए।

Physics

भौतिकी

(312)

Assignment - II

मूल्यांकन पत्र - II

(Lessons 13-23)

(पाठ 13 से 23 तक)

Max. Marks: 25
कुल अंक : 25

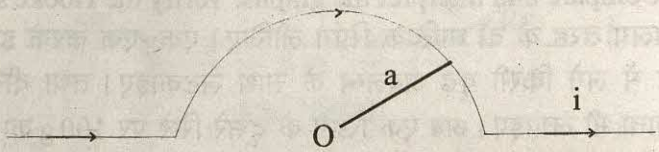
Note: (i) All questions are compulsory. Each question carries equal marks.
टिप्पणी: सभी प्रश्नों का उत्तर देना अनिवार्य है। प्रत्येक प्रश्न के अंक समान हैं।

(ii) Write your name, enrolment number, AI name and subject etc. on the top of the first page of the answer sheet.
उत्तर पुस्तिका के प्रथम पृष्ठ पर ऊपर की ओर अपना नाम, अनुक्रमांक, अध्ययन केन्द्र का नाम, विषय आदि स्पष्ट शब्दों में लिखिए।

1. Answer any **two** of the following questions.
निम्नलिखित प्रश्नों में से किन्हीं **दो** प्रश्नों के उत्तर दीजिए।

(a) State and prove Gauss' law in electrostatics. Use this law to derive Coulomb's law.
विद्युतस्थैतिकी में गाउस का नियम लिखिए और इसे सिद्ध कीजिए। इसका उपयोग करके कूलॉम के नियम को व्युत्पन्न कीजिए।

(b) A long wire has a semi-circular loop of radius a and carries a current i. Find the magnetic field at the centre O of the loop.
एक लम्बे तार में a त्रिज्या का अर्ध–वृत्ताकार लूप है। तार में से i धारा प्रवाहित हो रही है। लूप के केन्द्र O में चुंबकीय क्षेत्र का मान ज्ञात कीजिए।



(c) An air-cored solenoid 30 cm long and 2.5 cm in diameter contains 1500 closely wound turns. Calculate the self inductance of the solenoid. ($\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1})
किसी वायु क्रोडित परिनालिका की लम्बाई 30 cm और व्यास 2.5 cm है तथा इसमें सघन कुंडलित 1500 फेरे हैं। परिनालिका के आत्म प्रेरकत्व का परिमाण परिकलित कीजिए। ($\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1})

2. Answer any **two** of the following questions
निम्नलिखित प्रश्नों में से किन्हीं **दो** के उत्तर लिखिए।

(a) A transverse harmonic wave on a string is given by
$y(x, t) = 4.0 \sin(48\,t + 0.016\,x)$
Find (i) amplitide of particle velocity, and (ii) wave velocity

किसी डोरी में अनुप्रस्थ आवर्ती तरंग को निम्नलिखित समीकरण द्वारा व्यक्त किया गया है–

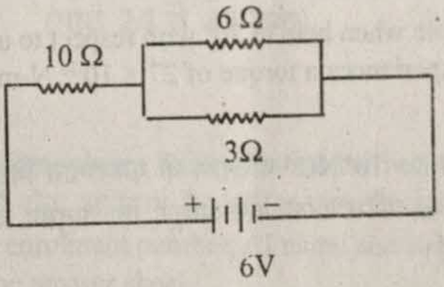
$y(x, t) = 4.0 \sin(48\,t + 0.016\,x)$

(i) इस तरंग के लिए कणों के वेग का आयाम,

(ii) तरंग वेग ज्ञात कीजिए।

(b) For the circuit shown below, calculate power dissipation in each of the three resistors.

नीचे दिखाए गए परिपथ में तीनों प्रतिरोधों में से प्रत्येक में क्षयित शक्ति की गणना कीजिए।



(c) State and explain Brewster's law. Mention some applications of this law in daily life.

ब्रूस्टर का नियम लिखिए और इसकी व्याख्या कीजिए। दैनिक जीवन में इस नियम के कुछ उपयोगों का उल्लेख कीजिए।

3. A 4.0 μF air capacitor is charged to a potential difference of 16.0 V by connecting it to a battery. The battery is kept connected and a slab of dielectric constant k = 8 is made to fill the space between the capacitor plates. Calculate the energy stored in the capacitor in each case.

एक वायु संधारित्र, जिसकी धारिता 4.0 μF है, को एक बैटरी से जोड़कर 16.0 V के विभवान्तर तक आवेशित किया गया है। बैटरी को जुड़ी रहने देकर संधारित्र के प्लेटों के बीच के खाली स्थान पर k = 8 परावैद्युतांक की एक पट्टी रख दी जाती है। दोनों स्थितियों में संधारित्र में संग्रहित ऊर्जा का परिकलन कीजिए।

Or (अथवा)

In a closed circuit of resistance 10 Ω, the flux (in weber) varies with t (in seconds) according to the equation $\phi = 6t^2 - 5\,t + 1$. What is the magnitude of induced current at $t = 0.15$ s?

एक बंद परिपथ, जिसका प्रतिरोध 10 Ω है, में अभिवाह (Wb में) का t (s में) के साथ परिवर्तन समीकरण $\phi = 6t^2 - 5\,t + 1$ के अनुसार होता है। ज्ञात कीजिए कि $t = 0.15$ s पर प्रेरित धारा का परिमाण क्या होगा?

4. A series LCR circuit with $R = 50\Omega$, $L = 30$ mH and $C = 40$ μF is driven by an ac source of emf ε = 80 sin (15000) volt. Determine

(a) the reactances of the capacitor and the inductor and the impedance of the circuit.

(b) the current amplitude.

(c) resonant frequency of the circuit

$R = 50\Omega$, $L = 30$ mH तथा $C = 40$ μF युक्त कोई श्रेणी ∠CR परिपथ एक प्रत्यावर्ती धारा स्रोत द्वारा चलित है। इस स्रोत का विद्युत वाहक बल $\varepsilon = 80 \sin (15000)$ वोल्ट है। निर्धारित कीजिए

(a) संधारित्र और प्रेरक के प्रतिघात तथा परिपथ की प्रतिबाधा

(b) धारा आयाम

(c) परिपथ की अनुनादित आवृत्ति

Or (अथवा)

An electric dipole when held at 30^0 with respect to a uniform electric field of strength 3×10^4 NC^{-1} experiences a torque of 27×10^{-26} N-m. Calculate the dipole moment of the dipole.

कोई विद्युत द्विध्रुव 3×10^4 NC^{-1} परिमाण के एकसमान विद्युत क्षेत्र में 30^0 के कोण पर रखे जाने पर 27×10^{-26} N-m परिमाण के बल आघूर्ण का अनुभव करता है। द्विध्रुव के द्विध्रुव आघूर्ण को परिकलित कीजिए।

5. Collect details about refracting and reflecting telescopes and find out which, as on date, is (i) the largest refracting telescope, and (ii) the largest reflecting astronomical telescope of the world.

The reflecting telescope made by Newton, called Newtonian reflector, is of Cassegrain type in which one mirror (objective) is concave and the other convex. There is another kind of reflecting telescope, called Gregorian type, in which both the mirrors are concave. Make a ray diagram of such a telescope.

You may also have seen a binocular which is mainly used for the purpose of bird watching. However, you may also have seen people using it for viewing flying aeroplanes on the occasion of Republic Day. Find out how a binocular is different from an astronomical telescope.

परियोजना कार्य

अपवर्ती और परावर्ती दूरदर्शकों के बारे में जानकारी इकट्ठा कीजिए तथा पता लगाइए कि इस समय विश्व का (i) सबसे बड़ा अपवर्ती दूरदर्शक, (ii) सबसे बड़ा परवर्ती खगोलीय दूरदर्शक कौन सा है।

न्यूटन द्वारा निर्मित परावर्ती दूरदर्शक, जिसे न्यूटनी परावर्तक कहते हैं, कैसेग्रेन प्रकार का है जिसमें एक दर्पण (अभिदृश्यक) अवतल तथा दूसरा उत्तल है। एक अन्य प्रकार का परावर्ती दूरदर्शक भी है, जिसे ग्रेगोरी दूरदर्शक कहते हैं। इसमें दोनों ही दर्पण अवतल हैं। इस दूरदर्शक का किरण आरेख बनाइए।

आपने द्विनेत्री (बाइनाक्यूलर) भी देखा हागा जिसका उपयोग मुख्य रूप से पक्षिओं को निहारने के लिए किया जाता है। किंतु गणतंत्र दिवस के अवसर पर इसकी सहायता से लोगों को उड़ते हवाई जहाजों का अवलोकन करते भी आपने देखा होगा। पता लगाइए कि बाइनोक्युलर खगोलीय दूरदर्शक से किस प्रकार भिन्न है।

Physics
भौतिकी
(312)
Assignment - III
मूल्यांकन पत्र - III
(Lessons 24-31)
(पाठ 24 से 31 तक)

Max. Marks: 25

कुल अंक : 25

Note: (i) All questions are compulsory. Each question carries equal marks.

टिप्पणी: सभी प्रश्नों का उत्तर देना अनिवार्य है। प्रत्येक प्रश्न के अंक समान हैं।

(ii) Write your name, enrolment number, AI name and subject etc. on the top of the first page of the answer sheet.

उत्तर पुस्तिका के प्रथम पृष्ठ पर ऊपर की ओर अपना नाम, अनुक्रमांक, अध्ययन केन्द्र का नाम, विषय आदि स्पष्ट शब्दों में लिखिए।

1. Answer any **two** of the following questions in about 60-80 words.

निम्नलिखित प्रश्नों में से किन्हीं **दो** के उत्तर लिखिए। प्रत्येक उत्तर 60-80 शब्दों में होना चाहिए।

(a) Consider motion of the single electron of the hydrogen atom in the nth orbit around the nucleus. Using Bohr's postulate show that the length of the circular orbit (i.e. circumference) is equal to the de-Broglie wavelength associated with the orbital electron.

हाइड्रोजन परमाणु की nवीं कक्षा में एक इलेक्ट्रॉन नाभिक के इर्द–गिर्द एक वृत्ताकार कक्षा में घूम रहा है। बोहर की अभिधारणाओं का प्रयोग करते हुए दर्शाइए कि इस वृत्ताकार कक्षा की परिधि इलेक्ट्रॉन की द'ब्राग्ली तरंगदैर्ध्य के बराबर होगी।

(b) The total energy of the single electron in hydrogen atom is given by $E_n = -\frac{13.6}{n^2} eV$. Select an appropriate scale to draw first five energy levels (n ranging between 1 and 5) and show emission of all possible spectral lines.

हाइड्रोजन परमाणु में एक इलेक्ट्रान की ऊर्जा निम्न समीकरण द्वारा दी जाती है

$$E_n = -\frac{13.6}{n^2} eV$$

एक सुनिश्चित पैमाने का प्रयोग करते हुए पहले पाँच (यानि n = 1 से 5) ऊर्जा स्तरों को बनाइए तथा सभी उत्सर्जित स्पेक्ट्रमी रेखाओं को दर्शाइए।

(c) Using Bohr's postulates, obtain a relation between r_n, the radius of nth orbit of the H-atom, and n. From this relation we get the radius of the orbital path of electron of H-atom in ground state (n = 1) to be $r_1 = 0.53$ A^0. Calculate the radii of next two orbits in terms of r_n.

बोहर की अभिधारणाओं का प्रयोग करते हुए, nवें वृत्ताकार कक्षा की त्रिज्या और n के बीच संबंध ज्ञात कीजिए। इस संबंध से हमें इलेक्ट्रान के निम्नतम स्तर (n = 1) के लिए त्रिज्या $r_n = 0.53$ A^0 मिलती है। अगली दो कक्षाओं की त्रिज्या r_1 के पदों में निकालिए।

2. Answer any **two** of the following questions in about 60-80 words.
निम्नलिखित प्रश्नों में से किन्हीं **दो** के उत्तर लिखिए। प्रत्येक उत्तर 60-80 शब्दों में होना चाहिए।

(a) Define unified atomic mass unit, u. (i) Express mass of an electron, m_e, in terms of u given that $m_e = 9.1 \times 10^{-31}$ kg. (ii) Mass of a neutron is $m_n = 1.008665$ u. Express it in kg.
एकीकृत परमाणु द्रव्यमान (u) को परिभाषित कीजिए। (i) एक इलेक्ट्रान का द्रव्यमान (m_e), एकीकृत परमाणु द्रव्यमान के पदों में निकालिए। दिया है $m_e = 9.1 \times 10^{-31}$ kg.
(ii) न्यूट्रान का द्रव्यमान $m_n = 1.008665$ u है। इसके द्रव्यमान का परिकलन किलोग्राम में कीजिए।

(b) Some nuclei such as ${}^{4}_{2}He$, ${}^{12}_{6}C$, ${}^{16}_{8}O$ have relatively higher B.E./nucleon. Keeping in mind their high B.E./nucleon, plot the variation of B.E./nucleon with mass number A for $A \leq 20$.
कुछ परमाणुओं जैसे ${}^{4}_{2}He$, ${}^{12}_{6}C$, ${}^{16}_{8}O$ की बंधन ऊर्जा प्रति न्यूक्लिऑन आस पास के अन्य परमाणुओं से अधिक है। इस बात को ध्यान में रखते हुए बंधन ऊर्जा प्रति न्यूक्लिऑन का द्रव्यमान संख्या ($A \leq 20$) के साथ परिवर्तन का एक ग्राफ बनाइए।

(c) Half life of a certain radioactive nuclei is 20 days. What % of it will decay in

(i) 20 days?

(ii) 40 days?

(iii) 10 days?

एक रेडियोधर्मी नाभिक की अर्धआयु 20 दिन है। इस नाभिक का कितने प्रतिशत क्षय हो जाएगा।
(i) 20 दिनों में?
(ii) 40 दिनों में?
(iii) 10 दिनों में?

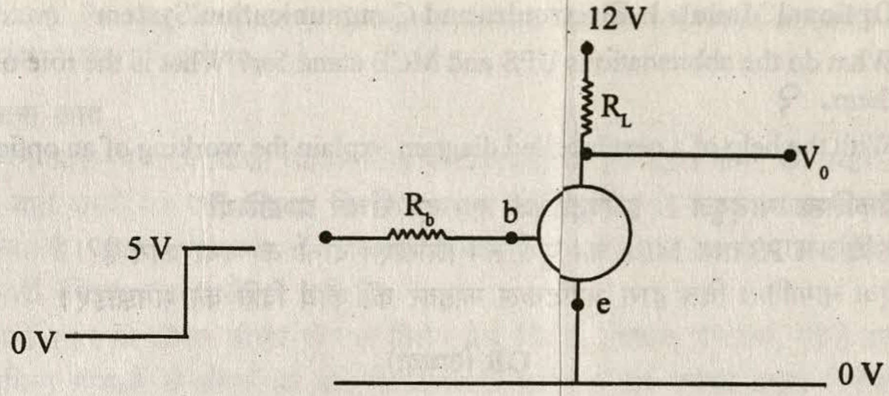
3. (a) How is the working of a transistor as a switch different from its working as an amplifier?

एक स्विच के रूप में ट्रांजिस्टर की कार्य पद्धति किस प्रकार एक आवर्धक की कार्यपद्धति से भिन्न है।

(b) With the help of a labelled circuit diagram, explain the working of a full-wave rectifier.

एक नामांकित परिपथ आरेख द्वारा पूर्ण–तरंग दिष्टकारी के कार्य को समझाइए।

(c) In the circuit diagram of transistor as a switch shown here identify whether the transistor used is a PNP transistor or an NPN transistor. Give justification for your answer.
नीचे दिए गए परिपथ आरेख में एक ट्रांजिस्टर स्विच की तरह कार्य कर रहा है। बताइए कि ट्रांजिस्टर PNP है या NPN? अपने उत्तर के लिए समुचित कारण बताइए।



OR (अथवा)

(a) For the same input ac rms voltage, how do the output voltages of a half-wave rectifer and a full-wave rectifier compare?

एक समान प्रत्यावर्ती वोल्टेज के वर्ग माध्यमूल मान (rms) के लिए पूर्ण–तरंग दिष्टकारी और अर्ध–तरंग दिष्टकारी के निर्गत वोल्टेजों की तुलना कीजिए।

(b) Figure shows two ideal diodes D_1 and D_2.

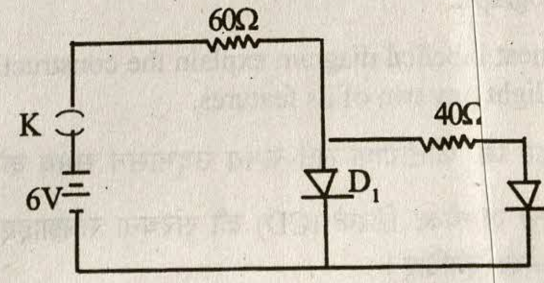
(i) When the key K is closed, how much current will be drawn from the 6 V battery?

(ii) Which of the two diodes must be reversed so that the current drawn from the battery is 0.06 ampere?

नीचे दिए गए परिपथ आरेख में दो आदर्श डॉयोड D_1 और D_2 हैं।

(i) कुंजी K को बंद करने पर 6V की बैटरी से कितनी धारा प्रवाहित होगी?

(ii) परिपथ में किस डायोड को उल्टा करने पर बैटरी से ली गई धारा 0.06 एम्पीयर हो जाएगी?



(c) With the help of a labelled circuit diagram, explain the working of a Zener diode as a voltage regulator.

एक नामांकित परिपथ आरेख द्वारा जेनर डायोड की वोल्टेज नियंत्रक के रूप में कार्यविधि समझाइए।

4. **[A] Optional Module I: Electronics and Communication System**

(a) What do the abbreviations UPS and MCB stand for? What is the role of each of them. ?

(b) With the help of a neat labelled diagram explain the working of an optical fibre.

[A] वैकल्पिक मॉड्यूल I: इलेक्ट्रॉनिकी एवं दूर संचार प्रणालियाँ

(a) संक्षिप्त UPS तथा MCB का पूर्ण रूप लिखिए। दोनों का क्या कार्य है?

(b) एक नामांकित चित्र द्वारा आप्टिकल फाइवर की कार्य विधि को समझाइए।

OR (अथवा)

(a) What does LCD stand for? How does it function?

(b) What is the necessity of modulation of signals for the purpose of their transmission from one place to another place. Give any two reasons. What do you understand by AM and FM?

(a) LCD का पूर्ण रूप लिखें। यह कैसे कार्य करता है?

(b) एक स्थान से दूसरे स्थान तक सूचना प्रसारण के लिए माडुलन की आवश्यकता क्यों है, कोई दो कारण बताइए। AM और FM से आप क्या समझते हैं?

[B] Optional Module II: Photography and Audio Videograpy

(a) Distinguish between a 'Normal lens' and a 'Wide angle lens' of a camera.

(b) Explain the construction and working of a magnetic tape used for audio recording.

[B] वैकल्पिक मॉड्यूल II: फोटोग्राफी एवं ऑडियो-वीडियोग्राफी

(a) किसी कैमरा के 'सामान्य लैंस' तथा 'वृहत् कोणीय लैंस' में अन्तर स्पष्ट कीजिए।

(b) ध्वनि रिकार्ड करने के लिए प्रयोग होने वाले चुम्बकीय टेप की संरचना एवं कार्यपद्धति को समझाइए।

Or (अथवा)

(a) List any four factors which affect the duration of exposure of the camera film while taking a photograph.

(b) With the help of a neat labelled diagram explain the construction of a Compact Disc (CD) and highlight any two of its features.

(a) कोई चार कारक बताइए जो फोटोग्राफ लेते समय उद्भासन समय को प्रभावित करते हैं।

(b) एक नामांकित चित्र द्वारा काम्पैक्ट डिस्क (CD) की संरचना समझाइए और इसकी किन्हीं दो विशेषताओं का उल्लेख कीजिए।

5. **Project Work**

(a) Talk to a professional photographer in your area who is now using a digital camera for commercial purpose. Talk to him to find out how technology has affected him commercially. With his help, list the features of a digital camera which are not available with the conventional cameras employing a film roll.

(b) Visit a camera dealer in your town/ city and collect pamphlets/brochures containing information about various models of cameras of a particular brand (such as Canon,

Nikon, Pentax, Sony etc.). Compare salient features of these cameras with the price range of each model.

परियोजना कार्य

(a) अपने आस–पास के किसी व्यावसायिक फोटोग्राफर, जो डिजिटल कैमरे का प्रयोग करता है, से बात करके यह पता लगाइए कि किस प्रकार नई तकनीक ने उसके व्यवसाय को प्रभावित किया है। उसकी सहायता से डिजिटल कैमरा के उन अवयवों का पता लगाइए जो पुराने कैमरों (फिल्म रोल वाले) में नहीं है।

(b) अपने शहर के कैमरा डीलर से एक विशेष ब्रैंड (कैनन, निकान, पेन्टेक्स, सोनी इत्यादि) के विभिन्न मॉडलों के कैमरों के बारे में जानकारी एकत्रित कर उनकी प्रमुख विशेषताओं की उनके मूल्यों के अनुसार तुलना कीजिए।

Complete and Post the feedback form today

Final fold and seal

Feed back on Lessons

Lesson No.	Lesson Name	Content			Language		Illustrations		What You Have Learnt	
		Difficult	Interesting	Confusing	Simple	Complex	Useful	Not useful	Very helpful	Not helpful
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2.										
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---Third fold----

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Thank you
Coordinator
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Feed back on Questions

Lesson No.	Lesson Name	Intext Questions		Terminal Questions		
		Useful	Not useful	Easy	Diff.	V. diff.
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Second Fold.

Yours suggestions

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Did you consult any other book to study Physics Yes/No

If Yes, give reason for consulting it

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Deputy Director (Academic)
National Institute of Open Schooling
A-24/25, Institutional Area,
Sector-62, NOIDA (UP)

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FUNDAMENTAL DUTIES

Part IVA (Article 51 A)

It shall be the duty of every citizen of India-

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forest, lake, rivers and wild life, and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that nation constantly rises to higher levels of endeavour and achievement.

A brief Guide to NIOS web site

The success of open learning and distance education very much depends upon the harnessing of the new and latest technology. The emerging Internet and Web technology help in effective dissemination of knowledge breaking all geographical boundaries. The web-site is a dynamic source of latest information and is also electronic information guide. The contents in the NIOS web site are open to all.

The learners can have an access to NIOS web-site at the following address:

http:/www.nos.org & nios.ac.in

Clicking the site address will bring the user to NIOS home page that will further guide them to visit different information pages of NIOS. NIOS is also developing a school network through Internet known as Indian Open Schooling Network (IOSN). The network will provide a common communication platform for learners and educators. NIOS is offering Certificate in Computer Applications (CCA) through selected AVI. This course is also offered through Internet on NIOS Web-Site.

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